Physics 218C

Lecture 7: Momentum Transport and Intrinsic Rotation; with ITB's

i.) Loose Ends

1.) Additional Postings (avail. Thursday, PDT)

- annotated version of previous lecture notes → gaps filled in.

- annotated notes on Predator-Prey Model (HUST-PKU '20-'21)

- "Feedback for Physicists" - J. Bechhoefer (R.M.P.)
  → important!

and see also:

- R. May, "Stability and Complexity in Model Ecosystems". (highly recommended) → simple, useful book

+

posted articles.

iii) Questions

→ Relation to 'inverse cascade'?
→ constraint of strong magnetic field!

In 2D fluid ($\nabla\langle PV\rangle = 0$) drag - control large scale

$$\partial_t \nabla_\perp^2\phi + \nabla\phi\times\hat{z}\cdot\nabla\,\nabla_\perp^2\phi = -\mu\nabla_\perp^2\phi + \nu\nabla_\perp^2\nabla_\perp^2\phi + \tilde{f}$$

vorticity stirring

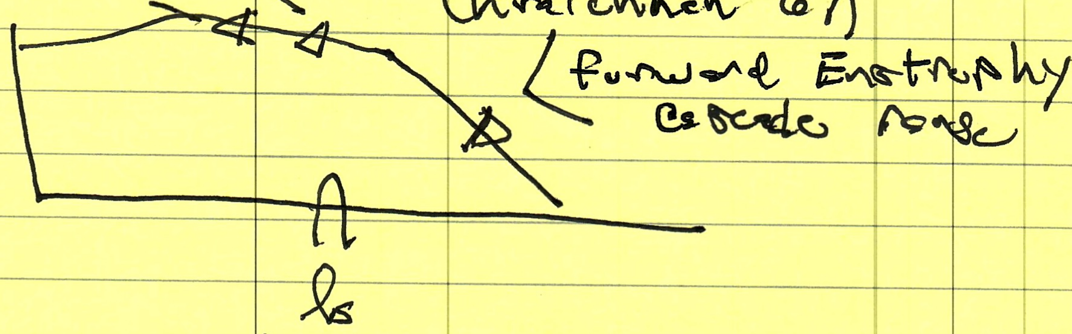
2 conservation laws (quadratic) - inviscid

→ Energy $\cong \int d^3x\, (\nabla\phi)^2/2$

Enstrophy $= \int d^3x\, (\nabla_\perp^2\phi)^2/2$

so if stir at some intermediate scale l_s, have: 2 ranges of self-similar transfer.

i.e.



(see Fluids notes)

here: "cascade" = self-similar transfer

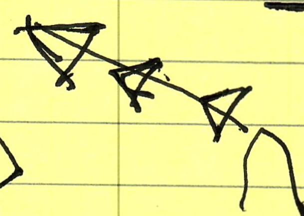
i.e. for inverse cascade range

$$\epsilon_s = \frac{v(\ell)^2}{\tau(\ell)} = \frac{v(\ell)^3}{\ell}$$

rate of energy stirring

$$E(k) \cong \epsilon_s^{2/3} k^{-5/3}$$

well documented in basic computer simulations

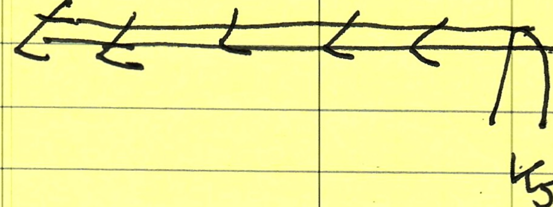


— spectrum grows from $1/\ell_s$

— continue to box.

Note:

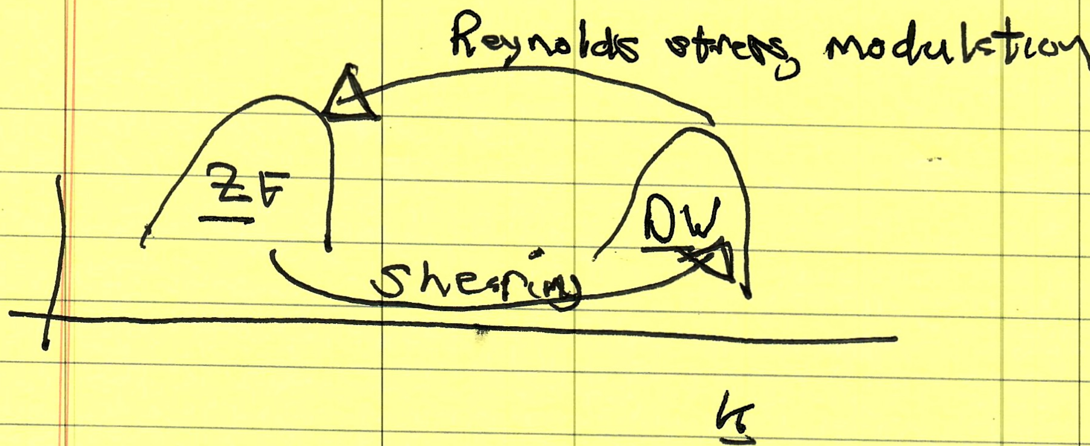
- Inverse cascade is continuous transfer (i.e. flow in scale space)



k_s

local transfer in k space

- should be contrasted to disparate scale interaction



non-local transfer in $\underline{k}$ - space

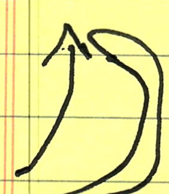
- "Inverse cascade" chanted as a mantra, but bicoherence experiments indicate

$\boxed{\text{ZF - DW transfer is non-local}}$

- primacy of strong-B constraint is fundamental to both mechanisms, but beyond - quite different.

→ Flow Damping - Zonal (Tokamak)

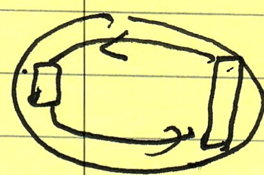
- Depends on collisionality (ion)
  - Banana / Plateau → Friction of flow with (stationary) 'banana' of trapped particles.



$\sim \epsilon \nu_{ii}$

– Pfirsch-Schluter (very edge, only)

Magnetic pumping – collisions + geometry



compression + collisions

and

■ νk^2, $D_0 k^2$ → scale?

and

Charge exchange – neutral friction (scale independent)

Other:

– ZF goes unstable ⇒ "tertiary instability"

⇒ how quantify vorticity transport?

→ $\langle \tilde{V}_r \nabla^2 \tilde{\phi} \rangle$ → expect a turbulent viscosity

→ tertiary "seen" in some computer simulations not in experiment (i.e. tokamak).

- $\langle v_\theta \rangle$ flow develops to allow

wave - flow resonance

i.e.

$$\partial_t \langle \nabla_\perp^2 \phi \rangle = \partial_r \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle + \text{lin. damping}$$

then, in HW: $\quad \longmapsto$ from n eqn.

$$\nabla_\perp^2 \tilde{\phi} = \frac{-\tilde{v}_r \partial_r \langle \nabla_r^2 \phi \rangle}{-i(\omega - k_\theta V_E)} + \frac{\alpha \tilde{h}}{-i(\omega - k_\theta V_\theta)}$$

resonance, with ExB flow

$$\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle = -\sum_k |\tilde{v}_{r_k}|^2 \pi \delta(\omega - k_\theta V_E) \, \partial_r \langle \nabla_r^2 \phi \rangle$$

$+ \quad \ldots$

$\omega = \omega_{DW}$

$\approx$ "turbulent diffusion" of vorticity

$\Rightarrow$ stronger as $V_E \to \omega / k_\theta$ from $V_E \sim 0$.

$\Leftrightarrow$ collisionless damping!

Does not require "instability".

→ Coincidences in Power ?

Why relation $\left\{ \begin{array}{ll} P_{OH} & \text{LOC} - \text{SOC} \\ P_{LH\,crit} & \text{min} \end{array} \right\}$? [cf Y. Ma, Jerry Hughes et. al.]

Both related to collisional electron - ion coupling !

i.e. LOC - SOC:

$$\sim n \nu_E (T_e - T_i) \sim n^2$$

$$\frac{\partial}{\partial t} \mathcal{E}_{\text{Electrons}} = P_{OH} - \underset{\substack{\text{due EDW} \\ \sim 1/n}}{\text{Transport}} - P_{e-c}$$

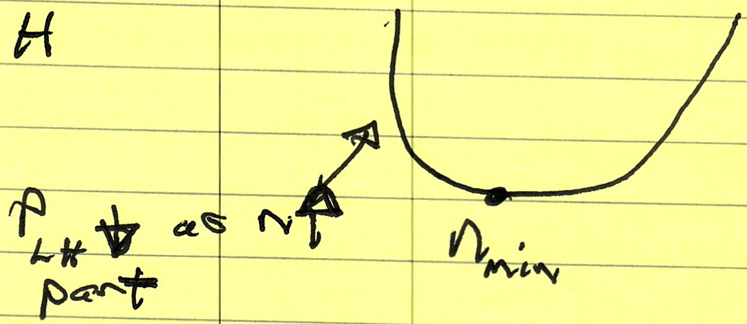
$$\frac{\partial}{\partial t} \mathcal{E}_{\text{Ions}} = P_{ei} - \underset{\substack{\text{due ITG} \\ \text{(with threshold)}}}{\text{Transport}}$$

coupling $\sim \underbrace{n^2}$ ⇒ ions / ITG excited as n rises !

$$\frac{\partial}{\partial t} \mathcal{E}_{\text{Total}} = P_{OH} - \underset{\text{ITG}}{\text{Transp}} - \text{Transp EDW}$$

saturates LOC regime.

and $L \to H$
(Ryter)



- Generally decreasing P_{crit} part of the curve is electron heated (OH, ECH)

- need $\underbrace{\nabla P_i}_{\uparrow}$ for $\langle E_r \rangle$ for barrier

so $P_{ei} \sim n^2$

$$\frac{\partial}{\partial t} \mathcal{E}_{electron} = P_{cn} - P_{e \to i} - \text{Transport}$$

$$\frac{\partial}{\partial t} \mathcal{E}_{ions} = P_{e \to i} - \text{Transport}$$

(Bifurcative)

$$\to Q = -\left[\chi / (1 + \alpha V_E'^2)\right] \nabla T - \chi_{neo} \nabla T$$

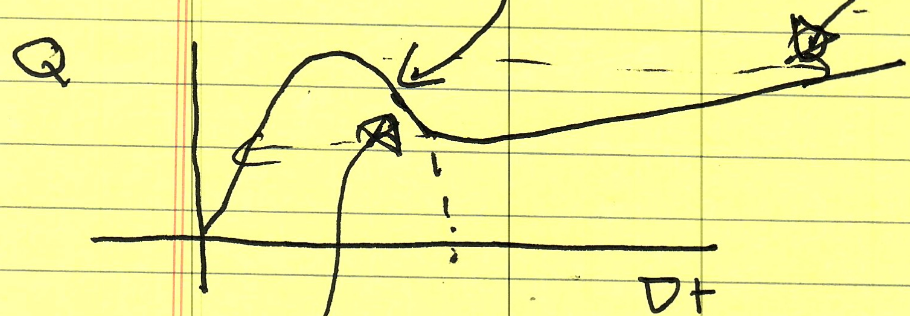
need sufficient power coupling to ions.

Exact / precise correspondence is TBC.

$\Longrightarrow$ important question!

- S-curve revisited

$$Q = \frac{-\chi_0 \nabla T}{\left[1 + \alpha\left(V_E'^2/\omega_0^2\right)\right]^{\gamma}} - \chi_{neo} \nabla T$$



$\delta Q / \delta \nabla T < 0$ → negative incremental χ.

c.f.: Hinton '91
Malkov, PD '10
also Hubbard, in PPCF '02.

Rice : Alcator - C Mod (again ...)

- noted bulk (including central) toroidal rotation, in H-mode, with ICRF (4 pts, initial)
- X-ray spectroscopy
- subsequently: "Rice Scaling"

$$\boxed{\Delta V_\phi \cong \Delta W / I_p}$$ → driven much of cntr. rotation rsch.

ΔV_ϕ → increment in toroidal rotation at L→H

ΔW → increment in stored energy at L→H

$$\tau_E \sim W / P_{in}$$

(i.e. largely pedestal formation).

$V_\phi |_{axis}$ subsonic, but significant.

Increment - co - current

• 10.) Momentum Transport and Intrinsic Rotation

c.f. Real study of momentum transport began with discovery of Intrinsic Rotation – "Prehistory" later

– K. Ida '95 – inferred non-diffusive stress, contributes to momentum balance

– John Rice '97

– noted tokamak plasma rotates at significant speed w/o any external torque.

[redacted]

· Ida: JFT-2M, via $\left\{\begin{array}{l}\text{Power} \\ \text{Momentum}\end{array}\right.$ Balance Analyses.

$$\frac{\partial}{\partial t}\langle V_\phi \rangle = -\underline{\nabla} \cdot \underline{\underline{\Pi}}$$

$$\underline{\underline{\Pi}} = -\underbrace{\chi_\phi}_{\phi}\, \nabla \langle V_\phi \rangle + \underbrace{\text{Something Else}}_{\Pi_{\text{Resid}}}$$

→ CERS

— What does it mean?

Solomon experiment ('07)

- 3 co-beam → Peaked V_ϕ

- 1 co, 2 cntr-beam → <u>Flat</u> V_ϕ

<u>i.e.</u> plasma generates 1 beam-line of co-torque, <u>on</u> <u>its</u> <u>own</u>.

⇝ this is serious ..... { fusion / Confinement Physics }

— Why care for fusion?

$$\underline{\langle E \rangle} = \frac{\langle \underline{\nabla} P_i \rangle}{n} + \underline{\langle v \rangle} \times \underline{B}$$

sheared toroidal rotation is beneficial

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Table 1. Selected phenomenology of intrinsic torque

Phenomenon	Signature	Sym. breaking	Key physics	Issue
H-mode and I-mode ETB	spin-up at L $\to$ I or H, Rice scaling $v_\phi(0) \sim \nabla T_i, \nabla p_i$	Pedestal $\langle v_E \rangle'$, I'	Π_{res} and $\nabla v_\phi \uparrow$ as ∇p_i, $\langle v_E \rangle \uparrow$ and ETB forms. Cancellation experiment.	Quantitative? ∇T_i or ∇p_i? How achieve global cancellation?
ITB	∇v_ϕ steepens with ∇T_i in ITB with $\tau_{ext} = 0$	$\langle v_E \rangle'$, I' in ITB	π_{res} and $\nabla v_\phi \uparrow$ as ∇p_i, $\langle v_E \rangle' \uparrow$ Relative hysteresis of ∇T_i, ∇v_ϕ observed	Quantitative? Relative hysteresis? Role in de-stiffening?
OH inversions	Inversion of $v_\phi(r)$ around pivot for $\nu_* > \nu_{*sat}$. Hysteresis in n, I, B-ramp	Open question I', $\langle v_E \rangle'$, ...?	$v_\phi(r)$ invert at $\nu_* \sim \nu_{*OH}$ without observable change in n, T profiles. v_{gr} flip at TEM $\leftrightarrow$ ITG transition. $\to \pi_{res}$ flips	Symmetry breaker? Extended flip versus localized flip +spreading Interplay with bndry
Co-NBI H-mode +ECH	ECH + co-NBI $\to$ central flattening of v_ϕ	Open question I', $\langle v_E \rangle'$, ...?	ECH induces $\Delta \nabla v_\phi(0) < 0$ in NBI H-mode $\to$ co NBI + co intr. ped. + cntr ECH. v_{gr} flips at TEM $\leftrightarrow$ ITG transition	Density profile peaking? Effect? Extended flip versus localized flip +spreading
LSN $\leftrightarrow$ USN L-mode Inversions ∇B asymmetry in P_T	LSN $\leftrightarrow$ USN jog $\to$ SOL flow reversal $\to$ core flow reversal in L-mode ∇B asym. in P_T	SOL flow direction or eddy tilt due combination magnetic and electric field shear	Change in competition between B and E field shear in USL versus LSN. Core responds to bndry +SOL flows	Boundary flow penetration $\to$ 'Tail + dog' problem Role of SOL flows?

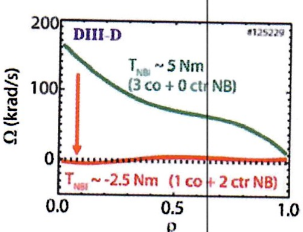
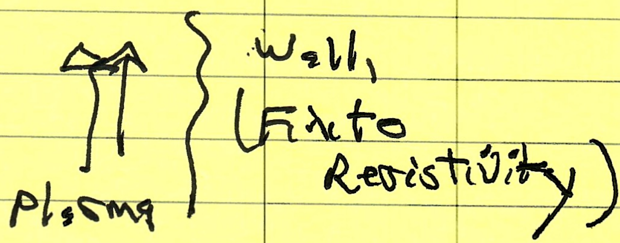


Figure 1. 'Cancellation' experiment of Solomon *et al* from DIII-D [21]. A mix of 1 co and 2 counter beams yield a flat rotation profile with $\langle v_\phi \rangle \cong 0$. This shows that the intrinsic torque for these parameters is approximately that of 1 neutral beam, in the co-current direction.

Regarding the phenomenology of intrinsic torque, an interesting selected subset we discuss here is: (a) H-mode edge transport barrier, (b) ITB, (c) OH-reversal, (d) co neutral beam injection (NBI) H-mode + ECH, (e) LSN$\leftrightarrow$USN L-mode rotation. This discussion and that of section 5 are summarized in table 1. Of course, the classic example of intrinsic torque and intrinsic rotation is the H-mode electron transport barrier (ETB) [3]. In the absence of external torque, a spin-up is initiated at the L$\to$H transition and builds inwards [3]. The basic trend is described by the Rice scaling $\Delta V_\phi(0) \sim \Delta W/I_p$ where W is energy content and Δ refers to the change across the L$\to$I or L$\to$H transition. The existence and location of the intrinsic torque have been rather convincingly established by the 'cancellation' experiment by Solomon *et al* [21]. The idea here was to exploit the asymmetry between co and counter-NBI H-modes due to the presence of a (hypothetical) 'intrinsic torque' τ. The result, shown in figure 1, is striking: a net counter-torque H-mode yields a rotation profile, which is *flat* (and zero) within the error bars! The implication is clear: the on-axis counter-NBI torque is exactly cancelled by a co-intrinsic pedestal torque! This result *strongly* argues for the viability of the intrinsic torque concept. It also suggests that intrinsic torque can give the appearance of a non-local intrinsic torque phenomenon, in that the intrinsic torque, situated in the pedestal, acts to flatten $\nabla \langle V_\phi \rangle$ in the core. To characterize the pedestal intrinsic torque, data base studies from Alcator C-Mod [8] indicate that central rotation in H-mode and I-mode tracks pedestal ∇T_i, i.e. $V_\phi(0) \sim \nabla T_{i,ped}$, suggesting that the pedestal intrinsic torque is ∇T_i-driven.

Intrinsic rotation in ITBs [22–25] has received *far* less attention than intrinsic rotation in ETBs. This is due in part to the fact that ITBs are usually formed in plasmas subject to external torque. However, since the interaction of external and intrinsic torques is important in low torque scenarios planned for ITER, intrinsic rotation in ITBs and 'de-stiffened' states should receive more attention. Here, a de-stiffened state is one with a stronger response of the temperature gradient to heat flux increments than that exhibited by a stiff state. De-stiffening can be achieved by enhanced $E \times B$ shear, for example. One recent experiment [10] obtained the scaling relation $\nabla V_\phi \sim \nabla T_i$ for intrinsic rotation gradients in ITBs. This is reminiscent of the similar result for ETBs and again suggests that the intrinsic rotation is temperature gradient driven, as in a heat engine. To look beyond correlation to causality, that study investigated *relative hysteresis* between ∇V_ϕ and ∇T_i. Results indicated that hysteresis in ∇V_ϕ was stronger than in ∇T_i, possibly due to the low residual Prandtl number (i.e. $Pr_{resid} \sim \chi_\phi/\chi_i$, in the ITB. Here, χ_ϕ and χ_i are the true, not effective, diffusivities) in the ITB. Since hysteresis of a transport barrier is a consequence of the disparity between transport in the normal and the barrier state, the fact that $\chi_i \gg \chi_\phi$ in the ITB implies that hysteresis will be stronger in ∇v_ϕ than in ∇T_i. Recall $\chi_i \sim \chi_\phi$ in L-mode.

A particularly compelling case for the need to consider intrinsic torque physics is the fascinating phenomenon of rotation reversals in OH or L-mode plasmas. Reversals refer to events in which the global rotation profile spontaneously reverses direction. First studied in detail in TCV [26] and Resistive Wall Modes (Kink)



Rotation of plasma relative to wall is good ⇒ stability
esp pedestal torque.

Intrinsic rotation provides that rotation, w/o NBI (dubious for ITER).

Some other important points:

- heating method, irrelevant. Settled confusion re: RF waves, orbit loss, ...

classic: Hutchinson, Rice et al. 2002

$$P_{OH} = P_{RF} \Rightarrow \text{same } (\Delta V)(I_p)$$

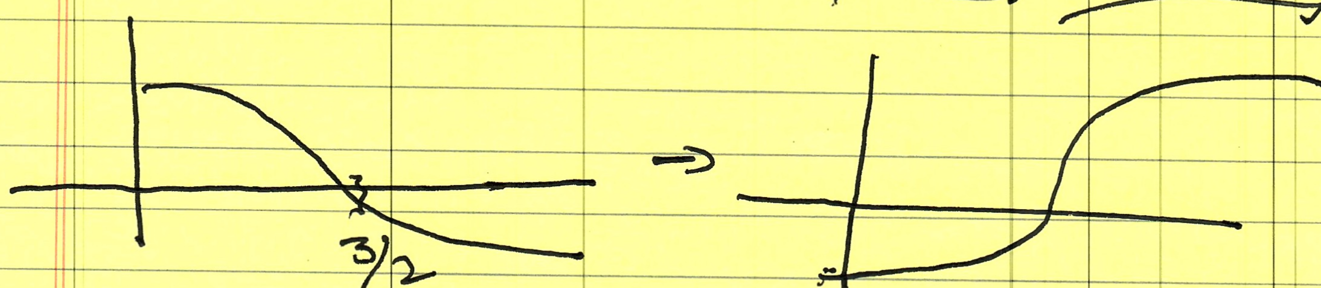
~ same story

- Cancellation experiments ✓

- Ohmic Reversals:
  - Bortolon, Duval '06, '08 TCV
  - Rico, et al. 2011, '12.

  - intrinsic rotation ~ universal, though strength varies
  - intrinsic rotation occurs in OH (L-mode).

but
  - intrinsic rotation changes, spontaneously



seen both co → cntr, cntr → co
(edge activity)

⇒ Ohmic reversal.

Reversal occurs for n s/t

LOC → SOC

and if LOC → SOC → IOC (AUG Angioni, McDermott)

⇒ Flips back

⇝ points at intrinsic rotation related to type of turbulence (CTEM vs ITG) [P.D. '08, P.D. '12]

$\boxed{\text{Ohmic reversal phenomenon nails down to connection of intrinsic rotation to turbulence, transport physics.}}$

see also: Beams + ECH — cast of thousands.

c.f. Hysteresis loop — $\langle V_\phi \rangle$ vs n.

— Beyond global / engineering scaling:

$\langle V_\phi \rangle$ vs ∇P, ∇T_i — Rice, Solomon.

and $\sim 1/B_\theta$

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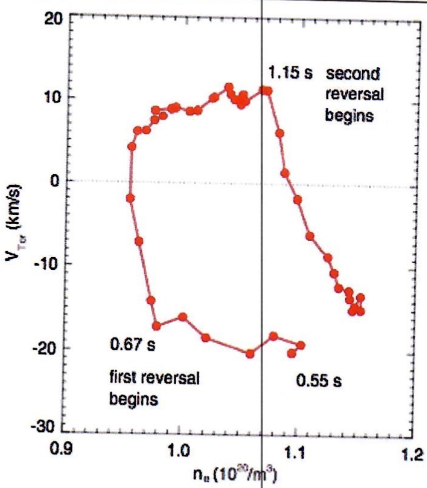


Figure 2. Density ramp hysteresis loop for reversals on Alcator C-Mod [28].

and C-Mod [27–29], reversals are spontaneous 'flips' in the toroidal rotation profile from co to counter (in C-Mod) which occur as n increases and exceeds n_{sat}, the density at which confinement transitions from the linear ohmic confinement (LOC) to saturated ohmic confinement (SOC) regime. During the reversal, the rotation profile effectively pivots around a fixed point inside $q \lesssim 3/2$. Interestingly, up–down density ramps reveal back flips, but with some hysteresis, i.e. the velocity versus density plot is a closed loop enclosing finite area, not a straight line, as shown in figure 2. In some cases, a rotation 'spike' (i.e. a transient, spatially localized bump in the toroidal rotation velocity profile) was observed near the edge just after the reversal [28]. Also, experiments on TCV do indicate some differences between reversals in limited and diverted discharges [30], suggesting that the effective boundary conditions play a role in reversal dynamics. Spikes are particularly interesting, as they may hold a clue to the *global* momentum balance and rotation profile dynamics. This is because spikes may reveal the dynamics of momentum ejection events which help understand how the total momentum balance of the core plasma is maintained. Building on the long standing idea that the evolution from LOC to SOC regimes is due to a transition from trapped electron mode (TEM) transport to ion temperature gradient (ITG) transport excited by collisional coupling, a speculation has arisen that inversions are a consequence of a change in the sign of $\Pi^{R}_{r\phi}$ as $n > n_{sat}$ or more generally $\nu_* > \nu_{*crit}$ [31, 32]. This change reflects the dependence of $\Pi^{R}_{r\phi}$ on v_{gr}, the group velocity of the underlying microinstability. Alcator C-Mod has pursued fluctuation studies, the results of which are consistent with the expected change in mode populations, but are not conclusive. Further work is needed.

A somewhat related phenomenon, related to the effect of ECH on co-NBI H-mode profiles, has been observed in JT-60U [33], AUG [34], DIII-D [35], KSTAR [36] and HL-2A [37]. Results indicate that ECH of NBI-driven H-modes tends to flatten the otherwise peaked velocity profile, and reduce central rotation speeds ($\Delta V/V \sim -40\%$, in KSTAR), while ∇T_e steepens. Profile studies indicate $\nabla V_\phi \sim \nabla T_e$ here, suggestive of a TEM counter-torque in the core. Correlation of ∇v_ϕ and ∇n is also indicated [38]. The H-mode pedestal rotation profile is unchanged by ECH, suggesting that the torque balance here is: co-NBI + pedestal co-intrinsic versus core counter torque related to ECH. KSTAR profiles with NBI and NBI + ECH are shown in figure 3. The data suggest a similar paradigm to that for the OH inversion, namely a change in the direction of the core intrinsic torque from co to counter, due to a flip in mode propagation direction from v_{*i} to v_{*e}, as ITG gives way to TEM. Comparative gyrokinetic stability analysis of NBI+ECH and NBI H-modes is, however, somewhat incomplete. This follows from the sensitivity of the results to density profile structure near the pivot radius, and from uncertainty concerning the spatial extent of the region where the mode population flips (according to purely linear analysis). Fluctuation measurements are not yet available. See [34, 36] for more details.

The importance of the edge in intrinsic rotation physics should already be apparent. A classic example of this is the LSN→USN jog experiments of LaBombard in C-Mod L-mode plasmas [39]. Here, 'jog' refers to the process of swing the null point from lower (LSN) to upper (USN) positions by controlled variation of the magnetic configuration. These are often described as a 'tail-wags-the-dog' phenomena, since changes from LSN to USN reverses not only scrape-off layer (SOL) flows, but also the direction of the core rotation. Interestingly, the effect on core rotation vanishes in H-mode, suggesting that the tail is 'cut-off' by the sheared flow in the ETB. The dynamics of this fascinating phenomenon are not understood. In particular, the issue of just how flow changes penetrate from the SOL and boundary to the core remains open. Note that this issue may be related to the long standing mystery concerning the ∇B-drift asymmetry in the L→H power threshold [40]. It is important to note here that at least two types of boundary effects are possible. One is due to SOL flows, produced by up–down SOL asymmetry (i.e. LSN versus USN) and driven by in-out asymmetry of edge particle transport [39]. The other is due to edge stresses, induced by eddy tilting [41].

3. Towards a fundamental theory: intrinsic rotation as the consequence of a heat engine

Recent work [7, 8] has developed a quite general theory of intrinsic rotation as the output of a heat engine, which exploits a heat flux-driven temperature differential (i.e. locally, a temperature gradient ∇T) to drive turbulence in a bounded domain. Magnetic geometry and boundary effects break symmetry and *total* momentum conservation, so that a net toroidal flow develops. Two heat engines, a car and a tokamak, are compared in table 2. The engine process effectively converts radial inhomogeneity into parallel flow via symmetry-breaking-induced non-diffusive component of the Reynolds stress $\langle \tilde{v}_r \tilde{v}_\parallel \rangle$, as shown in figure 4. The heat engine paradigm was developed to explain the formation of geophysical flows [42] and the solar differential rotation [43] (table 3). Both are prime examples of flows produced by heat flux-driven turbulence.

Here, we summarize the heat engine model, derived from the consideration of fluctuation entropy balance. This

→ Life before intrinsic rotation?
(Pre-History)

- CERS required for $\langle T_i \rangle$, $\langle V_\phi \rangle$ profiles

- first study - S. Scott, et. al, '88 TFTR

$$\frac{\partial \langle V_\phi \rangle}{\partial t} = \underset{\text{beams}}{\underset{\downarrow}{\tau_{ext}}} - \nabla \cdot \underset{\Pi = -\chi_\phi \partial \langle V_\phi \rangle / \partial r}{\underset{\downarrow}{\underline{\Pi}}}$$

Found $\chi_\phi \sim \chi_i$, as would expect from ITG + PVG (recall!)

repeated ad-nauseum .....

- theoretically, for ITG family turbulence, expect

$$\chi_\phi \cong \chi_i$$

but some studies showed significant deviation in ratio:

⇒ suggested something else hiding in

$$\underline{\Pi} = -\chi_\phi \nabla \langle V_\phi \rangle + \text{something else}$$

→ Two related:

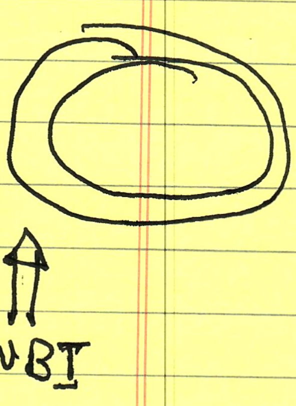
– what is "something else"?

naive bet: Pinch (like particle transport!)

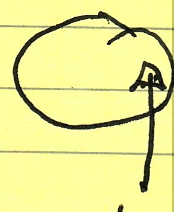
$$\underline{\Pi} = -\chi_\phi \frac{\partial \langle V_\phi \rangle}{\partial r} + V(r) \langle V_\phi \rangle$$

$V < 0$ – momentum pinch (?!)

How study? – off axis modulated NBI (Maiko Yoshida, et al.; JT-60U '08)

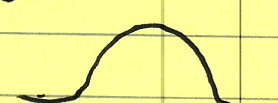


NBI



beam

but



peaked profile

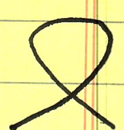
$V < 0$!?

For theory of momentum pinch, see Hahm, et. al., Peeters et. al. '07,

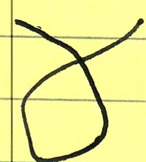
but, its not really/only pinch ....

— Tail wags Dog ? → B. LaBombard 04
(Boundary) (C-Mod)

L-mode:



vs.



→ SOL flow reverses
and
Core intrinsic rotation reverses

LSN USN

c.f. Jog experiments

but LSN vs USN no effect in H-mode

→ ETB cuts the tail off ?!

TBC.

Message: The boundary is dynamic.

The B.C. is non-trivial.

Not simple "no-slip".

→ So What <u>is</u> the "Something Else"?

⇒ $\boxed{\text{Residual Stress}}$ — <u>engine</u>

c.f. "Solar Differential Rotation"
— G. Rudiger

i.e. how convection generates solar differential rotation

i.e. $\boxed{\text{engine}}$

fusion (Power) → Fluid Turbulence → Differential Rotation

fusion (Power) → <u>gas</u>

Fluid Turbulence → <u>burn</u>

Mean field hydro. akin Dynamo Theory → symmetry breaking → <u>cylinder, cam</u>

Differential Rotation → <u>wheel</u>.

<u>So</u>

$Q \rightarrow \boxed{\text{Turbulence}} \rightarrow \boxed{\langle V_{\parallel} \rangle' \sim \Pi_{resid}/\chi_{\phi}}$

symmetry breaking

i.e.

mom. flux

$$\frac{\partial \langle V_\phi \rangle}{\partial t} = -\frac{\partial}{\partial r}\,\Pi_{r,V}$$ [redacted] $+ \ldots$

$\Rightarrow$

residual stress

$$\Pi_{r,V} = -\chi_\phi \frac{\partial \langle V_\phi \rangle}{\partial r} + V\langle V_\phi \rangle + \Pi_{resid}$$

non-diffusive stress

$\not\propto V_\phi,\ \nabla V_\phi$

$\sim \nabla P$.

N.B. For $\langle V_\phi \rangle$, $\frac{\partial \langle V_\phi \rangle}{\partial r} \to 0$,

$$\frac{\partial}{\partial t}\int dr\,\langle V_\phi \rangle = -\Pi_{resid}\Big|_0^{bndry}$$

$\Rightarrow$

$\Pi_{resid}\big|_{bndry} \neq 0$ required to accelerate plasma from rest.

Net spin-up $\Rightarrow$ bndry. | B.C. critical.

What is Π_{resid} ?

i.e. neglecting pinch (delicate),

Reynolds stress

$$\Pi_{r,v_\parallel} = \langle \tilde{v}_r \tilde{v}_{\parallel} \rangle$$

key → Parallel Reynolds stress.

Parallel counterpart $\langle \tilde{v}_r \tilde{v}_\theta \rangle$.

How calculate?

$$\partial_t \hat{v}_{\parallel k} + \frac{\tilde{v}_{\parallel k}}{\tau_{ck}} = -\tilde{v}_{r k} \frac{\partial \langle v_\parallel \rangle}{\partial r} - \frac{i k_\parallel \tilde{p}_{k}}{\rho_0} + \dots$$

χ_ϕ

$$\partial_t \tilde{p}_k + \frac{\tilde{p}_k}{\tau_{ck}} = -\tilde{v}_r \frac{\partial \langle p \rangle}{\partial r} - \rho_0 i k_\parallel \hat{v}_{\parallel k}$$

drives Π_{resid}

$$\Pi_{resid} \sim \langle \tilde{v}_r^2 \rangle_k (\ )\ \partial \langle p \rangle / \partial r$$

Ion Pressure Gradient (ITG) drives intrinsic rotation

Temperature

So

$$\langle \tilde{v}_r \tilde{v}_\parallel \rangle = -\overbrace{\chi_\phi}^{\text{visc.}} \underbrace{\frac{\partial \langle v_\parallel \rangle}{\partial r}} + \overbrace{d_{resid}}^{\text{resid.}} \underbrace{\frac{\partial \langle p \rangle}{\partial r}}$$

$$d_{resid} \sim \sum (\ ) \, k_\parallel |\phi_k|^2$$

converts heat to flow → thermodynamic engine.

→ odd spectral moment.

→ generic

→ requires symmetry breaking!

i.e. select direction $k_\parallel$!

analogy: α-effect in dynamo theory

i.e. net breaking of reflection symmetry.

[Convert radial inhomogeneity to parallel flow]

→ how? [tilting in eddy PIL]

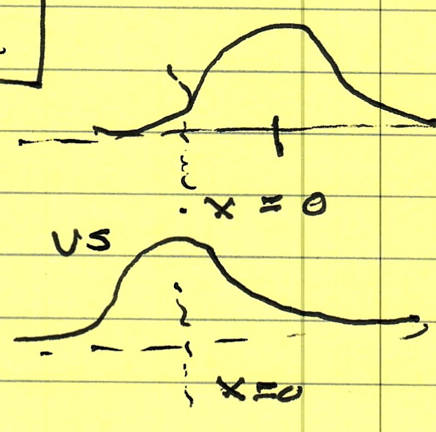
– spectral shift

mean E×B shear

↓

generic E×B shear

ubiquitous



22a.

Table 4. Physics of symmetry breaking mechanisms.

Relevant stress and mechanism	Spatial structure	Key physics	Macro implication
$\langle\tilde{v}_r\tilde{v}_\parallel\rangle$, $\langle v_E\rangle'$ (Electric field shear)	$k_\parallel$ from spectrum *shift* (config.) or *eddy tilt* (ballooning)	Centroid shift induces mean $\langle k_\parallel\rangle$ from parallel acoustic wave asymmetry	$\pi_{res}\sim\langle v_E\rangle'$. Intrinsic torque peaked at barriers, steep gradients π_{res} can flip with mode change
$\langle\tilde{v}_r\tilde{v}_\parallel\rangle$, I' (Intensity gradient) ($I\equiv$ intensity)	$k_\parallel$ from spectra *dispersion* due I'	Spectral dispersion from intensity gradient. Linked to $\perp$ Reyn. stress, also	$\pi_{res}\sim I'$. relevant to barriers but also for more general inhomogeneity. Can change with mode change. Ultimately tied to temp. profile curv.
Stress from polarization acceleration $\langle\tilde{E}_\parallel\nabla_\perp^2\tilde{\phi}\rangle$	$\langle k_r k_\parallel\lvert\phi_k\rvert^2\rangle$ stress due radial + parallel propagation, $(r,\parallel)$ tilting	Guiding centre stress from acceleration due polarization charge $\langle k_r k_\parallel\rangle\neq 0$ needed	As yet unclear. Merits further study. Linked to mode radial group velocity v_{gr} and can flip direction
Stress from $\partial_r\langle\tilde{v}_r\tilde{v}_\perp\rangle$ $\rightarrow\langle J_r\rangle\rightarrow B_\theta\langle J_r\rangle$ $\rightarrow$ toroidal torque	(r,θ) tilting, as for ZF Same physics for ZF	$\boldsymbol{J}\times\boldsymbol{B}$ torque originating from polarization flux $I'\neq 0$, $\langle k_r k_\theta\rangle\neq 0$ needed	$\sim$ universal mechanism, closely related to ZF, tied to I' and I_k structure. Flips with v_{gr}. Merits more study.

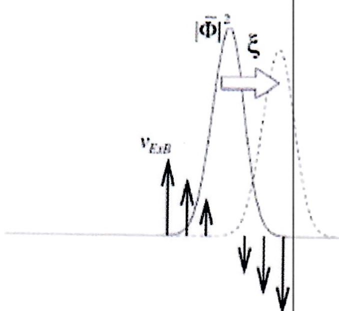


Figure 7. Symmetry breaking by $\langle V_E\rangle'$-induced spectral shift [53]. Finite $\langle V_E\rangle'$ renders the spectral centroid non-zero, and so yields $\langle k_\parallel\rangle$.

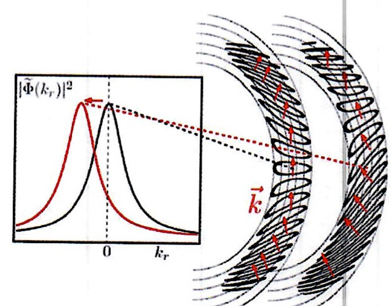


Figure 8. Shifted spectrum in real space and net eddy tilt in ballooning space. Note a Fourier transform directly relates the 'tilted' spectrum in ballooning space to the shifted spectrum in configuration space.

is necessarily proportional to $\langle V_E\rangle'$, and cannot be so large that the underlying shear turns the underlying instability off. The correspondence between the configuration and the ballooning space manifestations of shear flow induced symmetry breaking is shown in figure 8. Note the connection between mean $k_\parallel$ (i.e. $\langle k_\parallel\rangle$) and net eddy tilt. Clearly the real space and ballooning space approaches are equivalent.

A second, *equally important* mechanism for symmetry breaking in $\langle k_\theta k_\parallel\rangle$ is due to spatial spectral dispersion, with finite intensity *gradient* I' [54, 55]. This mechanism does not require a spectral *shift*. Rather, the requisite asymmetry is produced by the spatial profile of intensity. The origin of this effect can be seen from

$$\langle k_\theta k_\parallel|\tilde{\phi}_k|^2\rangle \simeq \left\langle k_y^2\frac{(r-r_0)}{L_s}\left\{|\tilde{\phi}_k|^2+(r-r_0)\frac{\partial}{\partial r}|\phi_k(\tilde{r}_0)|^2+\cdots\right\}\right\rangle \approx \left\langle k_y^2\frac{(r-r_0)^2}{L_s}\frac{\partial}{\partial r}|\tilde{\phi}_k|^2\right\rangle. \quad (7)$$

Figure 9 gives an instructive heuristic sketch related to this mechanism. Note that intensity gradients will surely be steep at the boundary between regions with different confinement properties (for example, at the 'corners', which bound transport barriers where profile curvature is large). Thus, strong intensity gradients will occur near regions with large changes in $\langle V_E\rangle'$. However, one can expect an intensity gradient in

– spectral intensity gradient

⋮

long list

– N.B. net zero flux

$$\frac{\partial \langle V_{\parallel} \rangle}{\partial r} = \frac{d_{res} \; \partial \langle p \rangle / \partial r}{\chi_\phi}$$

$d_{res} \sim \langle V_E \rangle'$

so in barrier, intensity ↓ but

$\dfrac{\partial \langle V_{\parallel} \rangle}{\partial r}$ ↑

(χ_ϕ neo negligible)

– Reversals ?

Change in mode population can force change in residual stress

c.f. McDevitt, P.D.; N. Cao, and many.