10.

$$\lambda = \frac{d\sin\phi}{2} = \frac{(0.215\text{ nm})(\sin 55°)}{2} = 0.0881\text{ nm}$$

$$pc = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{0.0881\text{ nm}} = 1.408\times 10^4\text{ eV}$$

$$K = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(1.408\times 10^4\text{ eV})^2}{2(0.511\times 10^6\text{ eV})} = 194\text{ eV}$$

To achieve this kinetic energy, the electrons must be accelerated through a potential difference of $\Delta V = +194$ V.

19. With $\Delta v = 2.8 \times 10^4$ m/s,

$$\Delta x \sim \frac{\hbar}{\Delta p} = \frac{\hbar}{m\,\Delta v} = \frac{1.05\times10^{-34}\ \mathrm{J\cdot s}}{(9.11\times10^{-31}\ \mathrm{kg})(2.8\times10^{4}\ \mathrm{m/s})} = 4.1\times10^{-9}\ \mathrm{m} = 5.8\ \mathrm{nm}$$

20. (a)

$$\Delta p \sim \frac{\hbar}{\Delta x} = \frac{h}{2\pi\,\Delta x} = \frac{1}{c}\frac{hc}{2\pi\,\Delta x} = \frac{1}{c}\frac{1240\ \mathrm{eV\cdot nm}}{2\pi(0.1\ \mathrm{nm})} = 2000\ \mathrm{eV}/c$$

(b)

$$K = \frac{(\Delta p)^2}{2m} = \frac{(c\,\Delta p)^2}{2mc^2} = \frac{(2000\ \mathrm{eV})^2}{2(0.511\times10^{6}\ \mathrm{eV})} = 4\ \mathrm{eV}$$

21.

$$\Delta E \sim \frac{\hbar}{\Delta t} = \frac{6.58\times10^{-16}\ \mathrm{eV\cdot s}}{2.0\times10^{-23}\ \mathrm{s}} = 33\ \mathrm{MeV}$$

Measurements of the Σ^+ rest energy are likely to fall in the range 1385 MeV ± 33 MeV, or from 1352 MeV to 1418 MeV.

22.

$$\Delta t \sim \frac{\hbar}{\Delta E} = \frac{6.58\times10^{-16}\ \text{eV}\cdot\text{s}}{120\times10^{6}\ \text{eV}} = 5.5\times10^{-24}\ \text{s}$$

25. As we did for electrons in Example 4.9, let's find the kinetic energy of an alpha particle with a momentum of 19.7 MeV/c:

$$K = \frac{p^2}{2m} = \frac{(pc)^2}{2mc^2} = \frac{(19.7\text{ MeV})^2}{2(3727\text{ MeV})} = 0.052\text{ MeV}$$

This is negligible compared with the typical kinetic energies of alpha particles emitted in radioactive decays. Therefore, the uncertainty principle does not limit the existence of these alpha particles inside the nucleus.

26. With Δx = 14 fm, we have

$$\Delta p_x = \frac{\hbar}{\Delta x} = \frac{1}{c}\frac{\hbar c}{\Delta x} = \frac{1}{c}\frac{197\text{ MeV}\cdot\text{fm}}{14\text{ fm}} = 14.1\text{ MeV}/c$$

With this uncertainty as an estimate for p_x,

$$K = \frac{p_x^2}{2m} = \frac{c^2 p_x^2}{2mc^2} = \frac{(14.1\text{ MeV})^2}{2(938\text{ MeV})} = 0.11\text{ MeV}$$

This is a very small contribution to the energy of protons or neutrons in a nucleus, which are typically 10-20 MeV.

27. $$y(x) = \int A(k)\cos kx\,dk = A_0 \int_{k_0-\Delta k/2}^{k_0+\Delta k/2} \cos kx\,dk = A_0 \left.\frac{\sin kx}{x}\right|_{k_0-\Delta k/2}^{k_0+\Delta k/2}$$

$$= \frac{A_0}{x}\left[\sin x(k_0 + \Delta k/2) - \sin x(k_0 - \Delta k/2)\right]$$

$$= \frac{A_0}{x}\left[\sin k_0 x \cos\frac{x\,\Delta k}{2} + \cos k_0 x \sin\frac{x\,\Delta k}{2} - \left(\sin k_0 x \cos\frac{x\,\Delta k}{2} - \cos k_0 x \sin\frac{x\,\Delta k}{2}\right)\right]$$

$$= \frac{2A_0}{x}\sin\left(\frac{\Delta k}{2}x\right)\cos k_0 x$$

29. $$y(x) = A\cos(2\pi x/\lambda_1) + A\cos(2\pi x/\lambda_2) = A[\cos(2\pi x/\lambda_1) + \cos(2\pi x/\lambda_2)]$$

Using the identity $\cos x + \cos y = 2\cos\tfrac{1}{2}(x+y)\cos\tfrac{1}{2}(x-y)$, we get directly

$$y(x) = 2A\cos\left(\frac{\pi x}{\lambda_1} + \frac{\pi x}{\lambda_2}\right)\cos\left(\frac{\pi x}{\lambda_1} - \frac{\pi x}{\lambda_2}\right)$$

31. (a) $v_{\text{phase}} = \omega/k$

$$v_{\text{group}} = \frac{d\omega}{dk} = \frac{d}{dk}(kv_{\text{phase}}) = v_{\text{phase}} + k\frac{dv_{\text{phase}}}{dk}$$

$$\frac{dv_{\text{phase}}}{dk} = \frac{dv_{\text{phase}}}{d\lambda}\frac{d\lambda}{dk} = \frac{dv_{\text{phase}}}{d\lambda}\frac{d}{dk}\left(\frac{2\pi}{k}\right) = \frac{dv_{\text{phase}}}{d\lambda}\left(-\frac{2\pi}{k^2}\right) = \frac{dv_{\text{phase}}}{d\lambda}\left(-\frac{\lambda}{k}\right)$$

$$v_{\text{group}} = v_{\text{phase}} - \lambda\frac{dv_{\text{phase}}}{d\lambda}$$

(b) The index of refraction n for light in glass decreases as λ increases (shorter wavelengths are refracted more than longer wavelengths); that is $dn/d\lambda < 0$. Because $n = c/v_{\text{phase}}$, $dn/d\lambda$ and $dv_{\text{phase}}/d\lambda$ have opposite signs and so $dv_{\text{phase}}/d\lambda > 0$. Thus $v_{\text{group}} > v_{\text{phase}}$.

32. $$v_{\text{phase}} = \sqrt{\frac{b}{\lambda}} = \sqrt{\frac{bk}{2\pi}} = \frac{\omega}{k} \qquad \text{or} \qquad \omega = \sqrt{\frac{b}{2\pi}}\,k^{3/2}$$

$$v_{\text{group}} = \frac{d\omega}{dk} = \sqrt{\frac{b}{2\pi}}\,\frac{3}{2}k^{1/2} = \frac{3}{2}\sqrt{\frac{bk}{2\pi}} = \frac{3}{2}v_{\text{phase}}$$

33. $$K = E - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2$$

$$\frac{dK}{dp} = \frac{1}{2}(p^2c^2 + m^2c^4)^{-1/2}(2pc^2) = \frac{pc^2}{\sqrt{p^2c^2 + m^2c^4}} = \frac{pc^2}{E} = c^2\,\frac{mv/\sqrt{1-v^2/c^2}}{mc^2/\sqrt{1-v^2/c^2}} = v$$

34. (a) With a node at each end (say, at $x = 0$ and $x = L$) and no other nodes, we must have one half-wave between the two nodes. Thus $L = \lambda_1/2$ or $\lambda_1 = 2L$. If there is an additional node at the midpoint ($x = L/2$), then there is a full wave between the two ends, and $L = \lambda_2$ or $\lambda_2 = 2L/2$. The next shorter wavelength has (in addition to the nodes at either end) nodes at $x = L/3$ and $x = 2L/3$, so there are three half-waves between the ends: $L = 3\lambda_3/2$ or $\lambda_3 = 2L/3$. Continuing in this way, we see that in the n^{th} case there are n half-waves in the length L, so $L = n(\lambda_n/2)$ or $\lambda_n = 2L/n$.

(b) With $p_n = h/\lambda_n = nh/2L$, we see that cp_n is of order keV, so nonrelativistic equations can safely be used:

$$K_n = \frac{p_n^2}{2m} = \frac{c^2 p_n^2}{2mc^2} = \frac{n^2 h^2 c^2}{8mc^2 L^2} = n^2 \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8(511{,}000 \text{ eV})(0.50 \text{ nm})^2} = n^2 (1.50 \text{ eV})$$

Thus $K_1 = 1.50$ eV, $K_2 = 6.00$ eV, $K_3 = 13.5$ eV.

36. (a) The mass of a nitrogen molecule is 14 u. The average molecular kinetic energy is $\frac{3}{2}kT$, so the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{2(14\ \text{u})(931.5\times10^6\ \text{eV/u})(1.5)(8.617\times10^{-5}\ \text{eV/K})(293\ \text{K})}} = 0.0279\ \text{nm}$$

(b) The number of nitrogen molecules per unit volume is

$$n = \frac{\rho N_\text{A}}{M} = \frac{(1.292\ \text{kg/m}^3)(6.02\times10^{23}\ \text{molecules/mole})}{(0.028\ \text{kg/mole})} = 2.78\times10^{25}\ \text{molecules/m}^3$$

and the average spacing between molecules is $n^{-1/3} = 3.30\times10^{-9}\ \text{m} = 3.3\ \text{nm}$. The de Broglie wavelength is 2 orders of magnitude smaller than the molecular spacing, so that quantum effects are unimportant in gases at room temperature.

(c) Let's estimate that quantum effects would be significant if the de Broglie wavelength were about 1/10 of the molecular separation (0.33 nm):

$$p = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\ \text{eV}\cdot\text{nm}}{0.33\ \text{nm}} = 3760\ \text{eV}/c$$

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(3760\ \text{eV})^2}{2(28\ \text{u})(931.5\times10^6\ \text{eV/u})} = 2.71\times10^{-4}\ \text{eV}$$

.

The molecules have this tiny amount of average kinetic energy at a temperature

$$T = \frac{2K}{3k} = \frac{2(2.71\times10^{-4}\ \text{eV})}{3(8.617\times10^{-5}\ \text{eV/K})} = 2.1\,\text{K}$$

Nitrogen is no longer a gas at this temperature, so our calculation using the formula for the mean molecular energy of gases is not correct. However, it does suggest that if quantum effects are to become important in gases, they will occur only at low temperatures. (Recall the discussion in Chapter 1 about how the equipartition of energy fails for the rotational and vibrational motions of some gases at even moderate temperatures, so other effects of quantum behavior may be observable at these temperatures.)

43. From Eq. 4.15 with $p_{x,\text{av}} = 0$, we have $(\Delta p_x)^2 = (p_x^2)_{\text{av}}$ and similarly in the y direction we have $(\Delta p_y)^2 = (p_y^2)_{\text{av}}$. The kinetic energy is $K = p^2/2m = (p_x^2 + p_y^2)/2m$, and using the minimum estimates for the uncertainties we have

$$K = \frac{(\Delta p_x)^2 + (\Delta p_y)^2}{2m} = \frac{(\hbar / 2\Delta x)^2 + (\hbar / 2\Delta y)^2}{2m} = \frac{\hbar^2 c^2}{8mc^2}\left[\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}\right]$$

$$= \frac{(197\ \text{eV}\cdot\text{nm})^2}{8(511{,}000\ \text{eV})}\left[\frac{1}{(1.25\ \text{nm})^2} + \frac{1}{(2.76\ \text{eV})^2}\right] = 7.3\ \text{meV}$$