PHYSICS 140B : STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) Turkey typically cooks at a temperature of $350^\circ\,\mathrm{F}$. Calculate the total electromagnetic energy inside an over of volume $V = 1.0\,\mathrm{m}^3$ at this temperature. Compare it to the thermal energy of the air in the oven at the same temperature.

(2) Let L denote the number of single particle energy levels and N the total number of particles for a given system. Find the number of possible N-particle states $\Omega(L,N)$ for each of the following situations:

(a) Distinguishable particles with $L = 3$ and $N = 3$.

(b) Bosons with $L = 3$ and $N = 3$.

(c) Fermions with $L = 10$ and $N = 3$.

(d) Find a general formula for $\Omega_{\mathrm{D}}(L,N)$, $\Omega_{\mathrm{BE}}(L,N)$, and $\Omega_{\mathrm{FD}}(L,N)$.

(3) A species of noninteracting quantum particles in $d = 2$ dimensions has dispersion $\varepsilon(\boldsymbol{k}) = \varepsilon_0 |\boldsymbol{k}\ell|^{3/2}$, where ε_0 is an energy scale and ℓ a length.

(a) Assuming the particles are $S = 0$ bosons obeying photon statistics, compute the heat capacity C_V.

(b) Assuming the particles are $S = 0$ bosons, is there an Bose condensation transition? If yes, compute the condensation temperature $T_{\mathrm{c}}(n)$ as a function of the particle density. If no, compute the low-temperature behavior of the chemical potential $\mu(n,T)$.

The following integral may be useful:

$$\int\limits_0^\infty \frac{u^{s-1}\,du}{e^u - 1} = \Gamma(s) \sum_{n=1}^\infty n^{-s} \equiv \Gamma(s)\,\zeta(s) \quad ,$$

where $\Gamma(s)$ is the gamma function and $\zeta(s)$ is the Riemann zeta-function.

(4) Hydrogen (H_2) freezes at $14\,\mathrm{K}$ and boils at $20\,\mathrm{K}$ under atmospheric pressure. The density of liquid hydrogen is $70\,\mathrm{kg/m}^3$. Hydrogen molecules are bosons. No evidence has been found for Bose-Einstein condensation of hydrogen. Why not?

(5) (Difficult) Consider a three-dimensional Bose gas of particles which have two internal polarization states, labeled by $\sigma = \pm 1$. The single particle energies are given by

$$\varepsilon(\boldsymbol{k},\sigma) = \frac{\hbar^2 \boldsymbol{k}^2}{2m} + \sigma\Delta \ ,$$

where $\Delta > 0$.

(a) Find the density of states per unit volume $g(\varepsilon)$.

(b) Find an implicit expression for the condensation temperature $T_{\mathrm{c}}(n, \Delta)$. When $\Delta \to \infty$, your expression should reduce to the familiar one derived in class.

(c) When $\Delta = \infty$, the condensation temperature should agree with the familiar result for three-dimensional Bose condensation. Assuming $\Delta \gg k_{\mathrm{B}} T_{\mathrm{c}}(n, \Delta = \infty)$, find analytically the leading order difference $T_{\mathrm{c}}(n, \Delta) - T_{\mathrm{c}}(n, \Delta = \infty)$.