

PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #4

(1) Consider the one-dimensional Ising model with next-nearest neighbor interactions,

$$\hat{H} = -J \sum_n \sigma_n \sigma_{n+1} - K \sum_n \sigma_n \sigma_{n+2},$$

on a ring with N sites, where N is even. By considering consecutive pairs of sites, show that the partition function may be written in the form $Z = \text{Tr}(R^{N/2})$, where R is a 4×4 transfer matrix. Find R . *Hint:* It may be useful to think of the system as a railroad trestle, depicted in fig. 1, with Hamiltonian

$$\hat{H} = - \sum_j \left[J \sigma_j \mu_j + J \mu_j \sigma_{j+1} + K \sigma_j \sigma_{j+1} + K \mu_j \mu_{j+1} \right].$$

Then $R = R_{(\sigma_j \mu_j), (\sigma_{j+1} \mu_{j+1})}$, with $(\sigma \mu)$ a composite index which takes one of four possible values $(++)$, $(+-)$, $(-+)$, or $(--)$.

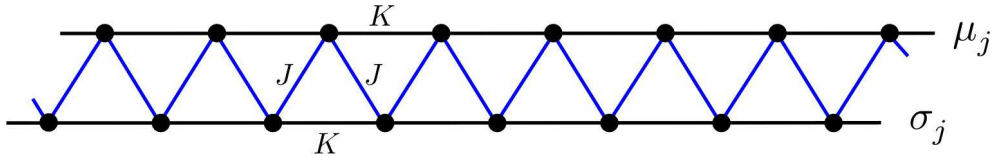


Figure 1: Railroad trestle representation of next-nearest neighbor chain.

(2) Compute the partition function for the one-dimensional Tonks gas of hard rods of length a on a ring of circumference L . This is slightly tricky, so here are some hints. Once again, assume a particular ordering so that $x_1 < x_2 < \dots < x_N$. Due to translational invariance, we can define the positions of particles $\{2, \dots, N\}$ relative to that of particle 1, which we initially place at $x_1 = 0$. Then periodicity means that $x_N \leq L - a$, and in general one then has $x_{j-1} + a \leq x_j \leq L - (N - j + 1)a$. Now integrate over $\{x_2, \dots, x_N\}$ subject to these constraints. Finally, one does the x_1 integral, which is over the entire ring, but which must be corrected to eliminate overcounting from cyclic permutations. How many cyclic permutations are there?

(3) For each of the cluster diagrams in Fig. 2, find the symmetry factor s_γ and write an expression for the cluster integral b_γ .

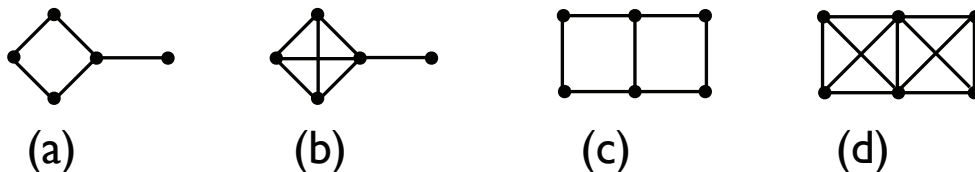


Figure 2: Cluster diagrams for problem 3.