19. $r_3 = 9a_0 = 9(0.0529\text{ nm}) = 0.476\text{ nm}$

$$v = \frac{n\hbar}{mr} = c\frac{n\hbar c}{mc^2 r} = c\frac{3(1240\,\text{eV}\cdot\text{nm})/2\pi}{(0.511\times 10^6\text{ eV})(0.476\,\text{nm})} = 2.43\times 10^{-3}c = 7.30\times 10^5\text{ m/s}$$

$$U = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{1.440\text{ eV}\cdot\text{nm}}{0.476\text{ nm}} = -3.02\text{ eV}$$

$$K = \frac{e^2}{8\pi\varepsilon_0}\frac{1}{r} = \frac{1.440\,\text{eV}\cdot\text{nm}}{2(0.476\text{ nm})} = 1.51\,\text{eV}$$

21. (a) From Equation 6.26, $v = \dfrac{n\hbar}{mr} = \dfrac{n\hbar}{mn^2 a_0}$. Using Equation 6.29 for a_0, we obtain

$$v = \frac{\hbar}{nm(4\pi\varepsilon_0\hbar^2/me^2)} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{n\hbar} = \frac{\alpha c}{n}$$

(b) When the nuclear charge is Ze, we must replace e^2 with Ze^2, so $v = Z\alpha c/n$.

24. The photon energy of the incident light is

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{53.0 \text{ nm}} = 23.4 \text{ eV}$$

When an atom in the ground state absorbs a 23.4-eV photon, the atom is ionized (which takes 13.6 eV). The excess energy, 23.4 eV – 13.6 eV = 9.8 eV, appears as the kinetic energy of the electron, which is now free of the atom. Neglecting a small recoil kinetic energy given to the proton, the electrons have a kinetic energy of 9.8 eV.

27. The Lyman series consists of transitions that end in the $n = 1$ level. The smallest energy difference, corresponding to the longest wavelength, is $n = 2$ to $n = 1$.

$$\Delta E = E_2 - E_1 = (-13.6\text{ eV})2^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 40.8\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{ eV}\cdot\text{nm}}{40.8\text{ eV}} = 30.4\text{ nm}$$

The largest energy difference would correspond to transitions from $n = \infty$ to $n = 1$:

$$\Delta E = E_\infty - E_1 = (-13.6\text{ eV})2^2\left(0 - \frac{1}{1^2}\right) = 54.4\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\text{ eV}\cdot\text{nm}}{54.4\text{ eV}} = 22.8\text{ nm}$$

28. Using Equation 6.38, we have $E_n = (-13.6\text{ eV})Z^2/n^2 = (-54.4\text{ eV})/n^2$,
so $E_1 = -54.40\text{ eV}$, $E_2 = -13.60\text{ eV}$, $E_3 = -6.04\text{ eV}$, $E_4 = -3.40\text{ eV}$. The possible transitions are:

$$4 \to 1: \quad \Delta E = E_4 - E_1 = 51.00\text{ eV} \qquad \lambda = hc/\Delta E = 24.31\text{ nm}$$

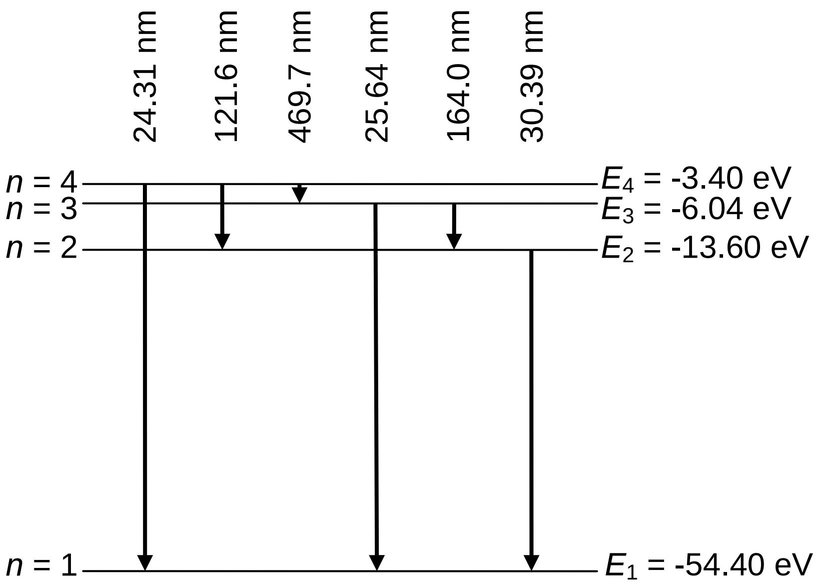
$$4 \to 2: \quad \Delta E = E_4 - E_2 = 10.20\text{ eV} \qquad \lambda = hc/\Delta E = 121.6\text{ nm}$$

$$4 \to 3: \quad \Delta E = E_4 - E_3 = 2.64\text{ eV} \qquad \lambda = hc/\Delta E = 469.7\text{ nm}$$

$$3 \to 1: \quad \Delta E = E_3 - E_1 = 48.36\text{ eV} \qquad \lambda = hc/\Delta E = 25.64\text{ nm}$$

$$3 \to 2: \quad \Delta E = E_3 - E_2 = 7.56\text{ eV} \qquad \lambda = hc/\Delta E = 164.0\text{ nm}$$

$$2 \to 1: \quad \Delta E = E_2 - E_1 = 40.80\text{ eV} \qquad \lambda = hc/\Delta E = 30.39\text{ nm}$$



30. (a) If the circumference is an integral number of de Broglie wavelengths ($2\pi r = n\lambda$), then after each orbit the waves will align, peak to peak and valley to valley, to give standing waves.

(b) $2\pi r = n\lambda = n\frac{h}{p} = \frac{nh}{mv}$ so $mvr = \frac{nh}{2\pi} = n\hbar$

35. (a) The frequency of revolution is given by Equation 6.41:

$$f_n = \frac{me^4}{32\pi^3\varepsilon_0^2\hbar^3}\frac{1}{n^3} = \frac{1}{\pi\hbar}\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^3} = \frac{13.6\text{ eV}}{\pi\hbar}\frac{1}{n^3} = \frac{6.58\times10^{15}\text{ Hz}}{n^3}$$

A similar calculation gives the radiation frequency from Equation 6.42:

$$f = \frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3}\frac{2n-1}{n^2(n-1)^2} = \frac{13.6\text{ eV}}{2\pi\hbar}\frac{2n-1}{n^2(n-1)^2} = (6.58\times10^{15}\text{ Hz})\frac{2n-1}{2n^2(n-1)^2}$$

For n = 10, we get $f_n = 6.58\times10^{12}$ Hz and $f = 7.72\times10^{12}$ Hz.

(b) For n = 100, $f_n = 6.58\times10^{9}$ Hz and $f = 6.68\times10^{9}$ Hz.

(c) For n = 1000, $f_n = 6.58\times10^{6}$ Hz and $f = 6.59\times10^{6}$ Hz.

(d) For n = 10,000, $f_n = 6.58\times10^{3}$ Hz and $f = 6.58\times10^{3}$ Hz. Note how f approaches f_n as n becomes large, in accordance with the correspondence principle.

36. The Rydberg constant in ordinary hydrogen is

$$R_{\text{H}} = R_\infty\left(1+\frac{m}{M_{\text{H}}}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\text{ u}}{1.007825\text{ u}}\right) = R_\infty(1.000544)$$

and in "heavy" hydrogen or deuterium:

$$R_{\mathrm{D}} = R_\infty\left(1+\frac{m}{M_{\mathrm{D}}}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\ \mathrm{u}}{2.104102\ \mathrm{u}}\right) = R_\infty(1.000272)$$

From Equation 6.33 the difference in wavelengths for the first line of the Balmer series (n = 3 to n = 2) is

$$\lambda_{\mathrm{D}} - \lambda_{\mathrm{H}} = \left(\frac{1}{R_{\mathrm{D}}} - \frac{1}{R_{\mathrm{H}}}\right)\left(\frac{3^2 2^2}{3^2 - 2^2}\right) = \frac{7.2}{1.09737\times10^7\ \mathrm{m}^{-1}}\left(\frac{1}{1.000272} - \frac{1}{1.000544}\right) = 0.178\ \mathrm{nm}$$