PHYSICS 110A : MECHANICS I

1. Introduction to Dynamics (Sept. 23 and 26)

   essential elements of dynamics
   discrete and differential equations
   deterministic *versus* stochastic
   dynamical systems
   examples

2. Motion in $d = 1$: Two-Dimensional Phase Flows (Sept. 28)

   (x, v) phase space
   dynamical system $\frac{d}{dt}\begin{Bmatrix} x \\ v \end{Bmatrix} = \begin{Bmatrix} v \\ a(x,v) \end{Bmatrix}$
   two-dimensional phase flows
   examples: harmonic oscillator and pendulum
   fixed points in two-dimensional phase space; separatrices

3. Solution of the Equations of One-Dimensional Motion (Sept. 30 and Oct. 3)

   potential energy $U(x)$
   conservation of energy
   sketching phase flows from $U(x)$
   solution by quadratures
   turning points; period of orbit

4. Linear Oscillations (Oct. 5)

   Taylor's theory and the ubiquity of harmonic motion
   the damped harmonic oscillator: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$
   - reduction to algebraic equation
   - generalization to all autonomous homogeneous linear ODEs

   solution to the damped harmonic oscillator: underdamped and overdamped behavior

5. Forced Linear Oscillations (Oct. 7)

   $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t)$
   solution for harmonic forcing $f(t) = A\cos(\Omega t)$
   presence of homogeneous solution: transients
   amplitude resonance and phase lag; Q factor

6. Green's Functions for Autonomous Linear ODEs (Oct. 10)

   Fourier transform
   physical meaning of $G(t - t')$; causality
   response to a pulse

7. Systems of Particles (Oct. 12 and 14)

kinetic, potential, and interaction potential energies
forces; Newton's third law
momentum conservation
torque and angular momentum
kinetic energy and the work-energy theorem

8. `MIDTERM EXAMINATION` (Oct. 17)

9. Calculus of Variations I (Oct. 19)

Snell's law for refraction at an interface
    continuum limit of many interfaces
functionals
variational calculus: extremizing $\int dx\, L(y, y', x)$
preview: Newton's second law from $L = T - U$

10. Calculus of Variations II (Oct. 21 and 24)

Examples
    surfaces of revolution
    geodesics
    brachistochrone
generalization to several dependent and independent variables
Constrained Extremization
Lagrange undetermined multipliers in calculus: review
systems with integral constraints
    hanging rope of fixed length
holonomic constraints

11. Lagrangian Dynamics (Oct. 26 and 28)

generalized coordinates
action functional
equations of motion: Newton's second law
    examples: spring, pendulum, *etc.*
double pendulum: Lagrangian and equations of motion
Lagrangian for a charged particle interacting with an electromagnetic field
    Lorentz force law

12. Noether's Theorem and Conservation Laws (Oct. 31 and Nov. 2)

continuous symmetries
"one-parameter family of diffeomorphisms" $q_i \to h_i^\lambda(q_1, \ldots, q_N)$
Noether's theorem and the conserved "charge" $Q = \sum_i \frac{\partial L}{\partial q_i} \frac{\partial h_i^\lambda}{\partial \lambda}\Big|_{\lambda=0}$
linear and angular momentum

13. Constrained Dynamical Systems (Nov. 4 and 7)

undetermined multipliers as forces of constraints
simple pendulum with $r = l$ or $x^2 + y^2 = l^2$ constraint
Examples

15. The Two-Body Central Force Problem (Nov. 9, 11, and 14)

CM and relative coordinates
angular momentum conservation and Kepler's law $\dot{\mathcal{A}}$ =const.
energy conservation
the effective potential
    radial equation of motion for the relative coordinate
    the effective potential and its interpretation
    phase curves
    solution for $r(t)$ and $\phi(t)$ by quadratures

16. The Shape of the Orbit (Nov. 16 and 18)

equation for $r(\phi)$, the geometric shape of the orbit
    $s = 1/r$ substitution
examples
almost circular orbits: bound *versus* closed motion, precession

17. Coupled Oscillations I: The Double Pendulum (Nov. 21 and 23)

review: Lagrangian for the double pendulum
equations of motion
linearization
solution of two coupled linear equations
normal modes

18. Coupled Oscillations II: General Theory (Nov. 25 and 28)

harmonic potentials
T and V matrices
normal modes
the mathematical problem: simultaneous diagonalization of T and V

19. Coupled Oscillations III: The Recipe (Nov. 30 and Dec. 2)

eigenvalues: $\det(\omega^2 T - V) = 0$
eigenvectors: $(\omega_s^2 T_{ij} - V_{ij}) a_j^{(s)} = 0$
normalization: $a_i^{(s)} T_{ij} a_j^{(s')} = \delta_{ss'}$
modal matrix: $A_{js} = a_j^{(s)}$
examples

- `COMPREHENSIVE FINAL EXAMINATION` (Dec. 6)