Chapter 4

1. (a) At 7 MeV, $K \ll mc^2$, so we use nonrelativistic kinetic energy.

$$p = \sqrt{2mK} = \frac{1}{c}\sqrt{2mc^2K} = \frac{1}{c}\sqrt{2(938.3\text{ MeV})(5\text{ MeV})} = 114.6\text{ MeV}/c$$

$$\lambda = \frac{hc}{pc} = \frac{1240\text{ MeV}\cdot\text{fm}}{114.6\text{ MeV}} = 11\text{ fm}$$

(b) In this case $K \gg mc^2$, so the extreme relativistic approximation $E = pc$ is valid.

$$\lambda = \frac{hc}{pc} = \frac{hc}{E} = \frac{1240\text{ MeV}\cdot\text{fm}}{45\times 10^3\text{ MeV}} = 0.028\text{ fm}$$

(c) The speed is small compared with c, so nonrelativistic formulas apply. With $v/c = (1.35\times 10^6\text{ m/s})/(3.00\times 10^8\text{ m/s}) = 0.00450$.

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{hc}{(mc^2)(v/c)} = \frac{1240\text{ eV}\cdot\text{nm}}{(511{,}000\text{ eV})(0.00450)} = 0.54\text{ nm}$$

2. (a) $K = \frac{3}{2}kT = \frac{3}{2}(8.6174\times 10^{-5}\text{ eV/K})(293\text{ K}) = 0.0379\text{ eV}$

(b) The neutrons are nonrelativistic, so

$$p = \sqrt{2mK} = \frac{1}{c}\sqrt{2mc^2K} = \frac{1}{c}\sqrt{2(939.6\times 10^6\text{ eV})(0.0379\text{ eV})} = 8.44\times 10^3\text{ eV}/c$$

$$\lambda = \frac{hc}{pc} = \frac{1240\text{ eV}\cdot\text{nm}}{8.44\times 10^3\text{ eV}} = 0.147\text{ nm}$$

3. (a) $p = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\text{ MeV}\cdot\text{fm}}{8.29\text{ fm}} = 149.6\text{ MeV}/c$

From $p = mv/\sqrt{1-v^2/c^2} = (1/c)mc^2(v/c)/\sqrt{1-v^2/c^2}$ we solve for v:

$$v = \frac{c}{\sqrt{1+(mc^2/pc)^2}} = \frac{c}{\sqrt{1+[(938.3\text{ MeV})/(149.6\text{ MeV})]^2}} = 0.157c$$

(b)

$$K = E - E_0 = \sqrt{(pc)^2 + (mc^2)^2} - mc^2$$
$$= \sqrt{(149.6\text{ MeV})^2 + (938.3\text{ MeV})^2} - 938.3\text{ MeV} = 11.9\text{ MeV}$$

This gain in kinetic energy requires a loss in potential energy of $\Delta U = -11.9$ MeV and thus a potential difference of $\Delta V = \Delta U / q = -11.9 \text{ MeV}/e = -11.9 \text{ MV}$.

4. With $\Delta U = q\Delta V = (+e)(-3.26\times 10^5 \text{ V}) = -0.326 \text{ MeV}$, we have $\Delta K = -\Delta U = +0.326 \text{ MeV}$. Then

$$p = \sqrt{2mK} = \frac{1}{c}\sqrt{2mc^2K} = \frac{1}{c}\sqrt{2(938.3 \text{ MeV})(0.326 \text{ MeV})} = 24.7 \text{ MeV}/c$$

$$\lambda = \frac{hc}{pc} = \frac{1240 \text{ MeV}\cdot\text{fm}}{24.7 \text{ MeV}} = 50.1 \text{ fm}$$

6. (a) The wavelength should be roughly the size of (or smaller than) the object we want to study, so $\lambda \le 0.10\ \mu$m.
(b) Corresponding to $\lambda \le 0.10\ \mu$m,

$$p = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\ \text{eV}\cdot\text{nm}}{100\ \text{nm}} = 12.4\ \text{eV}/c$$

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(12.4\ \text{eV})^2}{2(511{,}000\ \text{eV})} = 1.5\times10^{-4}\ \text{eV}$$

$$\Delta V = \Delta U/q = -\Delta K/q = +1.5\times10^{-4}\ \text{V}$$

This is a lower limit on the accelerating voltage. If ΔV is smaller than this value, the wavelength is too large and details of the particles could not be seen because of diffraction effects. As ΔV is increased above this value, finer details would be observed.

7. (a)

$$p = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\ \text{MeV}\cdot\text{fm}}{14\ \text{fm}} = 88.6\ \text{MeV}/c$$

For electrons $pc \gg mc^2$, so the extreme relativistic approximation is valid.

$$E \cong pc = 88.6\ \text{MeV}$$

$$K = E - mc^2 = 88.6\ \text{MeV} - 0.5\ \text{MeV} = 88\ \text{MeV}$$

(b) For neutrons, $pc \ll mc^2$ so

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(88.6\ \text{MeV})^2}{2(939.6\ \text{MeV})} = 4.2\ \text{MeV}$$

(c)

$$K = \frac{p^2}{2m} = \frac{p^2c^2}{2mc^2} = \frac{(88.6\ \text{MeV})^2}{2(3727.4\ \text{MeV})} = 1.1\ \text{MeV}$$

8. (a)

$$p = \sqrt{2mK} = \frac{1}{c}\sqrt{2mc^2K} = \frac{1}{c}\sqrt{2(3727\times10^6\ \text{eV})(0.020\,\text{eV})} = 1.22\times10^4\ \text{eV}/c$$

$$\lambda = \frac{hc}{pc} = \frac{1240\ \text{eV}\cdot\text{nm}}{1.22\times10^4\ \text{eV}} = 0.10\,\text{nm}$$

(b) The fringes are separated by about 9 μm.