

**PHYSICS 110A : MECHANICS 1**  
**PROBLEM SET #6**

[1] A point mass  $m$  slides frictionlessly, under the influence of gravity, along a massive ring of radius  $a$  and mass  $M$ . The ring is affixed by horizontal springs to two fixed vertical surfaces, as depicted in Fig. 1. All motion is within the plane of the figure.

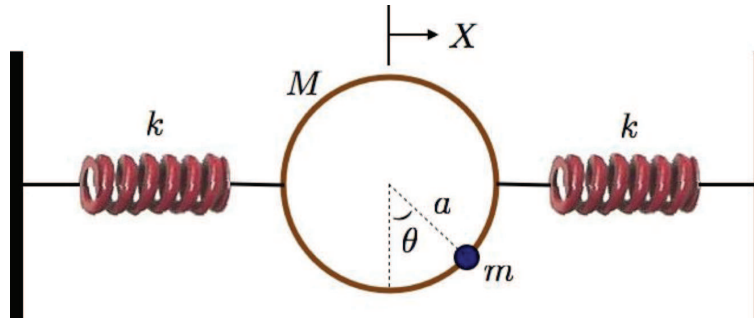


Figure 1: A point mass  $m$  slides frictionlessly along a massive ring of radius  $a$  and mass  $M$ , which is affixed by horizontal springs to two fixed vertical surfaces.

(a) Choose as generalized coordinates the horizontal displacement  $X$  of the center of the ring with respect to equilibrium, and the angle  $\theta$  a radius to the mass  $m$  makes with respect to the vertical (see Fig. 1). You may assume that at  $X = 0$  the springs are both unstretched. Find the Lagrangian  $L(X, \theta, \dot{X}, \dot{\theta}, t)$ .

(b) Derive the equations of motion.

(c) Find the eigenfrequencies for the small oscillations of this system. You may find it convenient to define  $\Omega^2 \equiv 2k/M$ ,  $\nu^2 \equiv g/a$ , and  $r \equiv m/M$ .

[2] [José and Saletan problem 3.11] Consider a three-dimensional one-particle system whose potential energy in cylindrical polar coordinates  $\{\rho, \phi, z\}$  is of the form  $V(\rho, k\phi + z)$ , where  $k$  is a constant.

(a) Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of the motion associated with it.

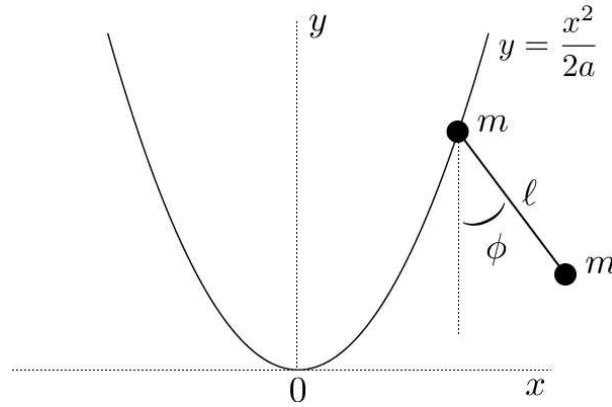
(b) Write down at least one other constant of the motion.

(c) Obtain an explicit expression for the vector field  $d\varphi/dt = \mathbf{V}(\varphi)$ , where

$$\varphi = \begin{pmatrix} \rho \\ \phi \\ z \\ \dot{\rho} \\ \dot{\phi} \\ \dot{z} \end{pmatrix},$$

and use it to verify that the functions found in (a) and (b) are indeed constants of the motion.

[3] A mass  $m$  moves frictionlessly under the influence of gravity along the curve  $y = x^2/2a$ . Attached to the mass is a massless rigid rod of length  $\ell$ , at the end of which is an identical mass  $m$ . The rod is constrained to swing in the  $(x, y)$  plane, as depicted in the figure below.



- Choose as generalized coordinates  $x$  and  $\phi$ . Find the kinetic energy  $T$  and potential energy  $U$ .
- Find the conjugate momenta  $p_x$  and  $p_\phi$ .
- Find the conjugate forces  $F_x$  and  $F_\phi$ .
- Write the Lagrangian for small oscillations, when  $x^2 \ll a^2$  and  $\phi \ll \pi$ .
- Find the coupled equations of motion for small oscillations.
- Find the eigenfrequencies for the two modes of oscillation.