PHYSICS 140A : STATISTICAL PHYSICS
HW ASSIGNMENT #7

(1) Recall how we derived the GCE probability distribution based on the maximization of the entropy S under the constraint of fixed average energy E and particle number N.

(a) Show that one obtains the Bose-Einstein distribution $n_\alpha = \big[e^{\beta(\varepsilon_\alpha - \mu)} - 1\big]^{-1}$ if one extremizes the entropy function

$$S = -k_{\rm B} \sum_\alpha \big[n_\alpha \ln n_\alpha - (1+n_\alpha)\ln(1+n_\alpha)\big]$$

subject to fixed average E and N.

(b) Show that one obtains the Fermi-Dirac distribution $n_\alpha = \big[e^{\beta(\varepsilon_\alpha - \mu)} + 1\big]^{-1}$ if one extremizes the entropy function

$$S = -k_{\rm B} \sum_\alpha \big[n_\alpha \ln n_\alpha + (1-n_\alpha)\ln(1-n_\alpha)\big]$$

subject to fixed average E and N.

(2) For a noninteracting quantum system with single particle density of states $g(\varepsilon) = A\,\varepsilon^r$ (with $\varepsilon \ge 0$), find the first three virial coefficients for bosons and for fermions.

(3) Consider a system with single particle density of states $g(\varepsilon) = A\,\varepsilon\,\Theta(\varepsilon)\,\Theta(W-\varepsilon)$, which is linear on the interval $[0,W]$ and vanishes outside this interval. Find the second virial coefficient for both bosons and fermions. Plot your results as a function of dimensionless temperature $t = k_{\rm B}T/W$.

(4) Using the argument we used in class and in §5.4.3 of the notes, predict the surface temperatures of the remaining planets in our solar system. In each case, compare your answers with the most reliable source you can find. In cases where there are discrepancies, try to come up with a convincing excuse.