PHYSICS 110A : MECHANICS 1
PROBLEM SET #9

[1] At perigee of an elliptical orbit, a satellite fires a rocket in the direction of its motion so as to effectively instantaneously increase its speed by a factor $\lambda > 0$.

(a) For what values of λ does the orbit remain bound?

(b) Assuming the orbit remains bound, find the new values of the semimajor axis length $\tilde{a}$, the eccentricity $\tilde{\varepsilon}$, and the location of periapsis $\tilde{\phi}_0$.

(c) For the unbound case, find the opening angle of the hyperbolic orbit.

[2] Evil space aliens send a probe into our solar system to observe the earth. The probe orbits the sun with its perihelion at distance $r_{\rm p} = \frac{3}{4} a_\oplus$, where its velocity is $v_{\rm p} = \frac{4}{3}\, v_\oplus$. (The quantities $a_\oplus$ and $v_\oplus$ are the orbital radius and velocity of the earth, respectively; you may neglect the eccentricity of earth's orbit.) The probe's orbit is coplanar with that of the earth, and you may neglect the interaction of the probe with all bodies other than the sun.

(a) Compute the eccentricity of the probe's orbit.

(b) Compute the probe's distance from the sun at aphelion, and its velocity at aphelion.

(c) Write down the geometric equation for the probe's orbit.

(d) Let perihelion occur when the azimuthal angle is $\phi = \pi$. At what value of ϕ does the probe cross the earth's orbit?

(e) Compute the period of the probe's orbit.

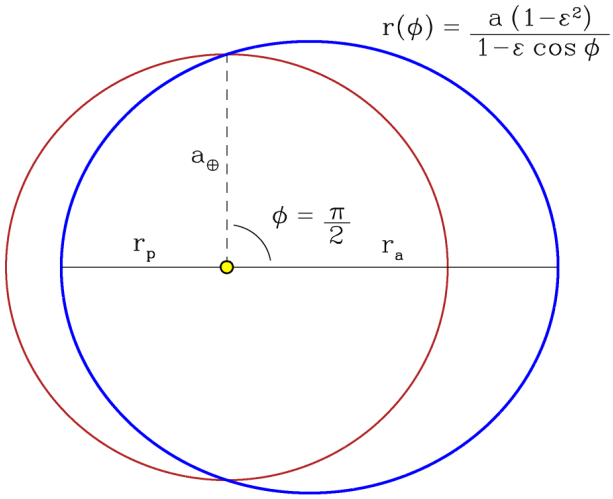

Figure 1: Orbits of the earth (red) and probe (blue).