1. $$\Delta y = y_{n+1} - y_n = \frac{\lambda D}{d} = \frac{(589.0\text{ nm})(2.604\text{ m})}{1.25\text{ mm}} = 1.23\text{ mm}$$

5. (a) $E = 10.0\ \text{MeV} = 1.60 \times 10^{-12}\ \text{J}$

$$p = \frac{E}{c} = \frac{10.0\ \text{MeV}}{c} = 1.00 \times 10^{7}\ \text{eV}/c$$

$$p = \frac{1.60 \times 10^{-12}\ \text{J}}{3.00 \times 10^{8}\ \text{m/s}} = 5.33 \times 10^{-21}\ \text{kg} \cdot \text{m/s}$$

(b) $E = 25\ \text{keV} = 4.0 \times 10^{-15}\ \text{J}$

$$p=\frac{E}{c}=\frac{25\text{ keV}}{c}=2.5\times10^{4}\text{ eV}/c$$

$$p=\frac{4.0\times10^{-15}\text{ J}}{3.00\times10^{8}\text{ m/s}}=1.3\times10^{-23}\text{ kg}\cdot\text{m/s}$$

(c) $\lambda=1.0\ \mu\text{m}=1.0\times10^{3}\text{ nm}$

$$p=\frac{h}{\lambda}=\frac{1}{c}\frac{hc}{\lambda}=\frac{1}{c}\frac{1240\text{ eV}\cdot\text{nm}}{1.0\times10^{3}\text{ nm}}=1.2\text{ eV}/c$$

$$p=\frac{6.6\times10^{-34}\text{ J}\cdot\text{s}}{1.0\times10^{-6}\text{ m}}=6.6\times10^{-28}\text{ kg}\cdot\text{m/s}$$

(d) $E=hf=(4.14\times10^{-15}\text{ eV}\cdot\text{s})(150\times10^{6}\text{ Hz})=6.2\times10^{-7}\text{ eV}=9.9\times10^{-26}\text{ J}$

$$p=\frac{E}{c}=\frac{6.2\times10^{-7}\text{ eV}}{c}=6.2\times10^{-7}\text{ eV}/c$$

$$p=\frac{9.9\times10^{-26}\text{ J}}{3.00\times10^{8}\text{ m/s}}=3.3\times10^{-34}\text{ kg}\cdot\text{m/s}$$

6. At $1\,\text{MHz} = 10^6$ Hz, $E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(10^6\text{ s}^{-1}) = 4\times10^{-9}\text{ eV}$

At $100\text{ MHz} = 10^8$ Hz, $E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(10^8\text{ s}^{-1}) = 4\times10^{-7}\text{ eV}$

The range is from 4×10^{-9} eV to 4×10^{-7} eV.

7. (a) $$\lambda = \frac{hc}{E} = \frac{1240\text{ eV}\cdot\text{nm}}{1.00\times10^4\text{ eV}} = 0.124\text{ nm}$$

(b) $$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{1.00\times10^6\text{ eV}} = 1.24\times10^{-3}\text{ nm}$$

(c) 350 nm: $$E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{350\text{ nm}} = 3.5\text{ eV}$$

700 nm: $$E = \frac{1240\text{ eV}\cdot\text{nm}}{700\text{ nm}} = 1.8\text{ eV}$$

The range is from 1.8 eV to 3.5 eV.

9.

$$\phi = \frac{hc}{\lambda_c} = \frac{1239.853\ \text{eV}\cdot\text{nm}}{352.8\ \text{nm}} = 3.514\ \text{eV}$$

$$eV_s = \frac{hc}{\lambda} - \phi = \frac{1239.853\ \text{eV}\cdot\text{nm}}{304.2\ \text{nm}} - 3.514\ \text{eV} = 0.561\ \text{eV}$$

$$V_s = 0.561\ \text{V}$$

11. (a) $$\phi = \frac{hc}{\lambda_c} = \frac{1240\text{ eV}\cdot\text{nm}}{254\text{ nm}} = 4.88\text{ eV}$$

(b) $\lambda < 254\text{ nm}$

12. (a) With $\phi = 4.31\text{ eV}$, $\lambda_c = \frac{hc}{\phi} = \frac{1240\text{ eV}\cdot\text{nm}}{4.31\text{ eV}} = 288\text{ nm}$

(b) $eV_s = \frac{hc}{\lambda} - \phi = \frac{1240\text{ eV}\cdot\text{nm}}{252.0\text{ nm}} - 4.31\text{ eV} = 0.61\text{ eV}$, so $V_s = 0.61\text{ volts}$

13. (a) The total number of oscillators is

$$\int_0^\infty n(E)\,dE = \int_0^\infty \frac{N}{kT} e^{-E/kT}\,dE = \frac{N}{kT}(-kT)e^{-E/kT}\Big|_0^\infty = N$$

(b) The average energy is (from Equation 3.31)

$$E_{\text{av}} = \frac{1}{N}\int_0^\infty E\,n(E)\,dE = \frac{1}{kT}\int_0^\infty E\,e^{-E/kT}\,dE = kT\int_0^\infty xe^{-x}\,dx \qquad \text{with } x = E/kT$$

The definite integral is a standard form that is equal to 1, so $E_{\text{av}} = kT$.

14. (a) The total number of oscillators at all energies is

$$\sum_{i=0}^{\infty} N_n = \sum_{n=0}^{\infty} A e^{-E_n/kT} = A\sum_{n=0}^{\infty} e^{-n\varepsilon/kT} = A\frac{1}{1-e^{-\varepsilon/kT}}$$

Setting this result equal to *N*, we obtain $A = N(1-e^{-\varepsilon/kT})$.

(b) On the left side $\frac{d}{dx}\sum_{n=0}^{\infty} e^{nx} = \sum_{n=0}^{\infty} ne^{nx}$ and on the right side $\frac{d}{dx}\frac{1}{1-e^x} = \frac{e^x}{(1-e^x)^2}$. Setting these equal to each other gives $\sum_{n=0}^{\infty} ne^{nx} = \frac{e^x}{(1-e^x)^2}$.

(c) $E_{\text{av}} = \frac{1}{N}\sum_{n=0}^{\infty} N_n E_n = (1-e^{-\varepsilon/kT})\sum_{n=0}^{\infty}(n\varepsilon)e^{-n\varepsilon/kT} = (1-e^{-\varepsilon/kT})\varepsilon\frac{e^{-\varepsilon/kT}}{(1-e^{-\varepsilon/kT})^2} = \frac{\varepsilon}{e^{\varepsilon/kT}-1}$

(d) For large λ, $e^{hc/\lambda kT} \approx 1 + hc/\lambda kT$ and thus $E_{\text{av}} = \frac{hc/\lambda}{e^{hc/\lambda kT}-1} \approx \frac{hc/\lambda}{1+hc/\lambda kT - 1} = kT$

As λ goes to 0, $e^{hc/\lambda kT} \to \infty$ and $E_{\text{av}} \to 0$.

15. $$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

$$\frac{dI}{d\lambda} = 2\pi hc^2\left[\left(\frac{-5}{\lambda^6}\right)\frac{1}{e^{hc/\lambda kT}-1} + \left(\frac{1}{\lambda^5}\right)\frac{(-e^{hc/\lambda kT})(-hc/\lambda^2 kT)}{(e^{hc/\lambda kT}-1)^2}\right]$$

Setting $dI/d\lambda$ equal to zero gives

$$-\frac{5}{\lambda} + \frac{(e^{hc/\lambda kT})(hc/\lambda^2 kT)}{e^{hc/\lambda kT}-1} = 0$$

or, with $x = hc/\lambda kT$,

$$(x-5)e^x + 5 = 0$$

This equation does not have an exact solution, but an approximate solution can be found by trial and error: $x = 4.9651 = hc/\lambda kT$, so

$$\lambda T = \frac{hc}{4.9651k} = \frac{1239.853\,\text{eV}\cdot\text{nm}}{4.9651(8.6174\times 10^{-5}\ \text{eV/K})} = 2.8978\times 10^{-3}\ \text{m}\cdot\text{K}$$

16. $$\int_0^\infty I(\lambda)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}\,d\lambda$$

With $x = hc/\lambda kT$ and $dx = (-hc/\lambda^2 kT)\,d\lambda$,

$$\int_0^\infty I(\lambda)\,d\lambda = 2\pi hc^2\left(\frac{kT}{hc}\right)^3\left(-\frac{kT}{hc}\right)\int_0^\infty \frac{x^3 dx}{e^x-1}$$

$$= 2\pi hc^2\left(\frac{k}{hc}\right)^4 T^4\int_0^\infty \frac{x^3 dx}{e^x-1} = \frac{2\pi k^4}{h^3c^2}T^4\,\frac{\pi^4}{15} = \sigma T^4$$

with $\sigma = 2\pi^5 k^4/15h^3c^2$

18. $$\lambda = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{6000\ \text{K}} = 483\ \text{nm}$$

This is in the middle of the visible spectrum, close to the peak sensitivity of the eye.

19. $$\lambda = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.7\ \text{K}} = 1.1\ \text{mm (microwave region)}$$

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{1.1\ \text{mm}} = 1.1\times10^{-3}\ \text{eV}$$

20. (a) $$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{307\ \text{K}} = 9.4\ \mu\text{m (infrared)}$$

(b) Assume a person can be represented as a cylinder, about 6 feet (1.83 m) tall and 1 foot (0.30 m) in diameter. The surface area is $2\pi rL = 2\pi(0.15\ \text{m})(1.83\ \text{m}) = 1.72\ \text{m}^2$.

$$I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(307\ \text{K})^4 = 504\ \text{W/m}^2$$

$$P = IA = (504\ \text{W/m}^2)(1.72\ \text{m}^2) = 870\ \text{W}$$

(c) For T = 20°C = 293 K,

$$I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(293\ \text{K})^4 = 418\ \text{W/m}^2$$

$$P = IA = (418\ \text{W/m}^2)(1.72\ \text{m}^2) = 719\ \text{W}$$

Thus the net power radiated by a person is about 150 W.

21. $I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(1750\ \text{K}) = 5.32 \times 10^5\ \text{W/m}^2$

$$P = IA = I(\pi r^2) = (5.32 \times 10^5\ \text{W/m}^2)\pi(0.62 \times 10^{-3}\ \text{m})^2 = 0.64\ \text{W}$$

23. (a) We consider an interval of width $d\lambda = 2.0$ nm at a central wavelength of 531.0 nm. At $T = 6000$ K, $kT = (8.6174\times10^{-5}\text{ eV/K})(6000\text{ K}) = 0.517\text{ eV}$. The intensity in this interval is

$$dI = I(\lambda)\,d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}d\lambda$$

$$= \frac{2\pi(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})^2(2.0\ \times10^{-9}\text{ m})}{(531.0\times10^{-9}\text{ m})^5(e^{(1240\text{eV}\cdot\text{nm})/(531.0\text{ nm})(0.517\text{ eV})}-1)} = 1.96\times10^5\text{ W/m}^2$$

(b) The total radiant intensity emitted by the Sun is

$$I = \sigma T^4 = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(6000\text{ K})^4 = 7.35\times10^7\text{ W/m}^2$$

The fraction is then

$$\frac{1.96\times10^5\text{ W/m}^2}{7.35\times10^7\text{ W/m}^2} = 0.0027 = 0.27\%$$

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(293\ \text{K})^4 = 418\ \text{W/m}^2$$

$$P = IA = (418\ \text{W/m}^2)(1.72\ \text{m}^2) = 719\ \text{W}$$