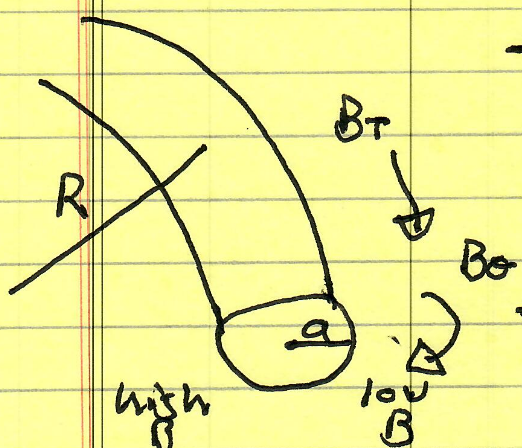
Physics 218c - MFE Theory

Lecture 1 - Overview a.

→ What is a tokamak?



- toroidal (donut) magnetic confinement device for plasma
- Russian acronym

i) Configuration:

- $R/a \sim 3 \to 4$
- B_T { strong, external / inhomogeneous } → { magnetically trapped particles }
- B_θ current { inductive / non-inductive CD / bootstrap - ∇P (self) }

$B_T \gg B_\theta$

- $q \equiv$ safety factor / Kruskal Shafranov factor

Safety Factor

$$q = \frac{B_T}{B_\theta}\frac{r}{R}$$

'pitch' of magnetic field lines

$q = q(r)$

typically $1 < q < 3\text{-}4$

Related: $\hat{s} = \frac{r q'}{q}$ = magnetic shear

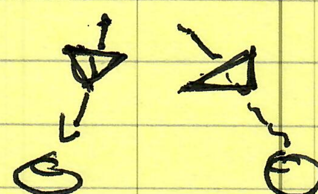
$\Leftrightarrow$

change in pitch of magnetic lines with radius

$\hat{s} \sim 1 \rightarrow 2$

$\Rightarrow$ stability

need twist ~~vortex~~ to displace

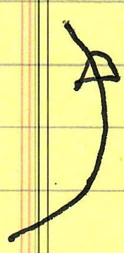


Important exception:

$\boxed{\text{reversed shear}}$
OACS
WNS
ERS
flat q

$\boxed{\text{enhanced confinement mode}}$

Why? Magnetic geometry $\rightarrow$ curvature $\rightarrow$ effective buoyancy



$\underline{B}$

$$\cancel{g} \equiv g_{eff} \equiv \frac{c_s^2}{R_c}$$

curvature of magnetic lines

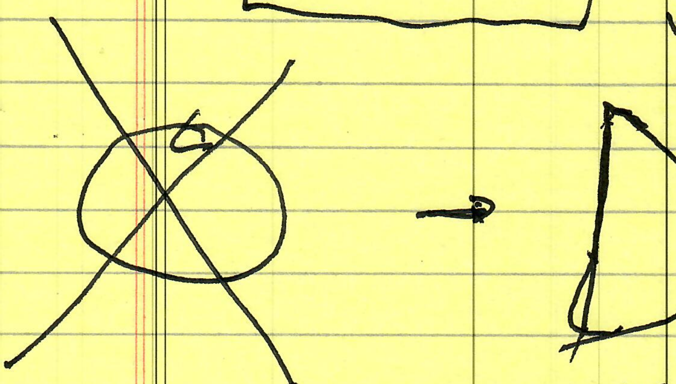
$g_{eff} \to g$ in Rayleigh-Benard
ITG, Ballooning ----
analogous

N.B. Appears either strong shear ($\hat{s} >> 1$) or "very weak shear ($\hat{s} \sim 0$) is "good".

(ii) Configuration control $\to$ $\boxed{\text{Boundary}}$

- First: $\boxed{\text{Shaping}}$ is critical

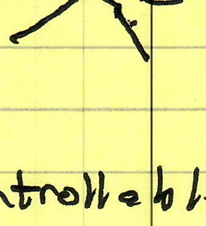
"D-shape" - DIII-D



circular cross section is ancient history

or

why:
→ stability (macro)
→ confinement and L-H transition (micro)



Xpt. → ← X-point → LSN
also: USN
DN.

N.B. shaping controllable in time.

N. B. : Rising star : "Negative Triangularity"
(TCV, DIII-D)

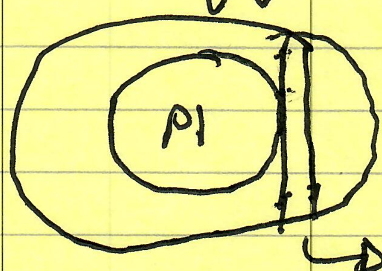
why? : - improved confinement
→ reduced trapped particle mode transport.

- Related : $\boxed{\text{Divertor}}$ - no ELMs.

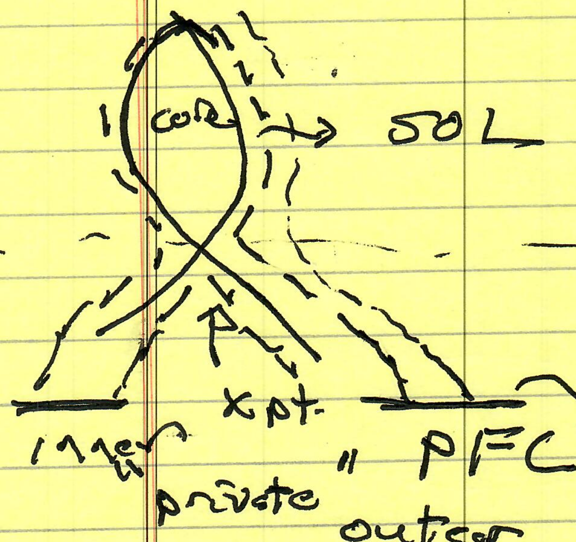
"divert" - change course of as in river flow

- why? - removes plasma boundary interaction from vicinity of plasma
⇓
control particles, impurities etc.

i.e. - ancient history : "limiter"



– modern history



core – closed field lines

SOL – open field lines
– boundary flow.

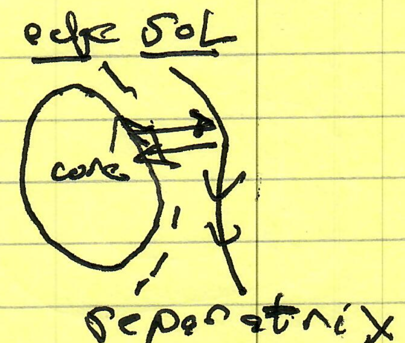
{ Divertor chamber

→ sheath

N.B.: [profile of SOL heat load (width) on PFC is key problem.

Should note:

– $\boxed{\text{boundary physics}}$ combines outer core and SOL dynamics



→ rapid transition in basic physics

→ critical to fueling

→ location L→H, ELMs.

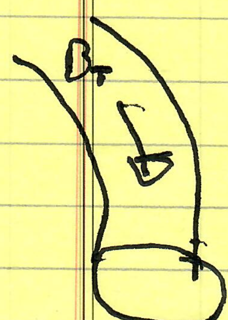
- Shaping + Divertor $\equiv$ challenge to magnetic control system

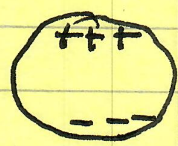
$\Rightarrow$ key part of tokamak operations.

iii) Configuration – Current

Why current?

a)



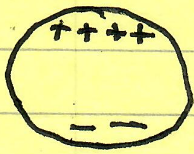


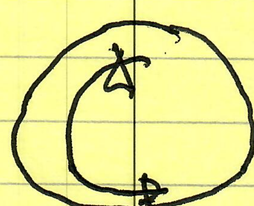
∇B/curvature drift $\underset{+}{\uparrow}$ $\underset{-}{\downarrow}$

$\Rightarrow$ rotational transform to short out charge separation i.e. $\to q = q(r)$

n.b. rotational transform needed in stellarator

i.e.





b.) heating $\Rightarrow$ ohmic

$$P_{OH} = \eta J^2$$

$\eta \equiv$ Spitzer resistivity

$\to \nu_{ec}$

c.) $\left\{\begin{array}{l} P_{OH} \text{ coefficient at high } T \\ \text{Transformer exhausted} \\ \text{(volt-sec.)} \end{array}\right.$ $\Rightarrow$ $\left\{\begin{array}{l} \text{heating} \\ \boxed{\text{current drive}} \end{array}\right.$

so Waves $\left\{\begin{array}{l} \text{LH} \\ \text{ECH} \\ \text{(beams)} \end{array}\right.$ $\xRightarrow{\text{drive}}$ $J(r)$ control

CD $\rightarrow$ $I(r)$ shape
$\Rightarrow$ stability

iv.) Profiles – distribution of thermodynamic quantities?
how controlled?

$$f_0 = \frac{n}{(v_{th}(r))^3} \exp\left[-\frac{(v - \langle v \rangle)^2 m}{T(r)}\right]_{e,i}$$

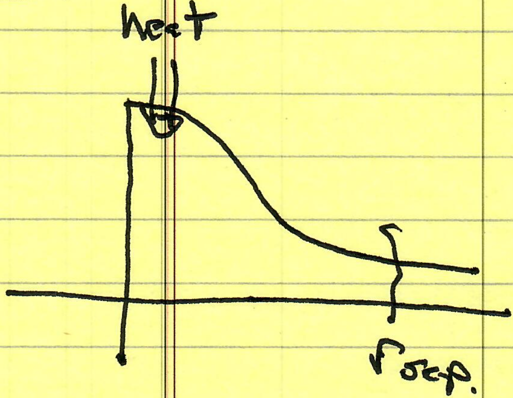
$\Rightarrow$ Temperature: T_i, T_e

– heating: NBI, ECH, ICRF, OH

– coupling via collisions, turbulence (!?)
$\nu n (T_e - T_i)$

– deposition $\rightarrow$ central, OA
etc.

<u>THE</u> key question: Given heating + params.
$\Rightarrow T(r)$



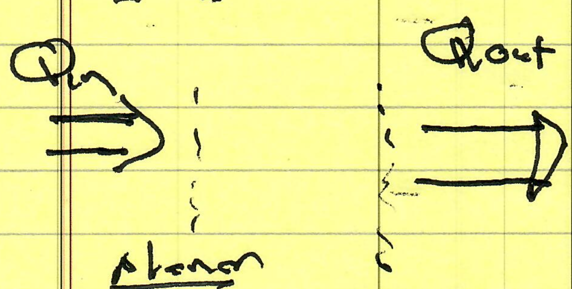
$P_{in} = \underline{\nabla} \cdot \underline{Q}$

$\Downarrow$

$\left\{ \begin{array}{l} \text{transport} \\ \text{stability} \end{array} \right.$

<u>n.b.</u> outside heating zone;

$\underline{\nabla} \cdot \underline{Q} = 0 \rightarrow$ "fixed flux"



$Q_{in} = Q_{out}$

$$Q = \underbrace{Q_{Turb}}_{-\chi_T \nabla T} + \underbrace{Q_{Macro}}_{?} + \underbrace{Q_{neo}}_{-\chi_{neo} \nabla T}$$

χ_{neo}: collisions

how calculate ?

(60's) $\chi_{e_T} \gg \chi_{neo,e}$

(80's) $\chi_{i_T} > \chi_{i\,neo,i}$

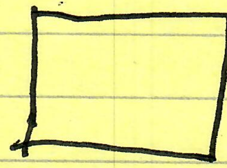
$Q_{Turb,\,e,i} \sim -\chi_{e,i} \nabla T$

Important Contrast:

Rayleigh–Benard
(classic – Chandra)

$\rightsquigarrow$ ① fixed gradient

$T_c \rightarrow$ (boundary)

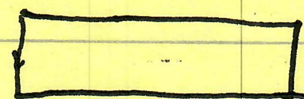


ΔT fixed

$T_h \rightarrow$ (heating)

(Chapman – Proctor)
(1982)

$\rightsquigarrow$ fixed flux



$Q = $ const

layer dynamics
s/t $Q = $ const
accross layer.

⇒ $\boxed{\text{Claim: Fixed Flux problem (Chapman + [redacted] Proctor) closer to tokamak confinement situation}}$

[redacted]

Caveat Emptor:

a.) Most of tokamak database, experience, intuition is with NBI

⇒ DUBIOUS relevance to ITER, CFETR etc.

Some effort at "low torque" studies.

b.) Boundary matters →

heat boundary condition and more.

c.) Anomalous coupling is insufficiently studied.

→ Density : n_e, n_i, n_Z

(mostly n_e) Q-N: $|e| n_e - Z_i |e| n_i - Z_Z |e| n_Z = 0$

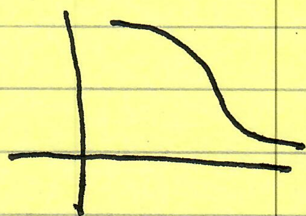
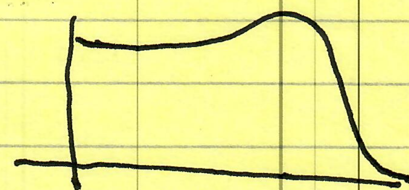
⇒ Key point:

– heating distributed (OH) or on axis (usually)

⇒ $T(r)$ peaked

– but, in basic scenario, fueling at edge → "gas puffing"

- yet density peaked

i.e.  not 

how? ⇒ up-gradient transport

→ pinch.

$$\Gamma_n = -\underbrace{D_n \nabla n}_{\text{turbulent diffusion}} + \underbrace{V n}_{\text{"pinch", convection (turbulent)}}$$

$V<0$

{ basic example of plasma self-organization }

akin 'chemotaxes'

- discovered by modulated gas-puffing (Strachan '78)

- necessarily, V driven

i.e.

$$V \sim \nabla T$$

i.e. heat flux drives T profile and V (others possible)

– fueling and density profile formation are <u>critical</u> to future tokamaks

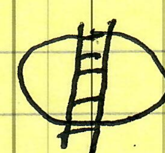
Also: <u>Density Limit.</u>

– one fundamental limit on tokamak performance is (Greenwald) Density limit.

– n appears in Lawson criterion

→ Greenwald: $\bar{n} \sim I_p$

line averaged



→ approaching $\bar{n}_g$ → <u>disruption</u>

(not good)

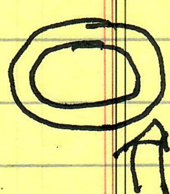
but:

$\bar{n}_g$ linked to transport of particles, especially at edge.

→ Toroidal, Poloidal Rotation

– Toroidal

Rotation
- external torque, NBI
- intrinsic



NBI

NBI → heating
→ rotation, flow
↓
confinement (shear)
→ $\langle V_\phi \rangle'$

$$\Gamma = -\chi_\phi \frac{\partial \langle V_\phi \rangle}{\partial r} + \ldots$$

↓

phenomenology $\chi_i \sim \chi_\phi \sim D_n$ (electrostatic drift waves)

but $\Gamma_{ext} = 0$, $\langle V_\phi \rangle \neq 0$
Plasma exhibits 'spontaneous' or 'intrinsic' rotation !

→ K. Ida, J. Rice (independent)
late 90's → recent history.
(JFT-2M, Alcator C-Mod)

What is going on?

$$\Pi = -\chi_\phi \frac{\partial \langle V_\phi \rangle}{\partial r} + \underbrace{\Pi_{resid}}_{\sim \nabla P_i,\ \nabla T_i,\ \nabla n}$$

~ Non-diffusive stress ("residual stress") driven by ∇P, etc. drives momentum flux ⇒ another key element of tokamak self-organization.

~ boundary important → momentum conservation

$\Delta V_\phi \sim \Delta W / I_p$

"Rice scaling"

~ analogous to engine

$$Q \to \text{turbulence} \to \underbrace{\Pi_{resid}}_{\substack{\text{includes symmetry breaking}\\ \text{(direction)}}} \to \text{flow}$$

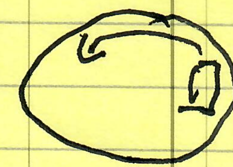
analogous Sun

fusion (ICF) → Convection → Differential Rotation

~ also
- Pinch
- Turbulent acceleration
- Intrinsic current

→ Poloidal Rotation

– neoclassical, due asymmetry



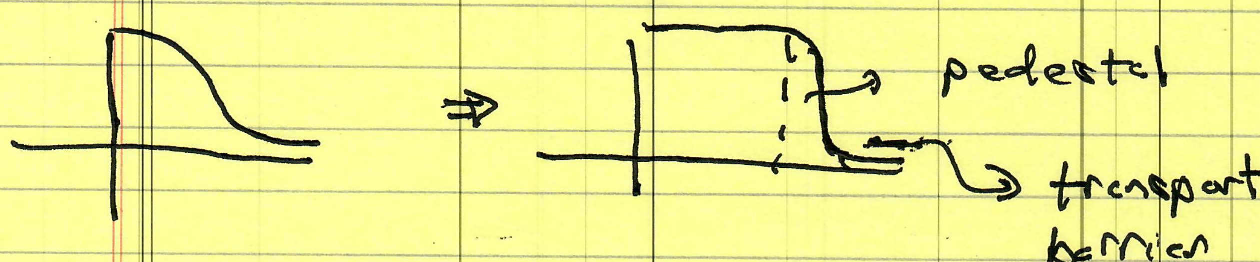
– some shift detected, absent T_θ.

⇒ <u>Important</u> Aside → $\boxed{\text{H-Mode}}$

⇒ H-mode, L-H Transition
Edge Transport Barrier

⇒ F. Wagner – ASDEX (1982) (now HL-2A)
(Divertor → boundary control)

– $P > P_{crit}$ $\left(Q_{i\,edge} > Q_{i\,crit} \right)$



spontaneous transition
to state of improved confinement
with edge transport barrier

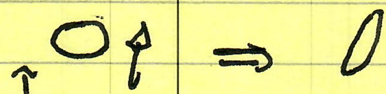
- Transport Barrier
  - region $w > \Delta x_c$ s/t
    Q_T, Γ_T etc. reduced dramatically
  - turbulence levels drop.

- How? → Shear Flow. (likely)
(BDT 1990 et seq) via radial force balance

$\underline{V}_{E\times B}$ from $0 = \left(+\frac{q}{m}\underline{E} + \frac{q}{mc}\underline{u}\times\underline{B} - \frac{\nabla P_i}{n m_i}\right)\cdot\hat{r}$

$\downarrow$

$\underline{E}\times B$ Flow

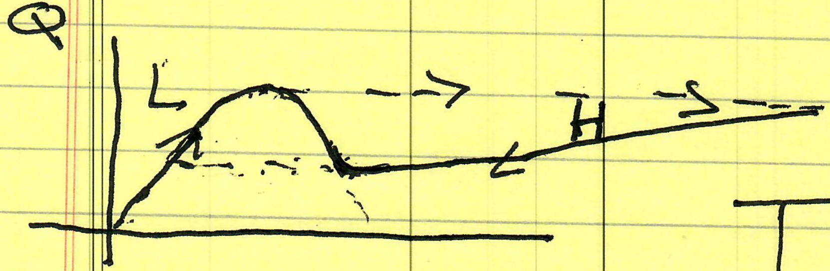
Classic cartoon:  etc.

Note: "Feedback loop" → critical concept

$$Q\uparrow \rightarrow \nabla P\uparrow \rightarrow V_E\uparrow \rightarrow V_E'\uparrow \rightarrow \chi_i\downarrow$$

change in self-consistent state.

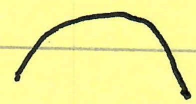
– Transport Bifurcation → $P_{crit}(n, B_T, \ldots)$



L → low

H – high

$\boxed{\text{Transport Barriers are Critical to Self-Organization}}$



quenched turbulence

+

neo/residual

Trigger → Flows ⇏ details ongoing

⇔ Variant : Internal Transport Barrier (ITB)

[localized by Σ – profile]

⇔ Variant : Zonal Flow (self-generated)

Not all .....

– ELMs – Edge Localized Modes (a misnomer)

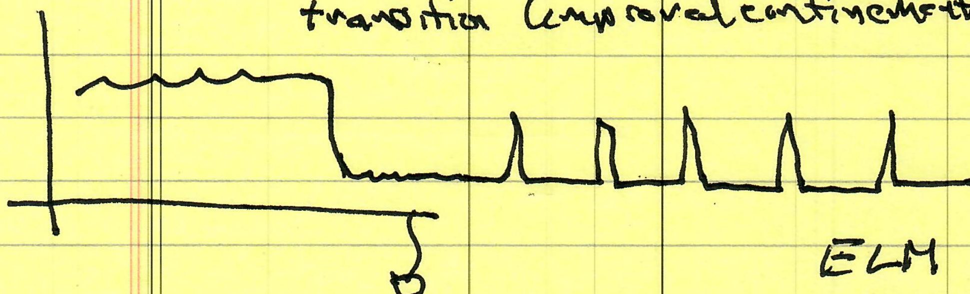
better

Edge Relaxation Phenomena (ERP – like hiccup)

ELM = 'small flame' in German.

L→H sequence H^α, D^α ; [ionization / radiation]

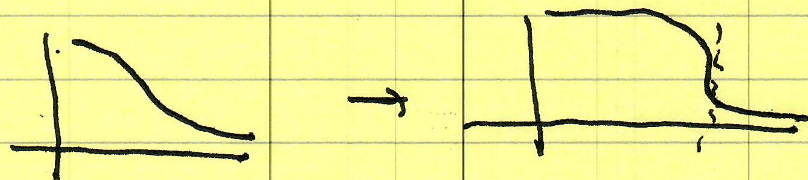
transition (improved confinement)



ELM-free period

ELM bursts

What?



"improved confinement needed to test β-limit"

$\nabla p \sim \nabla p_{crit}$

$\nabla J \quad \nabla J_{crit}$ for MHD instability:

δW [→ ballooning / → peeling (surface kink)]

⇒ ELM event relaxes pedestal

⇒ lots of energy released (ITER: 20 MJ)

⇒ Where does it go? ⇒ PFC's.

~ unacceptable transient heat loads.

→ N.B. ELM event related to proximity to P-B threshold

but ELM ≢ P-B mode. !
(n.b. some would disagree)

Nonlinear evolution, interaction etc.

Which brings us to: THE QUESTION

- we want good confinement → H-mode
- we don't want high transient heat loads on PFC

What to do? ~ a trip to the ZOO .....

- mitigate/supress ELMs — all current research

→ Resonant Magnetic Perturbation (Todd Evans)

coil → relieve ∇T ? → relieve ∇n
(stellarator-Tok hybrid) — pump out
but P_{LH} ↑

→ QH-mode (Garofalo, Burrell)

strong edge shear quenches/eliminates P-B → EHO (check saturated limit ?)

- Pace ELMs

→ SMBI, pellet pacing (provoke small ELMs) (AUG, HL-2A, DIII-D)

→ Density Limit ?

- Avoid ELMs.

→ I-mode, crested H-mode (C-Mod, AUG)
(T ped, not n ped;
τ_E ↗, τ_p const)

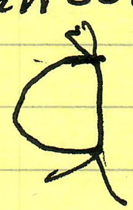
→ Relevance ?

or, forget H-mode ?! (M. Kikuchi)
which brings us back to....

→ Negative Triangularity ... (again)

- improved L-mode confinement, so far
No barrier needed CTEM↓ (Cameron)

- ballooning modes at corners



may prevent L→H transition ?!
(Marinoni, DIII-D)

— up-coming DIII-D experiments will be critical for negative T.

Don't want H-mode ...

And ...

→ improve Divertor ⇒ distribute heat Load

(beyond scope of this course)

Message: The self-organization of:

$$\boxed{\text{L} \rightarrow \text{H} \rightarrow \text{ELMs} \rightarrow \text{ELM mitigation} \rightarrow \text{Divertor (includes SOL width)}}$$

package is 1 of 2 critical problems in MFE today.

Other is Disruption.

? Is turbulence good or bad ?

→ More

Impurity transport X

EPG and AE's X → UCI ?

Disruption ✓ short

Details of RF heating, CD X

ITB's ✓

Divertor Physics X → others at UCSD ?

⇒ The $\boxed{\text{Magic Number}}$

Lawson Criterion:

$$nT\tau_E > \#\,\text{crit}$$

— n.b. Caveat Emptor re: claims about Lawson #

— interesting to re-write

re-writing Lawson:

why high field is attractive (Alcator, SPARC)

$$n T \tau_E = \beta B^2 \tau_E$$

$$= \beta \, \tau_{E,neo} \frac{\tau_E}{\tau_{E,neo}} B^2$$

MIT likes this

MIT ~~likes~~ understands this.

$$= \beta \, \frac{\tau_E}{\tau_{E,neo}} \, \tau_{E,neo} \, B^2$$

Physics (dimensionless) physics (d-less)

Engineering (Magnet Technology)

limited by understood Physics

$$\frac{\tau_E}{\tau_{E,neo}} \lesssim 1$$ rigorously

i.e. Boltzmann Eqn, H-Thm.
+
Chapman-Enskog
+
Particle Orbits
+
Field structure

So → all the issues in:

→ $\tau_E / \tau_{E\,neo}$ → confinement

→ β → beta limit
(includes density limit)

Rest → Engineering

N.B.: As emphasized by M. Kikuchi,
story of fusion has evolved from:
→ {quest for good confinement
⇒
→ quest for {good confinement + good power handling (boundary control)

My personal opinion:

So claims of victory in Lawson # must establish that good power handling is realizable, for that Lawson solution.

How quantify? → Kikuchi-X number?!

→ What are the Fundamental Physics <u>Limits</u> ?

- $\tau_{E_{neo}}$, $\tau_E / \tau_{E_{neo}} \le 1$

(suggests barriers)

- β Limit → stability (macro) Ballooning, Kinks (Troyon) → MHD

$$\boxed{\beta_N = \beta \frac{a B_T}{I_p}}$$

beta normal

$\beta_N \sim$ a few, some higher

- Density Limit — Greenwald

$\bar{n}_g \sim I_p$ (enters β)

N.B. - Current (I_p) is clearly <u>good</u>.

(confinement, too)

- but: disruptions, power handling.