Physics 218c

Lecture 4d → Dynamics of Collisionless Drift Waves and Trapped Electron Modes

Energetics

- Previously:
  → constructed drift wave and HW drift instability model via PV and its evolution/transport

  → Key Feature: Emergence of Zonal Flow (and zonal corrugation) as sink of fluctuation energy.

i.e. zonal structures (flow, corrugation) are:
- key element of mode spectrum
  i.e. $k_\theta, k_r, k_\parallel$ → waves

  k_r → zonal structures (defined by symmetry).

- why? Zonal modes are modes of minimal:
  - → inertia ($k_\perp^2 \rho_s^2$ vs. $1 + k_\perp^2 \rho_s^2$)
  - → transport → ZERO, by symmetry
  - → damping

  (weak damping → easily coupled to)

Now:

→ Collisionless DW + TEM

- resonant electrons + drive $\begin{cases} \omega - \omega_* < 0 \\ \text{n.b. } \nabla T \end{cases}$

i.e. $\omega = k_\parallel v_{Th}$

$$\omega = \overline{\omega}_D \, \epsilon \quad \text{(CTEM)}$$

ϵ → normalized energy

$\overline{\omega}_D$ → drift frequency

- in principle, resonant ions and ion Landau damping.

⇒ stability

flows → Intrinsic rotation.

So { Collisionless DW ? ('Form from Function')
① How understand energetics ⇒
with resonant particles effects on
Zonal structures ?

② Beyond collisionless drift waves

→ TEM (Collisionless DW 'variant' with $|k_\parallel v_{the}| = 1/\tau_T$ → $|\omega_D| = 1/\tau_{Drift}$ ⇒ Larger wrap correlation) (universal)

Why ?

- TEM is ∇n, ∇T_e driven ⇒ active when ITG stable

- Trapped electrons couple to ITG drive

- TEM limits ∇n, particle confinement.

- α's 'slow down on' / heat electrons ⇒ in burning plasma, electrons (bulk) heated preferentially.

N.B. electrons → ions < collisions (density) / collisionless transfer (TBD)

– so – ITG also relevant to Burning Plasma .....

Aside: Background on Energetics

– 1D Mean – Fluctuation Interaction

⇒ QLT

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} D \frac{\partial}{\partial v} \langle f \rangle$$

$$D = \sum_{\substack{k \\ \uparrow \\ \text{modes}}} \frac{q^2}{m^2} |E_k|^2 \left\{ \underbrace{\pi \delta(\omega - kv)}_{\text{resonant}} + \underbrace{\frac{|\gamma_k|}{\omega_k^2}}_{\text{non-resonant}} \right\}.$$

need: – island (phase space) overlap

– to validate use of unperturbed orbits

$$k \, \Delta(\omega/k) > 1/\tau_b.$$

and 2 key energy theorems:

$$\partial_t(\text{TWED}) + \partial_t(\text{RPKED}) = 0$$

Total Wave Energy Density

Resonant Particle KED

or

$$\partial_t(\text{TEED}) + \partial_t(\text{PKED}) = 0$$

total electric energy density

total particle kinetic energy density

Related to; 'Poynting' Thm:

$$\partial_t(\text{TWED}) + \nabla\cdot(\text{WEDF}) + \langle \underline{E}\cdot\underline{J} \rangle = 0$$

$$\text{TWED} = \omega_{\underline{k}} \frac{\partial \epsilon}{\partial \omega}\bigg|_{\omega_{\underline{k}}} \frac{|E_{\underline{k}}|^2}{8\pi}$$

$$\text{WEDF} = \underline{V}_{gr} \frac{|E_{\underline{k}}|}{8\pi}\left(\omega_{\underline{k}} \frac{\partial \epsilon}{\partial \omega}\bigg|_{\omega_{\underline{k}}}\right)$$

So expect for DW:

$$\partial_t(\text{TWED}) + \partial_t\overset{\text{electrons}}{(\text{REED})} + \partial_t\overset{\text{ions}}{(\text{RIED})} + \partial_t\underset{\text{zonal}}{(\text{ZED})} = 0$$

expect: $\partial_t \text{REED} < 0$ → resonant electrons drive wave

$\partial_t \text{RIED} > 0$ → resonant ~~[illegible]~~ ions damp wave.

$\partial_t(\text{ZED}) > 0$ → zonal modes damp wave.

n.b. ZF damping! ⇒ ZF level.

– Fluid electrons and ions support wave. (Boltzmann) ⇒ TWED.

– $\underline{3}$ constituent model
(also relevant EN)

- → resonant particles
- → modes
- ⇒ zonal structures

Historically: – Waves + Resonant Particles → Sagdeev + Galeev

– Waves + Zonal Flows → P.D. et. al. Reviews

② Zonal – Mean Conservation

Consider H-W Model:

$$\rho_s^2 \frac{d}{dt} \nabla_\perp^2 \phi = -D_\parallel \nabla_\parallel^2 \left(\tilde{\phi} - \frac{T}{|e|} \frac{\tilde{n}}{n} \right) + \text{visc}$$

$$\frac{1}{n_0} \frac{d}{dt} n + \tilde{V}_r \partial_r \frac{\langle n \rangle}{n_0} = -D_\parallel \nabla_\parallel^2 \left(\tilde{\phi} - \frac{T \tilde{n}}{|e| n_0} \right) + \text{diffn}$$

so, for zonal modes:

$$\underbrace{\frac{\partial}{\partial t} \left\langle \frac{\delta n}{n} \right\rangle}_{\text{zonal density}} + \frac{\partial}{\partial r} \underbrace{\left\langle \tilde{V}_r \frac{\tilde{n}}{n_0} \right\rangle}_{\text{waves}} = \langle \text{diffn} \rangle$$

$$\rho_s^2 \partial_t \langle \nabla_r^2 \phi \rangle + \frac{\partial}{\partial r} \langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle = \langle visc \rangle$$

then,

$$\underbrace{\mathcal{E}}_{\text{Energy}} = \int d^2x \, \rho c_s^2 \Big[\underbrace{\Big(\frac{\hat{n}}{n}\Big)^2}_{\text{internal}} + \underbrace{\rho_s^2 \Big(\nabla_\perp \Big(\frac{|e|\phi}{T}\Big)\Big)^2}_{\text{flow / mechanical kinetic}} \Big]$$

Energy → generic fcn DW

internal (wave + zonal density)

flow / mechanical kinetic (zonal flow + waves)

- drift wave system energy density
- obvious analogy to 1D ion-acoustic system
- internal ⇒ { density, temperature & ~~pressure~~, parallel KE } { more generally

absorbing normalization:

$$\partial_t \langle \delta n \rangle + \partial_r \langle \tilde{v}_r \tilde{n} \rangle = \langle diffn \rangle$$

so

$$\partial_t \int d^3x \; \begin{matrix}\text{Zonal} \\ \text{Internal}\end{matrix} = \partial_t \int d^3x \, \langle \delta n \rangle^2$$

$$= \boxed{+ \int \langle \tilde{v}_r \tilde{n} \rangle \, \partial_r \langle \delta n \rangle} + \langle \text{Diss}_{pa} \rangle$$

+ End pts

coupling to waves

but

$$\partial_t \int d^3x \, (\text{Wave Internal}) = \partial_t \int d^3x \, \langle \tilde{n}^2 \rangle$$

$$= \boxed{- \int \langle \tilde{v}_r \tilde{n} \rangle \, \partial_r \langle \delta n \rangle}$$

coupling to zonal $\langle \delta n \rangle$

$$+ \int \tilde{n} \, \tilde{v} \cdot \nabla \tilde{n} \quad + \int \tilde{n} \, \nabla_{\parallel}^2 (\tilde{n} - \tilde{\phi}) \, D_{\parallel}$$

[Wave-wave coupling]

[Linear coupling to vorticity. vanishes, summed when]

+ Diffn

N.B. Boxes $\leftrightarrow$ opposite signs!

Obvious that:

$$\partial_t \int d^3x \, [\text{Zonal Internal}] + \partial_t \int d^3x \, [\text{wave Internal}] = 0$$

→ likewise for [zonal kinetic] + [wave kinetic]

→ conserve independently ⇒ show this!

→ likewise for wave-wave coupling!
⇒ show this!

→ Note: All of zonal corrugation ↔ waves (and zonal flow ↔ waves) interaction contained in:

$\langle \tilde{v}_r \tilde{n} \rangle$ → flux — { conservation ↔ need apply <u>consistently</u>

– possible to be <u>grossly</u> <u>wrong</u> yet conserve energy

– For HW; EDW:

→ zonal density effect weak for $k_\parallel^2 v_{th}^2/\omega r \gg 1$. (phase on flux $\sim 1/\alpha$) show this!

⇒ Flow dominant there

(Adv students)

(*) → more generally: for CTEM

$$\langle \tilde{v}_r \tilde{n} \rangle \cong -\sum_{k,\omega} \langle |\tilde{v}_r|^2 \rangle_{k,\omega} \frac{\Delta\omega_k}{\omega_*^2} \nabla \langle n \rangle$$

①

$$\doteq D_{res} \nabla \langle n \rangle$$

②

① non-resonant. In steady state,

⇒ really NL wave-particle processes

i.e. $\sim \pi\, \delta(\omega + \omega' - \bar{\omega}_D \epsilon - \bar{\omega}_D' \epsilon)$

trapped electron NLLD.

(2) Resonant diffusion!

⇒ interesting coupling of resonant particles (electrons) to corrugation.

⇒ $O\left(\frac{e\phi}{T}\right)^2$ ⇒ stronger effect.

⇒ Propagating corrugation ⇔ avalanche!?

$$\partial_t \langle \delta n \rangle = -\partial_r \langle \tilde{v}_r \tilde{n} \rangle$$

$$= -\partial_r \left[-D_{res} \partial \langle \delta n \rangle \right]$$

⇒

$$\partial_t \langle \delta n \rangle - \underbrace{\left[\partial_r D_{res} \right]}_{C_{eff.}} \partial_r \langle \delta n \rangle - D_{res} \partial_r^2 \langle \delta n \rangle = 0$$

$C_{eff.}$ → corrugation speed.

$C_{eff} = -\partial_r D_{res} \equiv$ propagation speed.

Intensity gradients generate propagation.

⇒ Feedback!

→ Collisionless Drift Waves
Energetics with Resonant Particles.

Here: $\frac{\tilde{n}_e}{n} = \frac{|e|\hat{\phi}}{T} + \tilde{h}$

$\tilde{h}$: non-Boltzmann / Adiabatic Part.

For collisionless drift wave:

$$v_{thi} < \frac{\omega}{k_\parallel} < v_{the}$$
$$\Downarrow \qquad\qquad \Downarrow$$
$$\lesssim \qquad\qquad \ll$$

so take corrected HM ("$i\delta$") model

i.e. recall:

$$\frac{\partial}{\partial t} n + \nabla \cdot (n \underline{v}) = 0$$

so

$$\hat{\phi} = \frac{|e|\tilde{\phi}}{T}$$

Waves

$$\partial_t \hat{\phi} - \frac{d}{dt}\rho_s^2 \nabla_\perp^2 \phi + \partial_t \tilde{h} + V_* \partial_y \hat{\phi} = 0$$

$\llcorner$ includes shearing

Zonal Flow:

$$\rho_s^2 \frac{d}{dt} \nabla_\perp^2 \langle\phi\rangle = \rho_s^2 \partial_t \nabla_r^2 \langle\phi\rangle + \rho_s^2 \partial_r \langle \tilde{v}_r \nabla_\perp^2 \hat{\phi} \rangle = \text{Damp}.$$

$\tilde{h}$ from DKE:

$$\frac{\partial f}{\partial t} + v_\parallel \nabla_\parallel f - \frac{c}{B} \underline{\nabla}\phi \times \hat{z} \cdot \underline{\nabla} f$$

$$- \frac{|e|}{m_e} E_\parallel \frac{\partial f}{\partial v_\parallel} = 0$$

$$\tilde{h} = \int d^3v \, \tilde{f} - \frac{|e|\tilde{\phi}}{T}$$

n.b. here $\phi = \langle\phi\rangle + \tilde{\phi}$ $*$

but take $|k_\parallel v_{th}| > |\omega_{E \times B}|$

for simplicity

Then

$$\mathcal{E}_W = \int \frac{d^3x}{2} \left\{ (\tilde{\phi})^2 + (\nabla \tilde{\phi})^2 \right\}$$

so

$$\partial_t \, \mathcal{E}_W = -\int d^3x \, \langle \tilde{\phi} \, \partial_t \tilde{h} \rangle$$

resonant particle/electron coupling

a

$$+ \int d^3x \, \rho_s^2 \langle \tilde{\phi} \, \tilde{V}_r \, \partial_r \, \nabla_r^2 \langle \phi \rangle \rangle$$

b

zonal flow coupling.

b easier to calculate:

$$\partial_t \, \mathcal{E}_{W \, \text{Flow}} = - \, \partial_t \, \mathcal{E}_{Z \, \text{Flow}}.$$

$$\rho_s^2 \, \partial_t \, \nabla_r^2 \langle \phi \rangle \equiv -\frac{\partial}{\partial r} \langle \tilde{V}_r \, \rho_s^2 \, \nabla_\perp^2 \tilde{\phi} \rangle$$

ignore ST:

$$\partial_t \int d^3x \, \rho_s^2 \frac{(\nabla_r \langle \phi \rangle)^2}{2} = -\int d^3x \, (\partial_r \langle \phi \rangle) \langle \tilde{V}_r \, \rho_s^2 \, \nabla_\perp^2 \tilde{\phi} \rangle$$

Then, using Taylor Identity:

$$\partial_t \left[\int d^3x \, \rho_s^2 \frac{\left(\nabla_r \langle \phi \rangle \right)^2}{2} \right]$$

and IBP (ignore ST):

$$= \int d^3x \left(\rho_s \partial_r^2 \langle \phi \rangle \right) \langle \tilde{v}_r \, \rho_s \, \partial_r \tilde{\phi} \rangle$$

$$= + \int d^3x \, \langle V_E \rangle' \langle \tilde{v}_{r_E} \tilde{v}_{y_E} \rangle$$

so

$$\boxed{\partial_t \, \mathcal{E}_{w_{Flow}} = - \int d^3x \, \langle V_E \rangle' \langle \tilde{v}_{r_E} \tilde{v}_{y_E} \rangle}$$

N.B.: Surface terms relevant to edge/bndry application

so

$$\partial_t \mathcal{E}_w = \overset{(a)}{- \int d^3x \, \langle \tilde{\phi} \, \partial_t \tilde{h} \rangle} \underset{(b)}{- \int d^3x \, \langle V_E \rangle' \langle \tilde{v}_{r_E} \tilde{v}_{y_E} \rangle} + \langle \text{dissip} \rangle$$

(a) → resonant electron drive interaction

(b) → Reynolds power of waves expended on Zonal Flow

N.B. If seek include ion Landau damping:

$$\underline{\nabla} \cdot \underline{V} \rightarrow \underline{\nabla}_\perp \cdot \underline{V}_\perp + \nabla_\parallel V_\parallel$$

and have $\int d^3x \, \langle \hat{\phi} \, \nabla_\parallel \tilde{V}_\parallel \rangle$

calculate via ion GK or ion Drift kinetics

But: If resonant electrons produce wave energy evolution:

- need resonant electron kinetic energy evolution → QL Eqn. for Electrons
- Energy Balance Theorem.

— QL Eqn

$$\frac{\partial f}{\partial t} + v_\parallel \hat{n}\cdot\nabla f - \frac{c}{B}\nabla\phi\times\hat{z}\cdot\nabla f - \frac{|e|}{m_e}E_\parallel\frac{\partial f}{\partial v_\parallel} = 0$$

then: becomes ω in $\omega-\omega_*$

$$\tilde{f}_{\underline{k},\omega} = \frac{+\frac{|e|}{m_e}(-ik_\parallel)\phi_{\underline{k}}\frac{\partial\langle f\rangle}{\partial v_\parallel} + \frac{c}{B_0}ik_\theta\phi_{\underline{k}}\frac{\partial f}{\partial r}}{-i(\omega-k_\parallel v_\parallel)}$$

from parall accel.

$= \frac{|e|\phi}{T}\Big(1 -$ [redacted] $\frac{(\omega-\omega_*)}{\omega-k_\parallel v_\parallel}\Big)$ [redacted] $\langle f\rangle$ [redacted]

$$\boxed{\tilde{f}_{\underline{k},\omega} = \frac{|e|\phi_{\underline{k}}}{T}\langle f\rangle\left(1-\frac{(\omega-\omega_{*e})}{\omega-k_\parallel v_\parallel}\right)}$$

usual form.

$$\tilde{n}_{\underline{k}} = -\frac{(\omega-\omega_*)}{|k_\parallel v_{Th}|}\frac{|e|\phi_{\underline{k}}}{T}\left(-i\pi\langle f(\omega/k_\parallel v_{Th})\rangle\right)$$

$\to$ $\omega-\omega_*<0$ $\Rightarrow$ phase lag.

$$= \left[\frac{i\pi(\omega-\omega_*)}{|k_\parallel v_{Th}|}\right]\langle f\rangle\frac{|e|\phi_{\underline{k}}}{T}$$

more energy released by ∇n than lost in accel.!

then

$$\frac{\omega_*}{\omega} - k_\perp^2 \rho_s^2 = 1 + \left[\frac{i\pi(\omega-\omega_*)}{|k_\parallel v_{Te}|} \langle f \rangle \right]$$

$$\omega_r = \omega_* / (1 + k_\perp^2 \rho_s^2)$$

$$\frac{\omega_{IM}}{\omega_r} = -\frac{\pi(\omega_r - \omega_*)}{(k_\parallel v_{Te})(1 + k_\perp^2 \rho_s^2)} \sim O\left(\frac{\omega}{|k_\parallel v_{Te}|}\right)$$

Then QL eqn. :

$$\frac{\partial \langle f \rangle}{\partial t} + \frac{\partial}{\partial r} \langle \tilde{v}_r \tilde{f} \rangle + \frac{\partial}{\partial v_\parallel} \left\langle \frac{+|e|}{m_e} \tilde{E}_\parallel \tilde{f} \right\rangle = 0$$

Could integrate :

$$\underbrace{\frac{\partial}{\partial t} \varepsilon_p}_{\text{electron particle energy density}} + \underbrace{\partial_r Q_r}_{\text{turbulent heat flux}} - \langle E_\parallel J_{\parallel e} \rangle = 0 .$$

If integrate:

$$\partial_t \langle \mathcal{E}_p \rangle + Q_r \Big| - \int \langle E_\parallel J_{\parallel e} \rangle = 0$$

volume integrated kin energy density

bndry heat flux thru bndy

but also know: wave energy thm.

$$\partial_t \mathcal{E}_W + \partial_r S_{r\,\mathrm{wave}} + \langle \underline{E} \cdot \underline{J} \rangle = 0$$

but:

$$\langle \underline{E} \cdot \underline{J} \rangle = \langle \underline{E}_\perp \cdot \underline{J}_\perp \rangle + \langle E_\parallel J_\parallel \rangle$$

from electron energy/gain → electron cooling

Reynolds power → Z.F.

$$\int \langle V_E \rangle' \langle \tilde{V}_{Er} \tilde{V}_{E\theta} \rangle$$

↳ NL Polarization Drift

if integrate:

$$\partial_t \langle \mathcal{E}_w \rangle + S_r \Big|_{\text{bndry}} + \underbrace{\int d^3x \, \langle E_\perp \cdot J_\perp \rangle}_{\text{system integrated Reynolds work}} + \underbrace{\int d^3x \, \langle E_\parallel J_\parallel \rangle}_{\text{electron cooling}} = 0$$

N. B. In stationary state:

$$\underbrace{+ \langle Q_r \rangle \Big|_{\text{bndry}}}_{\text{heat flux thru bndry}} = \underbrace{\int d^3x \, \langle E_\parallel J_{\parallel e} \rangle}_{\text{electron cooling}}$$

and

$$\underbrace{S_r \Big|_{\text{boundary}}}_{\text{wave energy density flux thru bndry}} + \underbrace{\int \langle \underline{E}_\perp \cdot \underline{J}_\perp \rangle d^3x}_{\text{Reynolds power on ZF}} = \underbrace{- \int d^3x \, \langle E_\parallel J_{\parallel e} \rangle}_{\text{electron cooling}}$$

If $S_r \to 0$, then:

$$\int d^3x \langle \underline{E} \cdot \underline{J} \rangle = 0$$

$\leftrightarrow$ energy balance in steady state.

cooling of electrons onto waves

$\langle \underline{E}_\perp \cdot \underline{J}_\perp \rangle$

Reynolds waves work on ZF

and

$$\langle Q_r \rangle \Big|_{\text{bndry}} = -\int d^3x \langle \underline{E}_\perp \cdot J_\perp \rangle .$$

like heat flux thru bndry to Reynolds work.

N.B: Need evoln. eqn. for ZF to calculate saturated ZF $\leftrightarrow$ damping.

and for Q L equation:

$$\partial_t \langle f \rangle = \frac{\partial}{\partial r} D_{r,r} \frac{\partial}{\partial r} \langle f \rangle$$

$$+ \frac{\partial}{\partial r} D_{r,v} \frac{\partial}{\partial v_\parallel} \langle f \rangle + \frac{\partial}{\partial v_\parallel} D_{v_\parallel, r} \frac{\partial}{\partial r} \langle f \rangle$$

$$+ \frac{\partial}{\partial v_\parallel} D_{v_\parallel, v_\parallel} \frac{\partial}{\partial v_\parallel} \langle f \rangle$$

TBC ...