PHYSICS 110A : MECHANICS 1
PROBLEM SET #8

[1] Starting with the Hamiltonian for a charged particle in an electromagnetic field,

$$H = \frac{1}{2m}\Big(\boldsymbol{p} - \frac{q}{c}\,\boldsymbol{A}(\boldsymbol{x},t)\Big)^2 + q\phi(\boldsymbol{x},t) \quad ,$$

use Hamilton's equations of motion to derive the Lorentz force law.

[2] A particle moves in an elliptical orbit in an inverse square force field. If the ratio of its maximum angular velocity to its minimum angular velocity is λ, show that the orbit has eccentricity

$$\varepsilon = \frac{\sqrt{\lambda}-1}{\sqrt{\lambda}+1} \quad .$$

[3] Two point particles of masses m_1 and m_2 interact via the central potential

$$U(r) = U_0\,\ln\!\left(\frac{r^2}{r^2+b^2}\right) ,$$

where b is a constant with dimensions of length.

(a) For what values of the relative angular momentum ℓ does a circular orbit exist? Find the radius r_0 of the circular orbit. Is it stable or unstable?

(b) For the case where a circular orbit exists, sketch the phase curves for the radial motion in the $(r,\dot r)$ half-plane. Identify the energy ranges for bound and unbound orbits.

(c) Suppose the orbit is nearly circular, with $r = r_0 + \eta$, where $|\eta| \ll r_0$. Find the equation for the shape $\eta(\phi)$ of the perturbation.

(d) What is the angle $\Delta\phi$ through which periapsis changes each cycle? For which value(s) of ℓ does the perturbed orbit not precess?

[4] Two objects of masses m_1 and m_2 move under the influence of a central potential $U = k\,|\boldsymbol{r}_1 - \boldsymbol{r}_2|^{1/4}$.

(a) Sketch the effective potential $U_{\rm eff}(r)$ and the phase curves for the radial motion. Identify for which energies the motion is bounded.

(b) What is the radius r_0 of the circular orbit? Is it stable or unstable? Why?

(c) For small perturbations about a circular orbit, the radial coordinate oscillates between two values. Suppose we compare two systems, with $\ell'/\ell = 2$, but $\mu' = \mu$ and $k' = k$. What is the ratio ω'/ω of their frequencies of small radial oscillations?

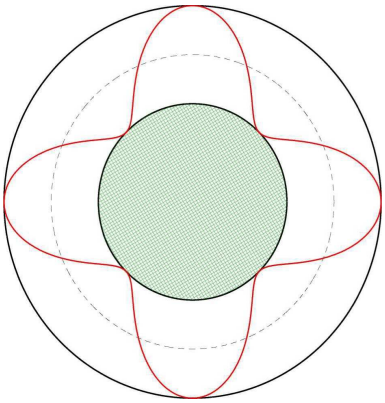

Figure 1: Closed precession in a central potential $U(r) = kr^n$.

(d) Find the equation of the shape of the slightly perturbed circular orbit: $r(\phi) = r_0 + \eta(\phi)$. That is, find $\eta(\phi)$. Sketch the shape of the orbit.

(e) What value of n would result in a perturbed orbit shaped like that in fig. 1?