PHYSICS 110A : MECHANICS 1
PROBLEM SET #4

[1] An electrical circuit consists of a resistor R and a capacitor C connected in series to an emf $V(t)$.

(a) Write down the differential equation for the charge $Q(t)$ on one of the capacitor plates.

(b) Solve the homogeneous equation for $Q(t)$, *i.e.* find $Q(t)$ when $V(t) = 0$ subject to arbitrary initial value of $Q(0)$.

(c) Solve for the current $I(t)$ flowing in the circuit when $V(t) = V_0\,\Theta(t)$. Assume $Q(0) = 0$.

(d) Solve for $I(t)$ when $V(t) = V_0\,\sin(\Omega t)\,\Theta(t)$ and $Q(0)$=0.

For parts (c) and (d), you should use the Green's function formalism in the time domain. The following integral may prove useful:

$$\int\limits_{-\infty}^{\infty} \frac{d\omega}{2\pi}\,\frac{e^{-i\omega s}}{1-i\omega\tau} = \frac{1}{\tau}\,e^{-s/\tau}\,\Theta(s) \quad .$$

[2] Do either of the following:

(a) A forced, damped harmonic oscillator obeys the equation of motion

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0\,e^{-\gamma t}\,\Theta(t) \quad .$$

Compute $x(t)$ assuming $x(0) = \dot{x}(0) = 0$.

(b) A forced, damped harmonic oscillator obeys the equation of motion

$$\Big(\frac{d}{dt} + \alpha\Big)\Big(\frac{d}{dt} + \beta\Big)x = f_0\,e^{-\gamma t}\,\Theta(t) \quad .$$

Compute $x(t)$ assuming $x(0) = \dot{x}(0) = 0$.