**PHYSICS 140A : STATISTICAL PHYSICS**
**HW ASSIGNMENT #8**

**(1)** Thanksgiving turkey typically cooks at a temperature of $350^\circ\,\mathrm{F}$. Calculate the total electromagnetic energy inside an over of volume $V = 1.0\,\mathrm{m}^3$ at this temperature. Compare it to the thermal energy of the air in the oven at the same temperature.

**(2)** In §5.4.4 of the lecture notes we derived the spectral energy density $\rho_\varepsilon(\nu, T)$ for a three-dimensional blackbody. We found that it was peaked at a frequency $\nu^* = s^* k_\mathrm{B} T/h$ where $s^* = 2.83144$ extremizes the function $s^3/(e^s - 1)$. Consider instead the function $\tilde{\rho}_\varepsilon(\lambda, T)$ as a function of wavelength $\lambda$ and temperature $T$, where $\lambda = c/\nu$. To relate $\rho_\varepsilon(\nu, T)$ and $\tilde{\rho}_\varepsilon(\lambda, T)$, set the fraction of energy of EM radiation between frequencies $\nu$ and $\nu + d\nu$ equal to the fraction of energy between wavelengths $\lambda$ and $\lambda + d\lambda$. Show that this is maximized at a wavelength $\lambda^* = t^* hc/k_\mathrm{B} T$, where $t^*$ is a constant. Find $t^*$ numerically. Is $t^* = 1/s^*$? Why or why not?

**(3)** A three-dimensional gas of particles obeys the dispersion relation $\varepsilon(\boldsymbol{k}) = A\,|\boldsymbol{k}|^{7/4}$. The internal degeneracy is $\mathrm{g} = 1$.

(a) Compute the single particle density of states $g(\varepsilon)$.

(b) For photon statistics, compute the pressure $p(n)$.

(c) For photon statistics, compute the entropy density $s(n) = S/V$.

(d) For Bose-Einstein statistics, compute the condensation temperature $T_\mathsf{BEC}(n)$.

**(4)** A branch of excitations for a three-dimensional system has a dispersion $\varepsilon(\boldsymbol{k}) = A\,|\boldsymbol{k}|^{2/3}$. The excitations are bosonic and are not conserved; they therefore obey photon statistics.

(a) Find the single excitation density of states per unit volume, $g(\varepsilon)$. You may assume that there is no internal degeneracy for this excitation branch.

(b) Find the heat capacity $C_V(T, V)$.

(c) Find the ratio $E/pV$.

(d) If the particles are bosons with number conservation, find the critical temperature $T_\mathrm{c}$ for Bose-Einstein condensation.