PHYSICS 200B : CLASSICAL MECHANICS
HOMEWORK SET #2

[1] Consider the standard map on the unit torus,

$$x_{n+1} = x_n + y_n \ \mathsf{mod}\ 1$$
$$y_{n+1} = y_n + \kappa \sin(2\pi x_{n+1}) \ \mathsf{mod}\ 1 \quad .$$

Find all the fixed points and identify their stability as a function of the control parameter κ.

[2] Write a computer program to iterate the map from problem [1]. For each value of κ you consider, iterate starting from N^2 initial conditions $(x_0, y_0) = (j/N, k/N)$, where j and k each run from 0 to $N-1$. You can take $N = 10$.

(a) By experimenting, see if you can find the value of κ where there are no unbroken KAM tori which span the x-direction $x \in [0,1]$.

(b) Next, consider the standard map on the cylinder,

$$x_{n+1} = x_n + y_n \ \mathsf{mod}\ 1$$
$$y_{n+1} = y_n + \kappa \sin(2\pi x_{n+1}) \quad ,$$

where the y variable now may take values on the entire real line. For each given κ, plot $\langle y_n^2 \rangle$ *versus* n, where the average is over the N^2 initial conditions. Assuming the evolution is diffusive in the chaotic regime, compute the diffusion constant $D(\kappa)$ from the formula $\langle y_n^2 \rangle = 2Dn$. Plot $D(\kappa)$ *versus* κ over the range $\kappa \in [1, 10]$. Compare to the value from the quasilinear approximation, $D_{\text{ql}} = \frac{1}{4}\kappa^2$.

[3] For the logistic map $x_{n+1} = f(x_n)$ with $f(x) = rx(1-x)$, plot the functions $f^{(n)}(x)$ for $n = 1$, 2, and 4 and plot the intersections of $y = f^{(n)}(x)$ with $y = x$. Show how varying the control parameter r results in bifurcations corresponding to the appearance of 2-cycles and 4-cycles.