PHYSICS 110A : MECHANICS 1
PROBLEM SET #3

There are four problems in all. Problems 1 and 2 constitute a practice midterm exam.

[**1**] A particle of mass m moves in the one-dimensional potential

$$U(x) = \frac{U_0}{a^4}\left(x^2 - a^2\right)^2 \quad . \tag{1}$$

(a) Sketch $U(x)$. Identify the location(s) of any local minima and/or maxima, and be sure that your sketch shows the proper behavior as $x \to \pm\infty$.
`[15 points]`

(b) Sketch a representative set of phase curves. Be sure to sketch any separatrices which exist, and identify their energies. Also sketch all the phase curves for motions with total energy $E = \frac{1}{2}\,U_0$. Do the same for $E = 2\,U_0$.
`[15 points]`

(c) The phase space dynamics are written as $\dot{\boldsymbol{\varphi}} = \boldsymbol{V}(\boldsymbol{\varphi})$, where $\boldsymbol{\varphi} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$. Find the upper and lower components of the vector field $\boldsymbol{V}$.
`[10 points]`

(d) Derive an expression for the period T of the motion when the system exhibits small oscillations about a potential minimum.
`[10 points]`

[**2**] An R-L-C circuit is shown in fig. 1. The resistive element is a light bulb. The inductance is $L = 400\,\mu\text{H}$; the capacitance is $C = 1\,\mu\text{F}$; the resistance is $R = 32\,\Omega$. The voltage $V(t)$ oscillates sinusoidally, with $V(t) = V_0\cos(\omega t)$, where $V_0 = 4\,\text{V}$. In this problem, you may neglect all transients; we are interested in the late time, steady state operation of this circuit. Recall the relevant MKS units:

$$1\,\Omega = 1\,\text{V}\cdot\text{s}\,/\,\text{C} \quad , \quad 1\,\text{F} = 1\,\text{C}\,/\,\text{V} \quad , \quad 1\,\text{H} = 1\,\text{V}\cdot\text{s}^2/\,\text{C} \quad .$$

(a) Is this circuit underdamped or overdamped? Why?
`[10 points]`

(b) Suppose the bulb will only emit light when the average power dissipated by the bulb is greater than a threshold $P_{\text{th}} = \frac{2}{9}\,W$. For fixed $V_0 = 4\,\text{V}$, find the frequency range for ω over which the bulb emits light. Recall that the instantaneous power dissipated by a resistor is $P_R(t) = I^2(t)R$. (Average this over a cycle to get the average power dissipated.)
`[20 points]`

(c) Compare the expressions for the instantaneous power dissipated by the voltage source, $P_V(t)$, and the power dissipated by the resistor $P_R(t) = I^2(t)R$. If $P_V(t) \neq P_R(t)$, where

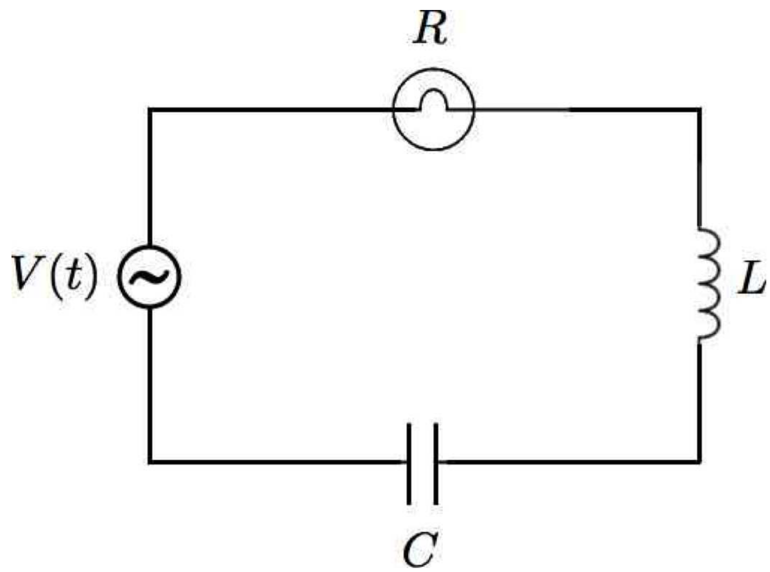

Figure 1: An R-L-C circuit in which the resistive element is a light bulb.

does the power extra power go or come from? What can you say about the averages of P_V and $P_R(t)$ over a cycle? Explain your answer.
`[20 points]`

(d) What is the maximum charge $Q_{\rm max}$ on the capacitor plate if $\omega = 3000\,{\rm s}^{-1}$?
`[100 quatloos extra credit]`

[3] The potential energy of two atoms in a molecule can sometimes be approximated by the Morse function,

$$U(r) = A\bigg[\Big(e^{(R-r)/\lambda} - 1\Big)^2 - 1\bigg] \quad ,$$

where r is the interatomic distance and A, R, and λ are positive constants.

(a) Sketch the function $U(r)$ for $0 < r < \infty$.

(b) Find the equilibrium separation r^* at which $U(r)$ is minimized.

(c) Assume the motion is one-dimensional. Writing $r = r^* + x$, so that x is the displacement relative to equilibrium, show that $U(r)$ takes the form $U(r^* + x) = U_0 + \frac{1}{2}kx^2$ for small $|x|$, so that Hooke's law applies. What do we mean by 'small'?

(d) What is the effective force constant k?

[4] An undamped oscillator has a period $T = 1.000\,{\rm s}$. Some damping is then introduced, causing the period of the damped oscillations to increase to $T' = 1.001\,{\rm s}$.

(a) What is the damping coefficient β?

(b) By what factor will the oscillation amplitude be decreased after ten cycles?

(c) Which effect of the damping would be more noticeable: the change in the period, or the change in the amplitude?