Physics 218C

Pred/Prey + Transitions → LH
Intr. Rot.
Mon 8L nol.

Wed – soc. Den Limit
Mon – DWI

Lecture 6: Confinement Transitions, especially Predator-Prey System and L→H Transition

a.) Predator-Prey ↔ Drift Wave-Zonal Flow System.

Recall derived the coupled equations for shear flow and turbulence:

WKE for DW Action Density:

$$\frac{\partial \langle N\rangle}{\partial t} - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N\rangle = \gamma\langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

or equivalently, in terms energy

$$\frac{\partial\langle \mathcal{E}\rangle}{\partial t} = -\int d^3k \left(\frac{\partial \omega_k}{\partial k_r}\right) \underbrace{\boxed{D_{k_r}}}_{O(\langle \tilde{V}^2\rangle)} \frac{\partial\langle N\rangle}{\partial k_r} + \underbrace{\int d^3k\, \omega \langle C(N)\rangle}_{\gamma\langle\mathcal{E}\rangle - \frac{1}{\tau_{NL}}\langle\mathcal{E}\rangle^2} + \boxed{S.T}$$

energy density field.

and, from Reynolds Stress:

Vorticity

$|\phi_q|^2$

Wave population

$$\partial_t |\phi_q|^2 = \Gamma_q \left[ \frac{\partial \langle N \rangle}{\partial k_r} \right] |\phi_q|^2 - \gamma_d |\phi_q|^2$$

ZF growth — ZF drag

$$\Gamma_q |\phi_q|^2 \simeq + \langle \tilde{V}'^2 \rangle$$

Energy conservation is straight forward!

Show this:

$$\partial_t \left[ \int d^3k \, \omega \langle \mathcal{E} \rangle + \sum_q \frac{1}{2} \rho_d |\tilde{V}_q|^2 \right] = 0$$

$+ \boxed{\gamma}$, $\Delta\omega$, $\gamma_d$ terms.

$\gamma(\nabla P, \nabla T, \nabla n)$

akin to energetics in QLT.

2 coupled populations

DW ⟺ "particles"

⟷ ecological pred-prey.

ZF ⟺ "waves" / "Fields"

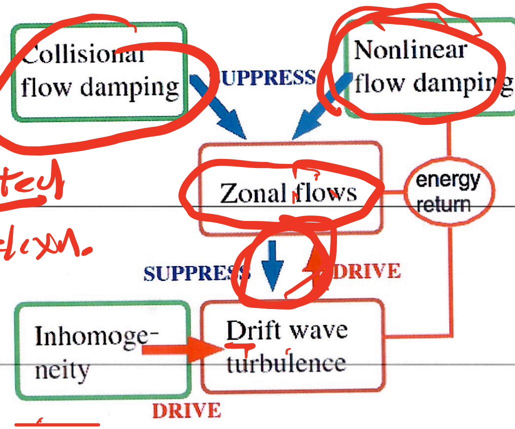
⟹ Coupled system for DW spectrum and ZF spectral Intensity.

# Feedback Loops I

- Closing the loop of shearing and Reynolds work

- Spectral 'Predator-Prey' equations

regulated by Predation.




Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k \langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 \cancel{- \gamma_{NL}[|\phi_q|^2]|\phi_q|^2}$$

$\rightarrow \gamma_{ZF} \rightarrow$ modulational instability

→ Some FAQ's

$\nabla \cdot \underline{J}_{pol}$

- What of Geometry?

$\frac{d}{dt}\rho_s^2 \nabla^2 \phi$

What sets $\underline{J}_{pol}$?

$\nabla \cdot \underline{J}_{pol} \sim$

H-M

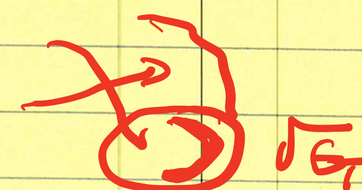
H-W

- FLR → $\rho_s^2 \nabla_\perp^2 \phi^2$ → e.s. screen length scale

torus

or

- Drifts + Particle Trapping




$\nabla B$, curv

Rosenbluth & Hinton

Length scale:

drift velocity

$\delta r \sim V_D \tau_b$

bounce time

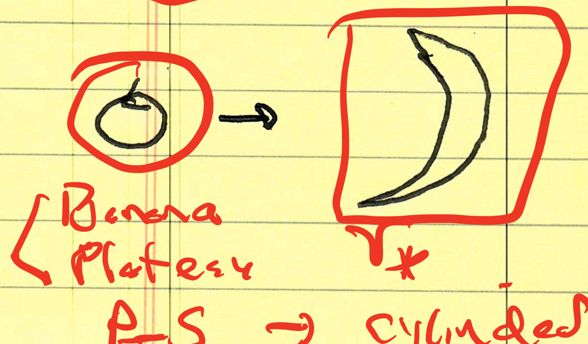
$\sim \frac{\rho_i v_{Ti}}{R} \frac{R\epsilon}{v_{Ti}\sqrt{\epsilon}}$

$\frac{d}{dt}\rho_{\theta i}^2 \nabla_r^2 \phi = \ldots$ ZF

length scale ZF

$\sim \sqrt{\epsilon}\, \rho_{\theta i}$ → poloidal gyro-radius

i.e. $\rho_{\theta i}$ sets polarization screening length




→ . $\rho_{\theta i} \gg \rho_i$

enhanced screening length and inertia

For full analysis, see $\boxed{\text{Rosenbluth and Hinton,}}$ 98.

→ what seeds / triggers Z.F.?

Answer: Nonlinear noise!

i.e. Can write:

$$\frac{\partial}{\partial t}(\nabla_\perp^2\phi)_q \sim -\underbrace{(\tilde{v}\cdot\nabla\nabla^2\tilde{\phi})_q}_{\text{beats of DW at } k}$$

noise field

$\tilde{v}_k \cdot \nabla \tilde{\phi}_{k+q}$

$\tilde{v}_k\cdot\nabla(\nabla^2\phi)_{q-k}$ etc.

then treat as Langevin equation, with $\tau_{c,q}$ set by coherence time of stochastically driven ZF field. → noise

then

$\langle \nabla^2\phi\,\nabla^2\phi \rangle$

$\langle\phi\phi\phi\phi\rangle$

$\langle\phi^2\rangle\langle\phi^2\rangle$

$$\frac{\partial}{\partial t}\langle(\nabla^2\tilde{\phi})^2\rangle \sim \sum_k (\tilde{v}\cdot\nabla\nabla^2\tilde{\phi})_q\,\tau_{c_q}\,(\tilde{v}\cdot\nabla\nabla^2\tilde{\phi})_q$$

5)

eddy Diffusion

$$\boxed{\langle (\nabla^2_\perp \phi)^2 \rangle \sim D t}$$

→ grow linearly in time!

and can add noise term to zonal mode growth eqn

→ can drive flow, absent modulational instability

A+H '98

→ also seeds zonal density

→ See R. Singh, P.D. PPCF '21 for a complete analysis.

Zonal noise is the answer to the question of "What triggers the trigger?".

→ what limits ZF ? / Zonal Mode ?

D, I°

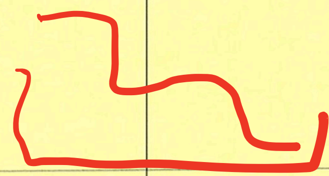
— Tertiary instability

$\omega$

d

thermo.

τ τ τ τ



i.e. DW → Zonal $\xrightarrow{\text{instability}}$ Tertiary

coupling  Modes

primary – secondary – Tertiary

1  2  3

Magnetic shear

i.e. ↓↑ → $\nabla V_{ZF}$ → KH type

or

tertiary → $\xi = 0$

$\xi \sim 1$

$\nabla n_Z$, $\nabla T_Z$ > Drift wave

limit ZF  visc.

Tertiary controversial, especially in magnetically sheared systems.

other? → turbulent viscosity (Li, PD) $\omega \sim \langle V_E \rangle$, $k_\theta$

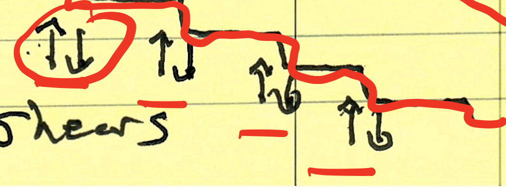
Key Question: How translate into effective NL ZF damping for coupled Spectra – Flow System? → open

→ ongoing  ? Limits Z. Modes

~ll years

→ Staircase – 1D spatial pattern.

steps + shear layer pattern



shears

→ Bistable mixing.

# Provocation: Staircase and Nonlocality
## (with G. Dif-Pradalier, et. al.)

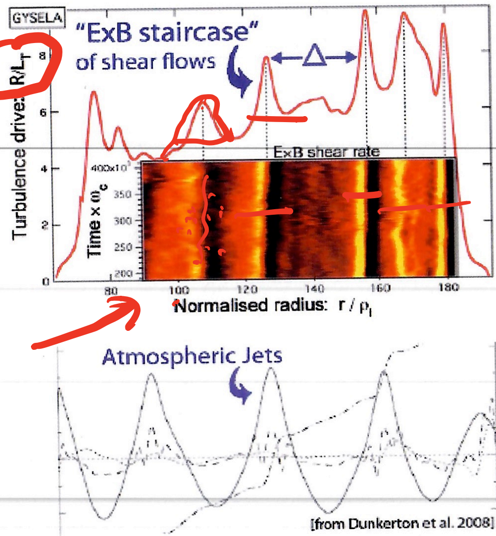
UCSD cea

## Analogy with geophysics: the '$\mathbf{E} \times \mathbf{B}$ *staircase*'

ZF → coord. energy therm.

$\frac{\partial \nabla T_i}{T_i}$

Flux Driven Sims




$$Q = -n\chi(r)\nabla T \;\Rightarrow\; \boxed{Q = -\int \kappa(r,r')\nabla T(r')\,dr'}$$

- '$\mathbf{E} \times \mathbf{B}$ staircase' width $\equiv$ kernel width $\Delta$
- coherent, persistent, jet-like pattern
  ➠ the '$\mathbf{E} \times \mathbf{B}$ staircase'
- staircase NOT related to low order rationals!

Dif-Pradalier, Phys Rev E. 2010

and many follow-ons.

Guilhem Dif-Pradalier APS–DPP meeting, Atlanta, Nov. 2009



10.

# Provocation, cont'd

- The point:
  - fit: $Q = -\int dr' \kappa(r,r') \nabla T(r')$ $\quad \kappa(r,r') \sim \dfrac{S^2}{(r-r')^2 + \Delta^2}$ $\rightarrow$ some range in exponent
  - $\Delta >> \Delta_c$ i.e. $\Delta \sim$ Avalanche scale >> $\Delta_c \sim$ correlation scale
  - Staircase 'steps' separated by $\Delta$ ! $\rightarrow$ stochastic avalanches produce quasi-regular flow pattern!?

    N.B.

    - The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)
    - What IS new is the connection to stochastic avalanches, independent of geometry
  - What is process of self-organization linking avalanche scale to zonal pattern step?

    i.e. How extend predator-prey feedback model to encompass both avalanche and zonal flow staircase? Self-consistency is crucial!

zonal noise

$\partial_t \langle A^2 \rangle = \sum K \langle A^2 \rangle \langle A^2 \rangle - \sum ()^2 \langle E \rangle \langle A^2 \rangle$

8.

model → expt.

# Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high $k_r$ { ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$ { ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work; ⇒ Analogous → inverse energy cascade

Zero D

- Mean Field Predator-Prey Model

(P.D. et. al. '94, DI²H '05)

$$\frac{\partial}{\partial t}N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t}V^2 = \alpha N V^2 - \gamma_d V^2 - \cancel{\gamma_{NL}(V^2)V^2}$$

Reduced Model (0D)

System Status

| State | No flow | Flow ($\alpha_2 = 0$) | Flow ($\alpha_2 \neq 0$) |
|---|---|---|---|
| $N$ (drift wave turbulence level) | $\frac{\gamma}{\Delta\omega}$ | $\frac{\gamma_d}{\alpha}$ | $\frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$ |
| $V^2$ (mean square flow) | 0 | $\frac{\gamma}{\alpha} - \frac{\Delta\omega}{\alpha}\frac{\gamma_d}{\alpha}$ | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$ |
| Drive/excitation mechanism | Linear growth | Linear growth | Linear growth; Nonlinear damping of flow |
| Regulation/inhibition mechanism | Self-interaction of turbulence | Random shearing, self-interaction | Random shearing, self-interaction |
| Branching ratio $\frac{V^2}{N}$ | 0 | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$ | $\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$ |
| Threshold (without noise) | $\gamma > 0$ | $\gamma > \Delta\omega\gamma_d\alpha^{-1}$ | $\gamma > \Delta\omega\gamma_d\alpha^{-1}$ |

DW → Prey

ZF → Pred.

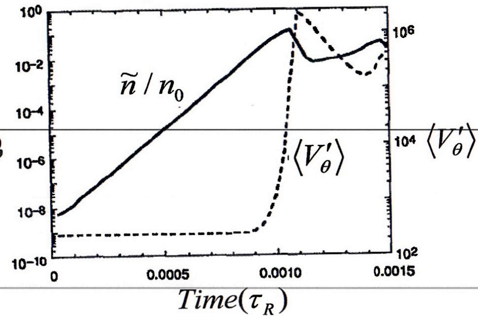
Pred - Prey : Ecology

$V^2 = [\gamma - \Delta\omega\gamma_d ...$

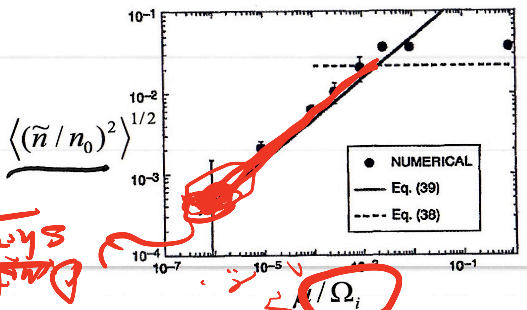
# Feedback Loops II

- Early simple simulations confirmed several aspects of modulational predator-prey dynamics

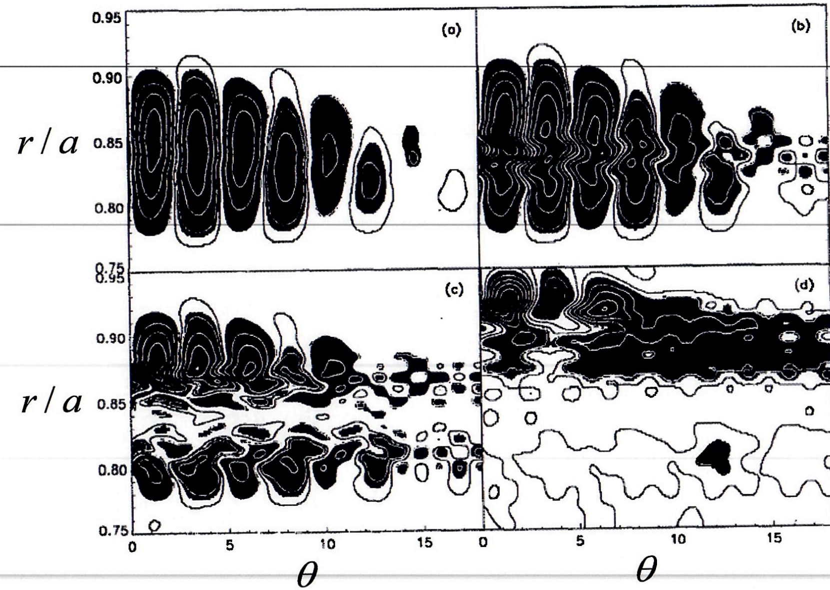
(L. Charlton et. al 94)




Shear flow grows above critical point




'With Flow' and 'No Flow'.
Scalings of $\langle(\tilde{n}/n_0)^2\rangle$ appear. Role of damping evident




Generic picture of fluctuation scale reduction with flow shear



$\mu \to 0 \Rightarrow$ Dimits shift

# Feedback Loops III

- $\nabla P$ coupling
  - $\gamma_L$ drive
  - $\langle V_E \rangle'$

i.e. $\gamma = \gamma(\nabla P)$

$$\partial_t \varepsilon = \varepsilon N - a_1 \varepsilon^2 - a_2 V^2 \varepsilon - a_3 V_{ZF}^2 \varepsilon$$

$$\partial_t V_{ZF} = b_1 \frac{\varepsilon V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF}$$

$$\partial_t N = -c_1 \varepsilon N - c_2 N + Q$$

transport

source, flux

$\mathcal{E} \equiv$ *DW energy*

$V_{ZF} \equiv$ *ZF shear*

$N \equiv \nabla \langle P \rangle \equiv$ *pressure gradient*

$V = dN^2$ *(radial force balance)*

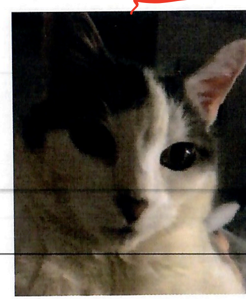
- Simplest example of 2 predator + 1 prey problem (E. Kim, P.D., 2003)

  i.e. prey sustains predators
  predators limit prey } usual feedback

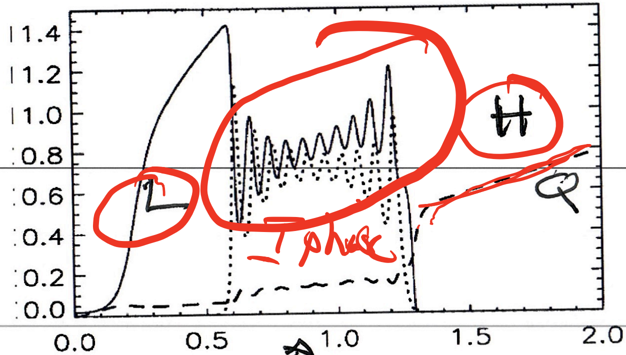
  now: 2 predators ( $ZF$, $\nabla \langle P \rangle$ ) compete
  $\nabla \langle P \rangle$ as both drive and predator

- Relevance: LH transition, ITB
  - Builds on insights from Itoh's, Hinton
  - ZF $\Rightarrow$ triggers
  - $\nabla \langle P \rangle \Rightarrow$ 'locking in'

Multiple predators are possible



# Feedback Loops III, cont'd

- 


I-phase (LCO)

*Solid* - $\mathcal{E}$

*Dotted* - $V_{ZF}$

*Dashed* - $\nabla\langle P\rangle$

— Simple, useful model of L–H transition,

— turbulence extinguished on H,

Causality

- Observations:
  - ZF's trigger transition, $\nabla\langle P\rangle$ and $\langle V\rangle$ lock it in
  - Period of dithering, pulsations .... during ZF, $\nabla\langle P\rangle$ oscillation as Q ↑
  - Phase between $\mathcal{E}$ , $V_{ZF}$, $\nabla\langle P\rangle$ varies as Q increases
  - $\nabla\langle P\rangle \Leftrightarrow$ ZF interaction $\Rightarrow$ effect on wave form

extended.

Intrinsic Rot.

What of Electromagnetics? – Neglected in this course....



# Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

## Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + \underbrace{B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A} + \tilde{f}$$

$$\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega \beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

# Progress II, cont'd

## Observation re: What happens?

- Turbulence → stretch field → $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$ ~~(ala Zeldovich)~~
- Cascades : - forward or inverse?
  - MHD or Rossby dynamics dominant !?
- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v}\tilde{q} \rangle$ ⟶ net change in charge content due PV/polarization charge flux

Now $$\frac{dQ}{dt} = -\int dA \left[ \langle \underbrace{\tilde{v}\tilde{q}}_{\text{PV flux}} \rangle - \langle \underbrace{\tilde{B}_r \tilde{J}_{\parallel}}_{\text{current along tilted lines}} \rangle \right] = -\int dA \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \underbrace{\tilde{B}_x \tilde{B}_y}_{\text{New players}} \rangle \right\}$$ ⟶ Reynolds mis-match ⟶ vanishes for Alfvenized state

PV flux

current along tilted lines

New players

Taylor: $\langle \tilde{B}_x \tilde{J}_{\parallel} \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

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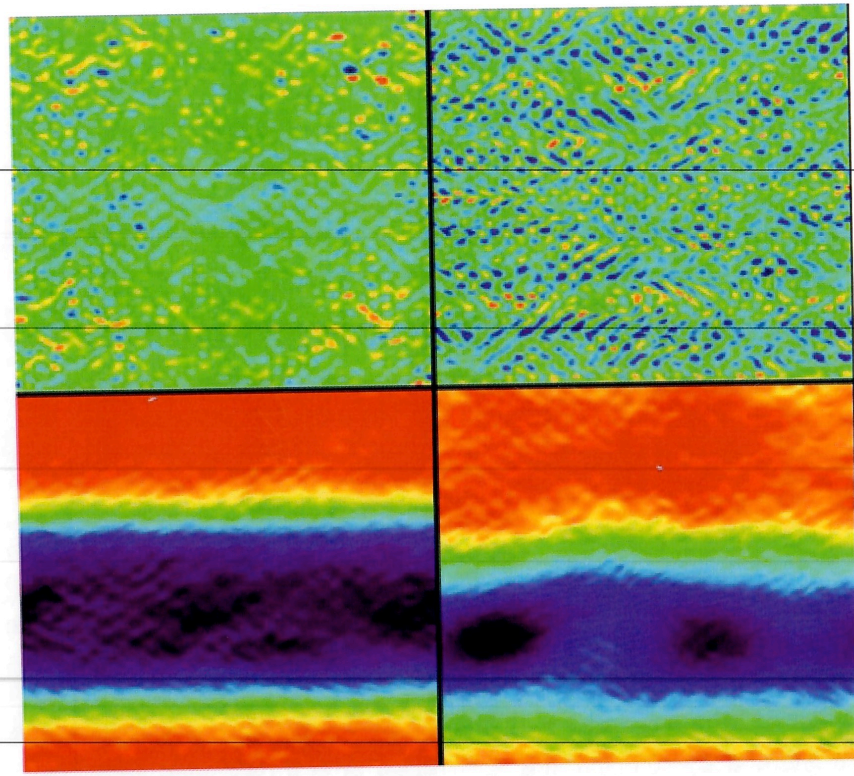
# Progress II, cont'd

- **With Field**



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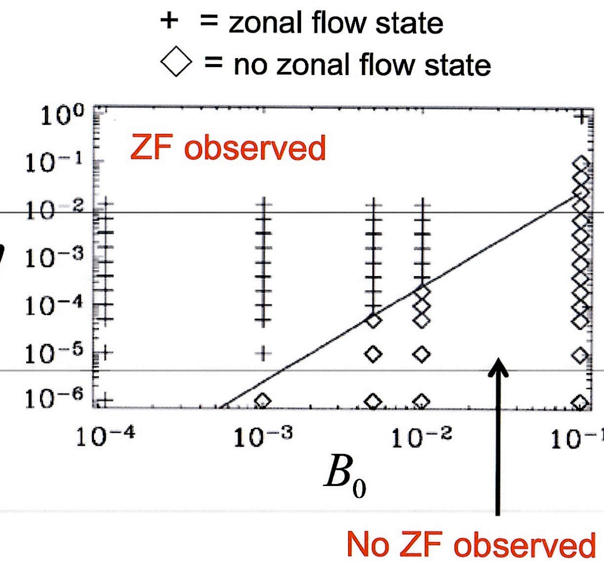
# Progress II, cont'd

- Control Parameters for $\tilde{\vec{B}}$ enter Z.F. dynamics

  ~~Like RMHD~~ Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m$ $\longrightarrow$ origin of $B_0^2/\eta$ scaling !?

- Further study $\rightarrow$ differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution



## b.) L–H Transition

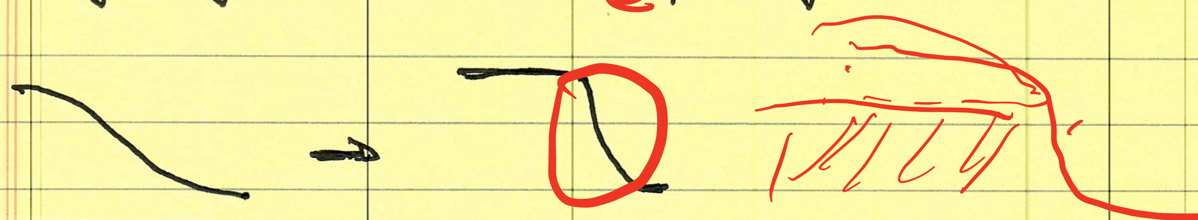
Phenomenology:

Converging from Wagner '82, but not yet fully converged

Caveat Emptor

⇒ LH characterized by:

→ edge gradient steepening



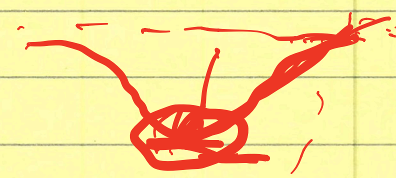
→ pedestal formation

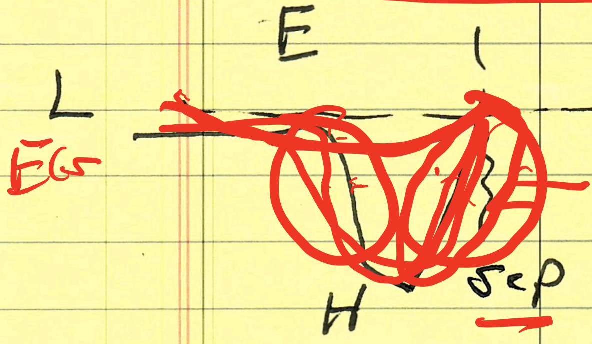
→ confinement improvement

→ fluctuation (low k) drop ↓

High k's persist in pedestal.

→ Increase in E×B shear, formation of Er well





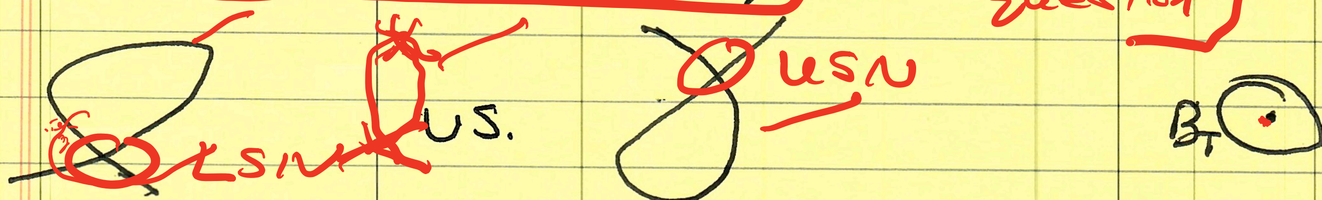

barrier can develop from inner, outer layer. (Schmitz 2021)

⇒ Power threshold $\boxed{(P_{crit})}$

— $\boxed{\text{critical to ITER}}$

→ recall $P_{crit}$ – ε ⊕ Sawtooth
HPP Wagner

→ $\boxed{\nabla B \text{ drift asymmetry}}$ → major question



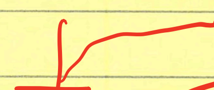

Lower $P_{crit}$ Higher $P_{crit}$

(unclear) cf. Fedorczak et al. 2012

→ $P_{thresh}$ – major concern ITER

$P_{crit}$


related $\boxed{\text{LOC–SOC}}$



flow damping
collisions
n
Radiation
Density
Limit

why? ⇒ coupling to ions

i.e. $\nabla P_i \sim E_r$

Radial F.B.
$F = n \ \underline{V \times B}$

increased n assures stronger

electron-ion coupling ~ $n \nu_e (T_e - T_i)$

flow damping radiation

$P_{output}$ → Radiation ~ $n^2$ $\Delta$ $(T_e - T_i)$

→ trans − $\theta_{ii}$

Some evidence $P_{Th} \sim n B_T$

Current ??

→ isotope ! − lower in D than H, etc.

DIII−D

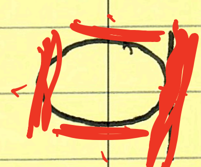
Relation to microphysics unclear.

→ Other points: $P_{Th}$

- universal to all heating method

- limiter and diverted plasmas, but:

→ never in outside limiter



→ $P_{Th}$ higher for limiter plasma.

→ observed in stellarators

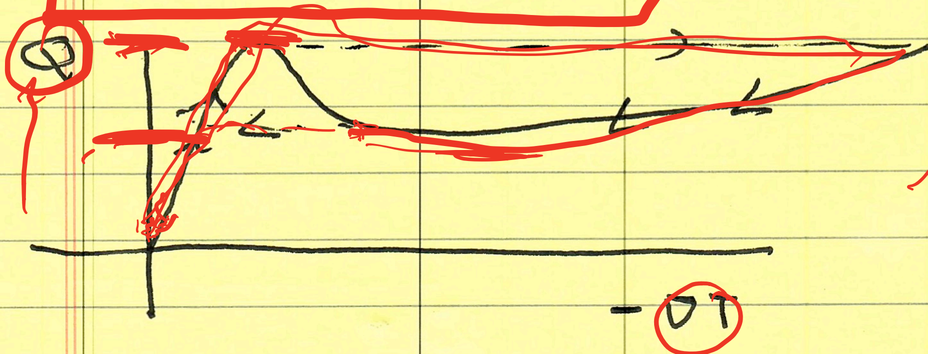
W7AS, TJ II, LHD,

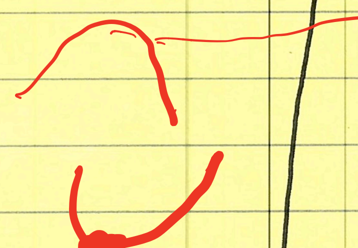
→ RFP ? → QSH ?! Padova / RFX

ITB

$P_{OH}$ big ! → Boundary & — limiter

→ Hysteresis happens



$\rho$

$P_{LH} > P_{HL}$.

Poorly understood → very important.

ELMs

→ $P_{LH} \uparrow$ in RMP plasmas

important for ELM mitigation.

How ?  Trigger + $\nabla P$ Feedback

Energy to Shear Flow (ZF)

↓

$\nabla P$ steepens

↓

Turbulence collapses

Implications for transition thresh, if any ?

→ Multi-step

⇒ Many examples. DIII-D, EAST, TJ-II, AUG, HL-2A, Textor, ....

⇒ A few dissenters .... not many

JFT-2M, AUG, HL-2A   LCO

→ ? Orbit loss ?

⇒ Is there a unique route to transition ?

# The Oscillating Flow Layer Widens Radially (Frequency Decreases) - Steady Flow after Final H-Mode Transition

Bifurcation

A weak ExB flow layer exists in L-mode (L-mode shear layer)

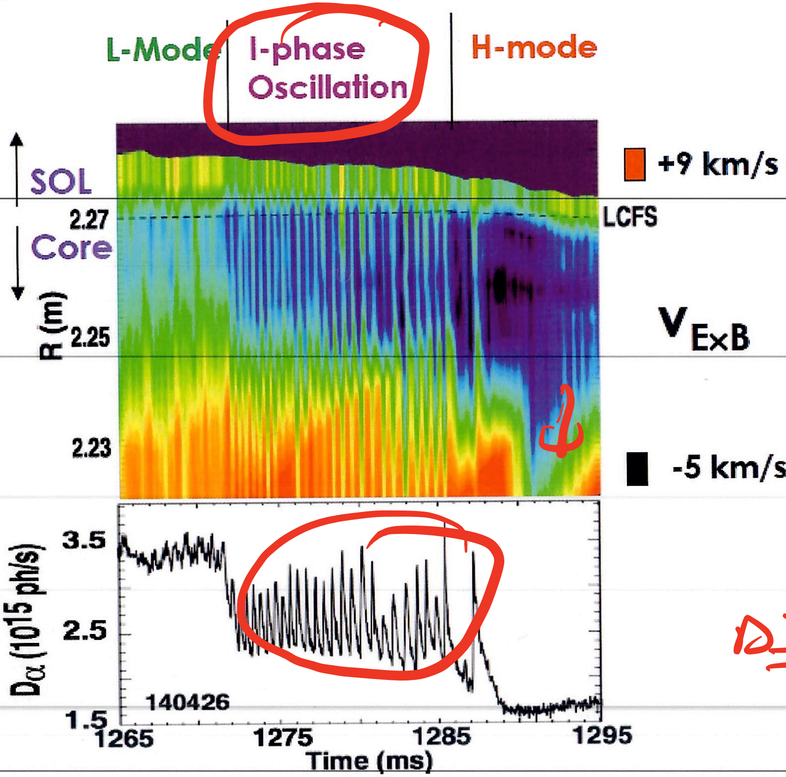
DBS

At the I-phase transition, the ExB flow becomes more negative first near the separatrix, flow layer then propagates inward

time

The flow becomes steady at the final H-mode transition (after one final transient)

LCO




DIII-D NATIONAL FUSION FACILITY SAN DIEGO

L. Schmitz TTF'11, PRL'12



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Intermediate Phase

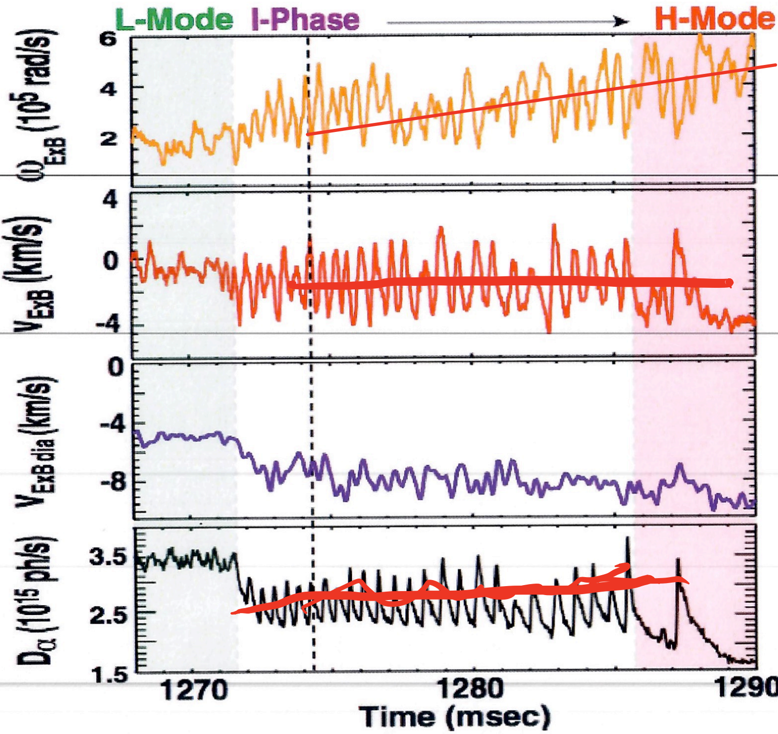
## During the I-phase, the Mean Shear $<\omega_{ExB}>$ Increases with Time and Eventually Dominates

Outer layer Shearing Rate (Mean flow+ ZF)

ExB Flow from DBS (includes ZF)

Diamagnetic component of ExB flow (from ion pressure Profile)

R~2.265m




OP

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