# Algebra in class today

We had: $$V_{ijlk} = \int d^3r\, d^3r'\, \varphi_i^*(r)\, \varphi_j^*(r')\, \frac{e^2}{4\pi\varepsilon_0 |r-r'|}\, \varphi_k(r')\, \varphi_l(r)$$

$$H_{ee} = \sum V_{ijlk}\, C_{i\sigma}^{+}\, C_{j\sigma'}^{+}\, C_{k\sigma'}\, C_{l\sigma}$$ , going to $k$-space

$$H_{ee} = \frac{1}{N^2} \sum V_{ijlk}\, e^{i(k_1 R_i + k_2 R_j - k_3 R_k - k_4 R_l)}\, C_{k_1\sigma}^{+}\, C_{k_2\sigma'}^{+}\, C_{k_3\sigma'}\, C_{k_4\sigma}$$

We showed that from translational invariance: $k_3 + k_4 = k_1 + k_2$

so we renamed the $k$'s as: $k_4 = k$, $k_1 = k+q$, $k_3 = k'$, $k_2 = k'-q$ which satisfy

then:
$$e^{i(k_1 R_i + k_2 R_j - k_3 R_k - k_4 R_l)} = e^{i[(k+q)R_i + (k'-q)R_j - k' R_k - k R_l]} =$$
$$= e^{i[k(R_i - R_l) + k'(R_j - R_k) + q(R_i - R_j)]}$$

$$H_{ee} = \frac{1}{N^2} \sum V_{ijlk}\, e^{i[k(R_i - R_l) + k'(R_j - R_k) + q(R_i - R_j)]}\, C_{k+q\sigma}^{+}\, C_{k'-q\sigma'}^{+}\, C_{k'\sigma'}\, C_{k\sigma}$$

For $V_{ijij} = V_{ij}$ matrix element: $R_l = R_i$, $R_k = R_j \Rightarrow$ we get
$$\sum V_{ij}\, e^{iq(R_i - R_j)}\, c^+ c^+ c c = \sum V(q)\, c^+ c^+ c c$$

For $V_{iiij} \equiv (\Delta t)_{ij}$ : $R_j \to R_i$, $R_l \to R_i$, $R_k \to R_j \Rightarrow$ we get
$$\sum (\Delta t)_{ij}\, e^{ik'(R_i - R_j)}\, c^+ c^+ c c = \sum \Delta t(k)\, c^+ c^+ c c$$

similarly for the others like this, i.e. $V_{iiji}$, $V_{ijii}$, $V_{jiii}$, we get in total
$$\sum \left[\Delta t(k) + \Delta t(k') + \Delta t(k+q) + \Delta t(k'-q)\right] C_{k+q\sigma}^{+}\, C_{k'-q\sigma'}^{+}\, C_{k'\sigma'}\, C_{k\sigma}$$