Physics 152B **Spring 2024**

Problem Set 5

<u>Problem 1</u>

Consider a superconducting cylinder of radius R=3cm and height h=15cm and carrier density 5×10^{22} electrons/cm^3 in the absence of applied magnetic field, initially at rest. Then, it is put into rotation around its symmetry axis with angular frequency ω.

(a) Show that a magnetic field B develops in its interior, find an expression for it.
<u>Hint:</u> follow the derivation given in lecture.

(b) Find an expression for the current density near the surface that gives rise to this magnetic field, assuming that the current circulates only in a layer of thickness λ_L next to the surface, with λ_L <<R. You may also assume h>>R.
<u>Hint:</u> use Ampere's law.

(c) What is the magnitude of the magnetic field in Gauss for ω=10,000 s^{-1} ?

(d) Find approximately the total current I (in Amps) that circulates when the angular velocity ω is 10,000 s^{-1}.

<u>Problem 2</u>

Consider a lattice of hydrogen-like ions of nuclear charge Ze. Each ion can have 0, 1 or 2 electrons of opposite spin in its 1s orbital. Assume that the wavefunction for 2 electrons in the 1s orbital is the one found in HW Set 1 problem 5 part (c). Assume that the hopping amplitude for an electron to hop to a nearest neighbor ion is (-t) as usual.

(a) Using the same reasoning as for Hw Set 4 problem 4 parts (a) and (b), calculate the effective hopping amplitude for an electron of spin σ at site i to hop to a nearest neighbor site j when there is another electron of spin –σ at site i and no other electron at site j.

(b) Same as (a) assuming there is another electron of spin –σ at site j and no other electron at site i.

(c) Same as (a) assuming there are electrons of spin –σ both at sites i and j.

(d) Consider a 2-site system. Find the effective Coulomb interaction U_{eff} between two electrons.
<u>Hint:</u> remember HW 2 problem 2(b) and HW 3 Problem 5.
Can U_{eff} be attractive? i.e. negative? Justify your answer.

(e) Same as (d) for two holes instead of two electrons.

Problem 3
Consider the BCS wavefunction for a superconductor for which the density of electronic states per spin per unit volume is given by the simple form

$$g(\varepsilon) = \frac{N_{atoms}}{V}\frac{1}{D} \qquad -\frac{D}{2} \le \varepsilon \le \frac{D}{2}$$

where D is the bandwidth, V is the volume, and N_{atoms} is the number of atoms. Assume the band is half-full
(a) From the BCS wavefunction, derive expressions for the average number of electrons in the system $< N >$ and for the standard deviation $\sigma = \sqrt{< (N - < N >)^2}$ (see lecture notes).
(b) Calculate approximately the ratio $\sigma / < N >$. How does it depend on N_{atoms} and on the energy gap Δ?

Problem 4
Consider a half-filled (1 electron per site) generalized Hubbard model described by the Hamiltonian (<ij> means nearest neighbor sites)

$$H = -t\sum_{\substack{<ij>\\ \sigma}}(c^{+}_{i\sigma}c_{j\sigma} + h.c.) + U\sum_{i} n_{i\uparrow}n_{i\downarrow} + J\sum_{\substack{<ij>\\ \sigma\sigma'}} c^{+}_{i\sigma}c^{+}_{j\sigma'}c_{i\sigma'}c_{j\sigma}$$

(a) Assume there are only 2 sites in the system. Extend the calculation of HW3 Problem 2 to include the interaction J (third term in above Hamiltonian) for the case when there are two electrons in the system. Find the difference in energy between ferromagnetic and antiferromagnetic states.
(b) Find the condition that J has to satisfy for the lowest state to be ferromagnetic for given U and t.
(c) Find a simplified form for that condition valid in the limit of large U/t.
(d) Show that for large U/t the effective Hamiltonian describing the low energy states of the system of N sites is a Heisenberg Hamiltonian:

$$H = -J_H \sum_{<ij>} \vec{S}_i \cdot \vec{S}_j + \mathrm{C}$$

, with $\vec{S} = \frac{1}{2}\vec{\sigma}$, with $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the 2x2 Pauli matrices. Find J_H in terms of t, U and J.
(e) Show that the condition you found in part (c) for the 2-site system to be ferromagnetic for large U/t is consistent with the result of (d).