Physics 218C

Lecture 4e: Electron Drift Wave Dynamics and TEM cont'd.

3 players
- Resonant electrons
- Waves
- Zonal Modes

Recall: Energy Theorem for Collisionless Drift Wave

$$\partial_t \underbrace{\mathcal{E}_w}_{\text{wave energy}} = \overset{ⓐ}{-\int d^3x \, \langle \tilde{\phi} \, \partial_t \underbrace{\tilde{h}}_{\text{non-ad. density}} \rangle}$$

$$- \int d^3x \, \underbrace{\langle V_E \rangle'}_{ⓑ} \langle \tilde{v}_{r_E} \tilde{v}_{y_E} \rangle$$

$$\overset{ⓒ}{+ \langle \text{dissp}^n \rangle}$$

ⓐ rate of transfer

ⓐ electron cooling into wave

ⓑ Reynolds Power → power wave stresses exert on flow.

ⓒ Dissp^n.

N.B. If include ion Landau damping:

$$\nabla \cdot \underline{v} \rightarrow \underline{\nabla}_\perp \cdot \underline{v}_\perp + \nabla_\parallel v_\parallel$$

and have:

$$-\int d^3x \, \langle \tilde{\phi} \, \nabla_\parallel \tilde{v}_\parallel \rangle \Rightarrow \text{compressional coupling to wave}$$

contribution to $\partial_t \mathcal{E}_W$.

calculate via:

$$-\int d^3x \, \langle \tilde{\phi} \, \nabla_\parallel \int d^3v \, v_\parallel \tilde{f} \rangle$$

response from ion drift kinetics

show this!

will include:

- Landau damping (ion) contribution to wave energy
- radiative piece:

$$\langle \tilde{\phi} \, \nabla_\parallel \tilde{v} \rangle = \langle \nabla_\parallel (\tilde{\phi} \tilde{v}_\parallel) \rangle - \langle \nabla_\parallel \tilde{\phi} \, \tilde{v}_\parallel \rangle$$

$$\int d^3x \, \nabla_\parallel (\tilde{\phi} \tilde{v}_\parallel) = \int d^2x \, \tilde{\phi} \tilde{v}_\parallel \Big|_{\parallel -}^{\parallel +}$$

$\Rightarrow$ radiation flux (acoustic)

Now, if resonant electrons produce wave energy evolution

- need resonant kinetic electron energy evolution $\Rightarrow$ QL Eqn for Electrons

- Energy balance theorem.

- So, QL Eqn:

$$\frac{\partial f}{\partial t} + v_{\parallel}\,\hat{n}\cdot\nabla f - \frac{c}{B}\nabla\phi\times\hat{z}\cdot\nabla f - \frac{|e|}{m_e}E_{\parallel}\frac{\partial f}{\partial v_{\parallel}} = 0$$

electron DKE

$$\Rightarrow \quad \frac{\partial f}{\partial t} - \frac{c}{B}\nabla\phi\times\hat{z}\cdot\nabla f = 0 \qquad \Rightarrow \text{@ GC Plasma}$$

$$+$$

$$\frac{\partial f}{\partial t} + v_{\parallel}\,\hat{n}\cdot\nabla f - \frac{|e|}{m_e}E_{\parallel}\frac{\partial f}{\partial v_{\parallel}} = 0 \qquad \Rightarrow \text{1D along lines.}$$

add $\underbrace{\omega_D}_{\text{magnetic drift}}\cdot\nabla f$ in torus

Then,

2D scattering operator

$$\Downarrow$$

$$\tilde{f}_{k,\omega} = \frac{\hat{\phi}_{k,\omega} \, L_k \langle f \rangle}{-i(\omega - k_\parallel v_\parallel)}$$

$$L_k = -i k_\parallel \frac{|e|}{m_e} \frac{\partial}{\partial v_\parallel} - i \frac{c}{B_0} k_\theta \frac{\partial}{\partial r}$$

and, anticipating resonance $\pi \delta(\omega - k_\parallel v_\parallel)$

$$= -\left[i \frac{\omega_k}{v_\parallel} \frac{|e|}{m_e} \frac{\partial}{\partial v_\parallel} + i \frac{c}{B} k_\theta \frac{\partial}{\partial r} \right]$$

then

from parallel accel.

$$\boxed{\tilde{f}_{k,\omega} = \frac{|e| \hat{\phi}_k}{T} \left(1 - \frac{(\omega - \omega_*)}{\omega - k_\parallel v_\parallel} \right) \langle f \rangle}$$

usual form

$\omega - \omega_* < 0$ for instability

→ fluctn gains more from relaxn in space than lost by heating then $\tilde{h} \cong \dfrac{i\pi(\omega-\omega_t)}{|k_\parallel v_{th}|} \langle F \rangle \Big/ \dfrac{\omega}{v_{th}} \; \dfrac{|e|\tilde{\phi}_a}{T}$

$\Downarrow$

transit rate thru mode

$\Rightarrow (\omega-\omega_t)\tilde{\tau}_\parallel \ll 1$

EDW weak, usually.

For QL and energetics:

$$\frac{\partial \langle f \rangle}{\partial t} + \frac{\partial}{\partial r}\langle \tilde{v}_r \tilde{f} \rangle + \frac{\partial}{\partial v_\parallel}\left\langle \frac{-|e|}{m_e} \tilde{E}_\parallel \tilde{f} \right\rangle = 0$$

and can integrate:

$$\frac{\partial}{\partial t} \mathcal{E}_p + \partial_r Q_r - \langle \tilde{E}_\parallel \tilde{J}_{\parallel e} \rangle = 0$$

$\mathcal{E}_p$: electron

$\Downarrow$ electron KE Density

$\Downarrow$ Turbulent electron heat flux

$\Downarrow$ turbulent heating/cooling of electrons to wave

And integrating:

$$\underset{\text{electron}}{\partial_t E_p} + Q_r\Big| - \int d^3x \, \langle \tilde{E}_\parallel \tilde{J}_{\parallel e} \rangle = 0$$

heat flux at bndry

volume integrated KED

But also know Wave Energy Theorem:

$$\partial_t \mathcal{E}_w + \partial_r S_{r\,\text{wave}} + \langle \underline{E} \cdot \underline{J} \rangle = 0$$

WED

wave energy density flux

wave-particle coupling

but:

$$\langle \underline{E} \cdot \underline{J} \rangle = \langle E_\parallel J_\parallel \rangle + \langle \underline{E}_\perp \cdot \underline{J}_\perp \rangle$$

electron cooling to wave ($J_\parallel \sim J_{\parallel e}$)

Reynolds power to ZF

NL polarization drift

$$\overset{\downarrow}{\langle E_\perp \cdot J_\perp \rangle} = \int d^3x \, \langle v_E \rangle \langle v_{Er} v_{\theta E} \rangle$$

Show: How related to "resonance"?
⇒ origin irreversibility

Q:

$$\underbrace{S_r \Big|_{\text{boundary}}}_{\text{wave energy density flux thru bndry}} + \underbrace{\int \langle E_\perp \cdot J_\perp \rangle d^3x}_{\text{Reynolds power to ZF}} = \underbrace{-\int d^3x \, \langle E_\parallel J_{\parallel e} \rangle}_{\text{electron cooling.}}$$

N.B. Estimate $\Big|_0$

If $S_r \Big|_{\text{bndry}} \to 0$ then:

$$\int d^3x \, \langle E \cdot J \rangle = 0$$

↓

steady state energy balance.

i.e. electron cooling (to wave) = Reynolds power (to ZF) and, $\langle Q_r \rangle \Big|_{\text{bndry}} = -\int d^3x \, \langle E_\perp \cdot J_\perp \rangle$

$\Rightarrow$ $\boxed{\text{heat flux thru bndry} \leftrightarrow \text{Reynolds Power, integrated.}}$ ✓

N.B.: Need address ZF damping to obtain ZF eqn. and stationary state ZF.

For QL eqn:

$$\frac{\partial \langle f_e \rangle}{\partial t} = \sum_k L_{\underline{k}} |\phi_k|^2 \left[\frac{i}{\omega - k_\parallel v_\parallel} \right] L_{\underline{k}} \langle f_e \rangle$$

$$= \partial_r D_{r,r} \partial_r \langle f \rangle + \partial_r D_{r,v_\parallel} \partial_{v_\parallel} \langle f \rangle$$

$$+ \frac{\partial}{\partial v_\parallel} D_{v_\parallel, r} \frac{\partial}{\partial r} \langle f \rangle + \frac{\partial}{\partial v_\parallel} D_{v_\parallel, v_\parallel} \frac{\partial \langle f \rangle}{\partial v_\parallel}$$

$\rightarrow$ 2D QL evolution

$\rightarrow$ Why ?

where:

$$\rightarrow \pi\delta(\omega - k_\parallel v_\parallel)$$

$$D_{r,r} = \text{re}\sum_k \frac{c^2}{B_0^2} k_\theta^2 |\phi_k|^2 \frac{i}{[\omega - k_\parallel v_\parallel]}$$

Basic
E×B
diffusivity $= \text{re}\sum_k k_\theta^2 \rho_s^2 c_s^2 \left|\frac{|e|\phi_k}{T}\right|^2 \frac{i}{[\omega - k_\parallel v_\parallel]}$

$$\sim \langle \tilde{v}_r^2 \rangle \tau_{T_k}$$

$$D_{v,v} = \text{re}\sum_k \frac{|e|^2}{m_0^2} k_z^2 |\phi_k|^2 \frac{i}{[\omega - k_\parallel v_\parallel]}$$

$\sim$ like ω

n.b. $k_\parallel \ll k_\perp$

and cross terms:

$$D_{r,v} = D_{v,r} = \text{re}\sum_k k_\parallel (k_\theta \rho_s c_s) v_{th_0} \left|\frac{e\phi_k}{T}\right|^2 \frac{i}{[\omega - k_\parallel v_\parallel]}$$

n.b. $D_{r,v} = D_{v,r} \neq 0$

requires: $\langle k_\theta k_\parallel \rangle \neq 0$ $\rightarrow$ spectral asymmetry (mediated by resonance).

What is the validity ?

Recall, for QL type $\left\{ \begin{array}{l} \text{transport} \\ \text{relaxation} \end{array} \right.$

$$1/\tilde{\tau}_{ac} > 1/\tau_{Tr}$$

Now, for $i/\omega_k - k_\parallel v_\parallel$ type resonance

$$1/\tilde{\tau}_{ac} \simeq \left| \frac{d\omega}{dk_z} - \frac{\omega}{k_z} \right| \Delta k_z + \left| \frac{d\omega}{dk_\theta} \right| \Delta k_\theta$$

but $d\omega/dk_z \to 0$ for drift waves

$$\omega_*/\omega - 1 - k_\perp^2 \rho_s^2 + \underbrace{\frac{k_\parallel^2 c_s^2}{\omega^2}}_{} = 0$$

so for resonant particles,

$$1/\tilde{\tau}_{ac} \cong \underbrace{\left| \frac{\omega}{k_z} \right| \Delta k_z}_{v_{Th} \Delta k_z} + \left| \frac{d\omega}{dk_\theta} \right| \Delta k_\theta$$

$\sim v_{*d} \Delta k_\theta$, except: $k_\theta \rho_s \sim 1$

Multi-dimensional decorrelation more robust than 1D

$$1/\tilde{\tau}_{ac} = \left| \frac{\omega}{k} - \frac{d\omega}{dk} \right| \Delta k. \quad \to \text{dispersion sensitive.}$$

Compare to: $1/\tilde{\tau}_{tr} \sim k_\perp \tilde{V}_\perp$

$$\sim \frac{c}{B}\tilde{\phi}\, k_\theta k_\perp$$

$$\sim k_\perp c_s k_\theta \rho_s \frac{|e|\tilde{\phi}}{T}$$

$$\left|\frac{d\omega}{dk_\theta}\right| \Delta k_\theta \simeq k_\perp c_s k_\theta \rho_s \frac{|e|\tilde{\phi}}{T}$$

$$\frac{\cancel{c_s}\cancel{\rho_s}}{L_n} \Delta k_\theta > k_\perp \cancel{c_s} k_\theta \cancel{\rho_s} \frac{|e|\tilde{\phi}}{T}$$

$$\boxed{\frac{|e|\tilde{\phi}}{T} < \frac{\Delta k_\theta}{k_\theta} \; 1/k_\perp L_n}$$

Interestingly, if $\Delta k_\theta \sim k_\theta$,

$$\frac{|e|\tilde{\phi}}{T} \sim \frac{\tilde{n}}{n} < 1/k_\perp L_n$$

But $\quad k_u \sim \frac{\tilde{v}\,\tau_c}{\Delta}$

$$\sim \frac{T}{|q|}\frac{c}{B} k_\theta \frac{|e|\tilde{\phi}}{T} \frac{1}{\Delta\omega} [k_\perp]$$

$$Ku \sim \cancel{\;} k_0 \cancel{\rho_s} \frac{|e\tilde{\phi}|}{T} \frac{1}{\Delta k \cancel{\rho_s} / L_n} |k_\perp|$$

$$\sim \frac{k_0}{\Delta k_\perp} \frac{|e\tilde{\phi}|}{T} |k_\perp| L_n$$

and $Ku < 1$ → diffusive approx.

$\Downarrow$

$$\left(\frac{e\tilde{\phi}}{T}\right) < \frac{1}{|k_\perp| L_n} \frac{\Delta k_\perp}{k_0}.$$

Need $Ku < 1$ ⇒ sub-MLT levels

$Ku \sim 1$ ⟺ "MLT level"

($n.b$ $\tilde{v} \cdot \nabla \tilde{n} \sim \tilde{v} \frac{\partial \langle n \rangle}{\partial r}$)

- obscure, as parallel streaming sets response time.

- MLT and auto-correlation constraint on ITG Levels ?
  What is "mixed" ?
  ~ exercise.

→ Evolution

How does $\langle f \rangle$ evolve to turn off growth?

Consider:

$$\partial_t \langle f \rangle = \sum_{\underline{k}} L_{\underline{k}} |\phi_{\underline{k}}|^2 \underbrace{R(\omega - k_\parallel v_\parallel)}_{\text{resonance}} L_{\underline{k}} \langle f \rangle$$

then,

$$\partial_t \int \frac{\langle f \rangle^2}{2} d^3v \, dr$$

$$= -\sum_{\underline{k}} |\phi_{\underline{k}}|^2 \int R(\omega - k_\parallel v_\parallel) \frac{\left(L_{\underline{k}} \langle f \rangle \right)^2}{2}$$

so $\langle f \rangle$ evolves until

$$L_{\underline{k}} \langle f \rangle = 0 \qquad \Rightarrow \left\{ \begin{array}{l} \text{QL} \\ \text{plateau} \end{array} \right.$$

$$L_k\langle f\rangle = -i\left[k_\parallel \frac{|e|}{m_e}\frac{\partial\langle f\rangle}{\partial v_\parallel} + \frac{c}{B}k_\theta\frac{\partial}{\partial r}\langle f\rangle\right]$$

$$\approx \left[\frac{|e|}{m_e}\frac{\omega}{v_\parallel}\frac{\partial\langle f\rangle}{\partial v_\parallel} + \frac{c}{B}k_\theta\frac{\partial\langle f\rangle}{\partial r}\right]$$

so $L_k\langle f\rangle \approx 0$ (for $\partial_t\langle f\rangle = 0$)

defines level curves of $\langle f\rangle$

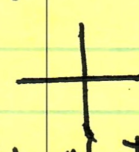
$$\frac{|e|}{m_e}\frac{\omega_k}{v_\parallel}\frac{\partial\langle f\rangle}{\partial v_\parallel}+\frac{c}{B}k_\theta\frac{\partial\langle f\rangle}{\partial r}\approx 0$$

$$\frac{c}{B}k_\theta\frac{\langle f\rangle}{\Delta r} + \frac{|e|\omega}{m_e v_\parallel}\frac{\langle f\rangle}{\Delta v_\parallel} = 0$$

$$\Delta r - \frac{k_\theta v_\parallel \Delta v_\parallel}{\omega_k \Omega_{ce}} = 0$$

$$\boxed{x - \frac{k_\theta v_\parallel^2}{2\omega_k\Omega_{ce}} = \text{const}}$$

defines



QL plateau.

- saturated $\langle f\rangle$ const along above curve.

– analogue of plateau in 1D.

⇒ Physics:

– imposes constraint on transport

↔ heating relation (reson. $\leftarrow (\omega - \omega_k)$)

– $$\boxed{\delta x \simeq \frac{k_\theta\, \delta V_{\parallel}^2}{2\,\cancel{\omega_k}\,\Omega_{ce}}}$$ displacement ↔ heating relation.

→ electrons @ adiabatic heating limited

$$\cong \frac{\cancel{k_\theta}\, \alpha V_{The}^2}{\cancel{k_\theta}\, c_s\, \Omega_e / L_n}$$

$\alpha \ll 1$
as τ_T short.

$$\simeq \left[(\alpha V_{The}^2) / c_s^2 \right] \frac{m_e}{m_i} L_n$$

$$\boxed{\delta x \cong \alpha L_n}$$

so excursion $\ll L_n$.

→ Comments:

- transport accompanied by heating

- QL plateau saturation weak

- plateau reset by collisions or external pumping / heat input avoids plateau.

- cross-terms essential.

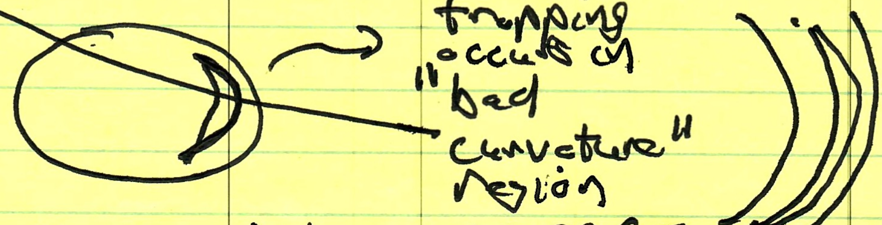
→ More generally, saturation is via zonal mode pumping and nonlinear ion heating.

→ EDW is prototype.

→ Trapped Electron Modes

→ TEM are electron drift waves, with
→ dissipative (mostly)
separate responses for trapped, circulating electrons.

→ Trapping : Variation B
$1/R$



trapping occurs on "bad curvature" region

- Particles with
  ⇒ trapped, on outside
  $v_{\parallel}^2 \lesssim \frac{2E}{M} - \mu B$
- bounce back and forth, toroidally
- electrons → negligible banana width
- $\frac{n_{Tr}}{n_0} \sim \sqrt{\epsilon}$ show!

trapped correspond to a fraction of the population.

→ Why important?

– trapped particles (electrons, here) have longer DW – electron coherence time

– larger contribution to growth!

→ more 'fluid' response.

– TEM, can in some cases resemble an interchange (hydro/reactive). Usually not relevant.

→ Its all in the time scales:

ω, ω_{De}, ν_{ee}, $k_\parallel v_\parallel$, ω_{be}

- ω – wave frequency
- ω_{De} – magnetic drift frequency $\left(\frac{k_\theta \rho_s c_s}{R}\right)$
- ν_{ee} – electron-electron collisions – de-trap
- $k_\parallel v_\parallel$ – streaming → bounce frequency for trapped
- $\omega_{be} \sim \sqrt{\epsilon}\,\omega_{Te}$

⇒ Ordering: Point is to average over fast time scale for T.P.

$\omega_{bo} > \omega > \omega_{De}, \nu \Rightarrow$ TEM

ω_{bo}: fast time scale

$\omega \sim \omega_{*e}$, as usual.

ω_{De} → drift frequency $\sim \epsilon$, energy

then:

$\omega_{De} > \nu \Rightarrow$ <u>C</u>ollisionless TEM

$\nu > \omega_{De} \Rightarrow$ <u>D</u>issipative TEM

⇒ Averaging:

instead: z, y, x

⇒ η, x / r → radial, y → binormal

η: along field line

→ y, θ is good enough.

then write eqn for $g = f - \frac{|e|\hat{\phi}}{T} F(\epsilon)$

g: non-adiabatic distribution

as:

Electron DKE:

$$-i(\omega - \omega_D(\theta))\hat{g} + \frac{v_\parallel}{R q}\frac{\partial \hat{g}}{\partial \eta} + \nu \hat{g}$$

$$= i\frac{|e|}{T}(\omega - \omega_*)\hat{\phi}\langle f\rangle$$

N.B.: η is Fourier Transform variable for $k_\parallel$.

shear $\rightarrow k_\parallel = k_\parallel(x)$

l.o.:

$$v_\parallel \frac{1}{Rq}\frac{\partial g^{(0)}}{\partial \eta} = 0$$

$g^{(0)}$ indep η

$$g = g^{(0)} + g^{(1)} + \cdots$$

1st order:

$$-i(\omega - \omega_D(\theta))\hat{g}^{(0)} + \frac{v_\parallel}{Rq}\frac{\partial}{\partial \eta}\hat{g}^{(1)} + \nu \hat{g}^{(0)}$$

$$= i\frac{|e|}{T}(\omega - \omega_{*e})\hat{\phi}\langle f\rangle$$

$$\int_{-\eta_b}^{\eta_b} d\eta + \int_{\eta_b}^{-\eta_b} d\eta \equiv \langle\ \rangle = \oint \frac{d\ell}{|v_\parallel|}$$

bounce average

$$-i\left(\omega - \overline{\omega_D(\theta)}\right)\langle g\rangle + \cancel{\left\langle \frac{v_\parallel}{R_0}\frac{\partial}{\partial\eta} g^{(1)} \right\rangle}$$

$v_\parallel$ flips

$$+ \nu\langle g\rangle = i\frac{|e|}{T}(\omega - \omega_*)\langle\phi\rangle\langle f\rangle$$

→ Bounce avg kinetic eqn !

⇒ bounce avg ϕ

$$\langle g\rangle^{(0)} = \frac{i|e|}{T_e}\frac{(\omega - \omega_*)\langle\phi\rangle\langle f\rangle}{-i\left(\omega - \overline{\omega_D(\theta)} + i\nu\right)}$$

and

$$\tilde{n}_{\substack{Tr\\NA}} \cong \int_{trapped} d^3v\, \langle g\rangle$$

strictly $\langle\phi\rangle \neq \phi$
⇒ eigenmode eqn. integral
→ n is good quantum #

$$\tilde{n}_{\substack{circ.\\NA}} \cong \frac{|e|\phi}{T}\frac{(\omega-\omega_*)\langle f\rangle}{|k_\parallel v_{th}|} \to 0$$

usually

i.e. small in $\omega/|k_\parallel v_{the}|$

so

$$\left[\frac{\omega_*}{\overline{\omega}} - k_\perp^2 \rho_s^2\right]\hat{\phi} = \hat{\Phi} - \int_{tr} d^3v \left[\frac{(\omega-\omega_*)\,\langle f\rangle}{(\omega - \overline{\omega}_D(\epsilon) + i\nu)}\right]\langle\phi\rangle$$

→ CTEM : $\overline{\omega}_D > \nu_{eff}$, $\langle\phi\rangle = \phi$

$\sqrt{\epsilon}$ → fraction trapped

$$\frac{\omega_*}{\omega} - k_\perp^2\rho_s^2 = \left[1 + \sqrt{\epsilon}\,\frac{i\pi(\omega-\omega_*)}{|\overline{\omega}_D|}\, e^{-\frac{\omega}{\overline{\omega}_D}}\sqrt{\frac{\omega}{\overline{\omega}_D}}\right]$$

$\sim(\omega-\omega_*)\tau_c$

tail of energy distribution

τ_c → drift time weighted

⇒ long correlation

N.B. $\frac{\gamma}{\omega} \sim \left[\frac{\omega-\omega_*}{|\omega_D|}\right] e^{-R/L_n}\sqrt{R/L_n}$ { from d^3v. DT weighted mode

CTEM is EDW, but more virulent.

Compare to $\left[\frac{|(\omega-\omega_*)|}{|k_\parallel v_{th}|}\right]$

Exercises:

- calculate χ_e, D_n for CTEM (QL)

- Calculate CTEM correction to ITG growth.

- Calculate ITG-driven particle flux using CTEM response

N. B. CTEM nonlinear evolution is prime candidate for study of strong wave-particle interaction. Why?

i.e. $\Delta |(\omega - \omega_{DG})|$ → dispersion on wave-particle resonance

$$= \frac{d\omega}{dk_\theta} \Delta k_\theta - \Delta \overline{\omega}_D G$$

resonant G

$$= \left| \frac{d\omega}{dk_\theta} - \overline{v}_D \frac{\omega}{\overline{\omega}_D} \right| \Delta k_\theta$$

$$= \left| \frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta} \right| \Delta k_\theta$$

- $1/\tau_{ac} \sim |V_{gr_0} - V_{phw_0}| \Delta k d$

→ modest dispersion ⇒

- similar to 1D resonance
  i.e. $1/\tau_c = |V_{gr} - V_{ph}| \Delta k_0$

⇒ DTEM : $\nu_{eff} > \overline{\omega_D}$

$$\frac{\gamma}{\omega} \sim \left[\frac{|\omega - \omega_*|}{\nu_{eff}} \right] \cong 1/n$$

(post pellet C-Mod)

- collisions shift η, ϕ phase → destabilize. Like H-W

Cross-over: ω_D vs ν_{eff}

$\sim T^2$ → regime sensitive.