

PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #5

(1) DC Comics superhero Clusterman and his naughty dog Henry are shown in fig. 1. Clusterman, as his name connotes, is a connected diagram, but the diagram for Henry contains some disconnected pieces.

(a) Interpreting the diagrams as arising from the Mayer cluster expansion, compute the symmetry factor s_γ for Clusterman.

(b) What is the *total* symmetry factor for Henry and his disconnected pieces? What would the answer be if, unfortunately, another disconnected piece of the same composition were to be found?

(c) What is the lowest order virial coefficient to which Clusterman contributes?

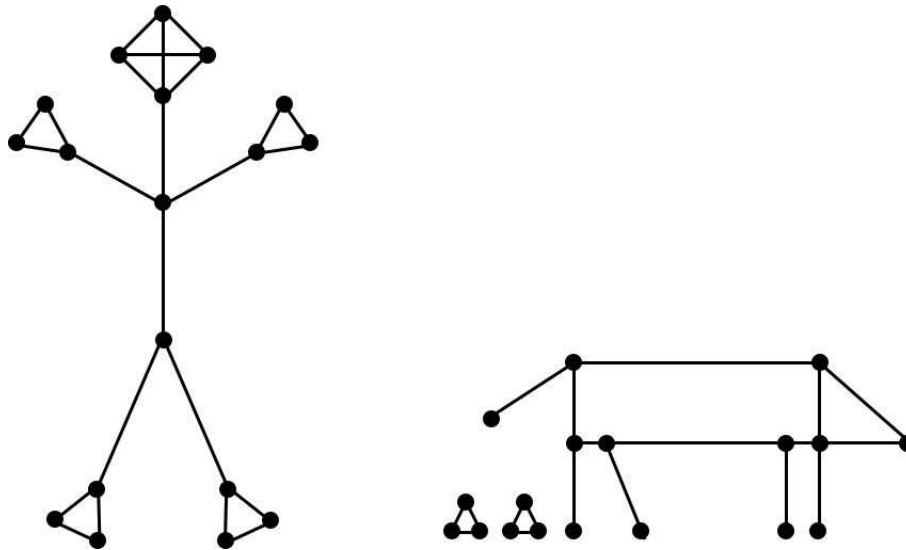


Figure 1: Mayer expansion diagrams for Clusterman and his dog Henry.

(2) Find an expression for the screened potential of a test charge Q in a two-dimensional system using an appropriate generalization of Debye-Hückel theory. The unscreened interparticle potential is $v(\mathbf{r}, \mathbf{r}') = -2qq' \ln(|\mathbf{r} - \mathbf{r}'|/a)$, where a is a constant. Assume two species of charge, with $q = \pm e$, for the plasma. Show that at asymptotically large distances the test charged is perfectly screened.

(3) Consider the equation of state

$$p\sqrt{v^2 - b^2} = RT \exp\left(-\frac{a}{RTv^2}\right).$$

(a) Find the critical point (v_c, T_c, p_c) .

(b) Defining $\bar{p} = p/p_c$, $\bar{v} = v/v_c$, and $\bar{T} = T/T_c$, write the equation of state in dimensionless form $\bar{p} = \bar{p}(\bar{v}, \bar{T})$.

(c) Expanding $\bar{p} = 1 + \pi$, $\bar{v} = 1 + \epsilon$, and $\bar{T} = 1 + t$, find $\epsilon_{\text{liq}}(t)$ and $\epsilon_{\text{gas}}(t)$ for $-1 \ll t < 0$.