PHYSICS 110A : MECHANICS 1
PROBLEM SET #5

[1] Extremize the functional

$$F[y(x)] = \int_0^{\ln 2} dx \left(\tfrac{1}{2} y'^2 + a y y' + \tfrac{1}{2} y^2 + y \right)$$

subject to the boundary conditions $y(0) = y_0$ and $y(\ln 2) = y_1$.

[2] Extremize the functional

$$F[y(x), z(x)] = \int_0^{\frac{\pi}{2}} dx \left(y'^2 + z'^2 + 2yz \right)$$

subject to the boundary conditions

$$y(0) = z(0) = 0 \quad , \quad y(\tfrac{\pi}{2}) = z(\tfrac{\pi}{2}) = 1 \ .$$

[3] Derive the equations of motion for the Lagrangian

$$L = e^{\gamma t} \left[\tfrac{1}{2} m \dot{q}^2 - \tfrac{1}{2} k q^2 \right] \ ,$$

where $\gamma > 0$. Compare with known systems. Rewrite the Lagrangian in terms of the new variable $Q \equiv q \exp(\gamma t / 2)$, and from this obtain a constant of the motion.