15. The shortest wavelength is the series limit. For the Lyman series, $n_0 = 1$ and Equation 6.21 becomes

$$\lambda = (91.13\ \text{nm})\frac{n^2}{n^2 - 1} \qquad n = 2, 3, 4, \ldots$$

which gives λ = 121.51 nm (n = 2), 102.52 nm (n = 3), 97.21 nm (n = 4).

18. From Figure 6.16 we see that only the Paschen series ($n_0 = 3$) has lines near 1000 nm. Using the series limit of 820.1 nm, we have from Eq. 6.21

$$1005\ \text{nm} = (820.1\ \text{nm})\frac{n^2}{n^2 - 9}$$

$$1.225(n^2 - 9) = n^2$$

Solving, we find $n = 7$, so the transition connects the $n = 7$ and $n = 3$ states.

19. $r_3 = 9a_0 = 9(0.0529\text{ nm}) = 0.476\text{ nm}$

$$v = \frac{n\hbar}{mr} = c\frac{n\hbar c}{mc^2 r} = c\frac{3(1240\,\text{eV}\cdot\text{nm})/2\pi}{(0.511\times10^6\text{ eV})(0.476\,\text{nm})} = 2.43\times10^{-3}c = 7.30\times10^5\text{ m/s}$$

$$U = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r} = -\frac{1.440\text{ eV}\cdot\text{nm}}{0.476\text{ nm}} = -3.02\text{ eV}$$

$$K = \frac{e^2}{8\pi\varepsilon_0}\frac{1}{r} = \frac{1.440\,\text{eV}\cdot\text{nm}}{2(0.476\text{ nm})} = 1.51\,\text{eV}$$

21. (a) From Equation 6.26, $v = \dfrac{n\hbar}{mr} = \dfrac{n\hbar}{mn^2 a_0}$. Using Equation 6.29 for a_0, we obtain

$$v = \frac{\hbar}{nm(4\pi\varepsilon_0\hbar^2/me^2)} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{n\hbar} = \frac{\alpha c}{n}$$

(b) When the nuclear charge is Ze, we must replace e^2 with Ze^2, so $v = Z\alpha c/n$.

22. The energy of the initial $n = 5$ state is $E_5 = \dfrac{-13.6\text{ eV}}{25} = -0.544\text{ eV}$. An electron in this state can make transitions to any of the lower states with $n = 4$ ($E_4 = -0.850$ eV), $n = 3$

(E_3 = –1.51 eV), n = 2 (E_2 = –3.40 eV), and n = 1 (E_1 = –13.6 eV). The transition energies are:

$$5 \rightarrow 4: \quad \Delta E = E_5 - E_4 = -0.544\text{ eV} - (-0.850\text{ eV}) = 0.306\text{ eV}$$

$$5 \rightarrow 3: \quad \Delta E = E_5 - E_3 = -0.544\text{ eV} - (-1.51\text{ eV}) = 0.97\text{ eV}$$

$$5 \rightarrow 2: \quad \Delta E = E_5 - E_2 = -0.544\text{ eV} - (-3.40\text{ eV}) = 2.86\text{ eV}$$

$$5 \rightarrow 1: \quad \Delta E = E_5 - E_1 = -0.544\text{ eV} - (-13.6\text{ eV}) = 13.1\text{ eV}$$

24. The photon energy of the incident light is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{53.0\ \text{nm}} = 23.4\ \text{eV}$$

When an atom in the ground state absorbs a 23.4-eV photon, the atom is ionized (which takes 13.6 eV). The excess energy, 23.4 eV – 13.6 eV = 9.8 eV, appears as the kinetic energy of the electron, which is now free of the atom. Neglecting a small recoil kinetic energy given to the proton, the electrons have a kinetic energy of 9.8 eV.

25. (a) The ionization energy is the magnitude of the energy of the electron. For the $n = 3$ level of hydrogen

$$\left|E_3\right| = \left|\frac{-13.6\text{ eV}}{9}\right| = 1.51\text{ eV}$$

(b) For singly ionized helium ($Z = 2$) we use Equation 6.38:

$$\left|E_n\right| = \left|\frac{(-13.6\text{ eV})Z^2}{n^2}\right| = \left|\frac{(-13.6\text{ eV})2^2}{2^2}\right| = 13.6\text{ eV}$$

(c)
$$\left|E_n\right| = \left|\frac{(-13.6\text{ eV})Z^2}{n^2}\right| = \left|\frac{(-13.6\text{ eV})3^2}{4^2}\right| = 7.65\text{ eV}$$

27. The Lyman series consists of transitions that end in the $n = 1$ level. The smallest energy difference, corresponding to the longest wavelength, is $n = 2$ to $n = 1$.

$$\Delta E = E_2 - E_1 = (-13.6 \text{ eV})2^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 40.8 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}\cdot\text{nm}}{40.8 \text{ eV}} = 30.4 \text{ nm}$$

The largest energy difference would correspond to transitions from $n = \infty$ to $n = 1$:

$$\Delta E = E_\infty - E_1 = (-13.6 \text{ eV})2^2\left(0 - \frac{1}{1^2}\right) = 54.4 \text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}\cdot\text{nm}}{54.4 \text{ eV}} = 22.8 \text{ nm}$$

28. Using Equation 6.38, we have $E_n = (-13.6 \text{ eV})Z^2/n^2 = (-54.4 \text{ eV})/n^2$, so $E_1 = -54.40 \text{ eV}, E_2 = -13.60 \text{ eV}, E_3 = -6.04 \text{ eV}, E_4 = -3.40 \text{ eV}$. The possible transitions are:

$$4 \to 1: \quad \Delta E = E_4 - E_1 = 51.00 \text{ eV} \qquad \lambda = hc/\Delta E = 24.31 \text{ nm}$$

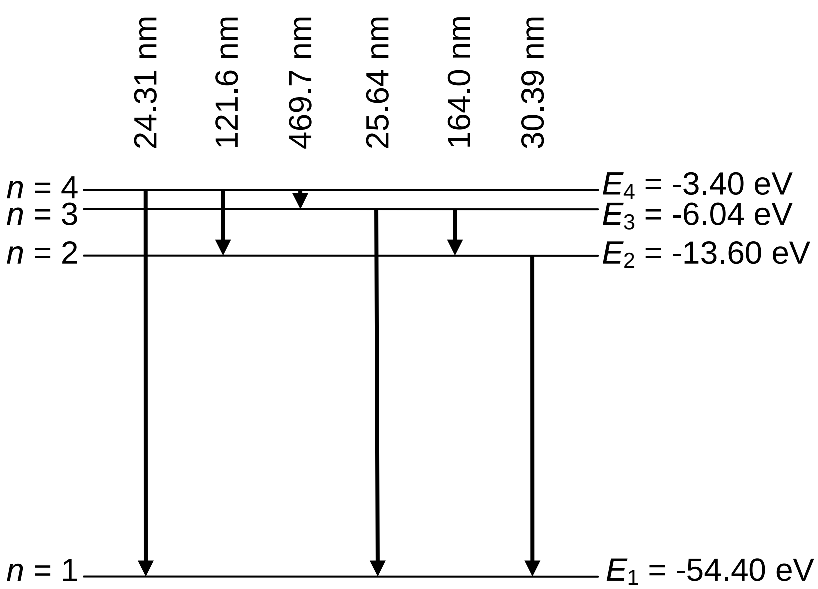
$$4 \to 2: \quad \Delta E = E_4 - E_2 = 10.20 \text{ eV} \qquad \lambda = hc/\Delta E = 121.6 \text{ nm}$$

$$4 \to 3: \quad \Delta E = E_4 - E_3 = 2.64 \text{ eV} \qquad \lambda = hc/\Delta E = 469.7 \text{ nm}$$

$$3 \to 1: \quad \Delta E = E_3 - E_1 = 48.36 \text{ eV} \qquad \lambda = hc/\Delta E = 25.64 \text{ nm}$$

$$3 \to 2: \quad \Delta E = E_3 - E_2 = 7.56 \text{ eV} \qquad \lambda = hc/\Delta E = 164.0 \text{ nm}$$

$$2 \to 1: \quad \Delta E = E_2 - E_1 = 40.80 \text{ eV} \qquad \lambda = hc/\Delta E = 30.39 \text{ nm}$$



30. (a) If the circumference is an integral number of de Broglie wavelengths ($2\pi r = n\lambda$), then after each orbit the waves will align, peak to peak and valley to valley, to give standing waves.

(b) $2\pi r = n\lambda = n\dfrac{h}{p} = \dfrac{nh}{mv}$ so $mvr = \dfrac{nh}{2\pi} = n\hbar$

35. (a) The frequency of revolution is given by Equation 6.41:

$$f_n = \frac{me^4}{32\pi^3\varepsilon_0^2\hbar^3}\frac{1}{n^3} = \frac{1}{\pi\hbar}\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2}\frac{1}{n^3} = \frac{13.6\text{ eV}}{\pi\hbar}\frac{1}{n^3} = \frac{6.58\times10^{15}\text{ Hz}}{n^3}$$

A similar calculation gives the radiation frequency from Equation 6.42:

$$f = \frac{me^4}{64\pi^3\varepsilon_0^2\hbar^3}\frac{2n-1}{n^2(n-1)^2} = \frac{13.6\text{ eV}}{2\pi\hbar}\frac{2n-1}{n^2(n-1)^2} = (6.58\times10^{15}\text{ Hz})\frac{2n-1}{2n^2(n-1)^2}$$

For n = 10, we get $f_n = 6.58\times10^{12}$ Hz and $f = 7.72\times10^{12}$ Hz.

(b) For n = 100, $f_n = 6.58\times10^{9}$ Hz and $f = 6.68\times10^{9}$ Hz.

(c) For n = 1000, $f_n = 6.58\times10^{6}$ Hz and $f = 6.59\times10^{6}$ Hz.

(d) For n = 10,000, $f_n = 6.58\times10^{3}$ Hz and $f = 6.58\times10^{3}$ Hz. Note how f approaches f_n as n becomes large, in accordance with the correspondence principle.

36. The Rydberg constant in ordinary hydrogen is

$$R_{\mathrm{H}} = R_\infty\left(1+\frac{m}{M_{\mathrm{H}}}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\text{ u}}{1.007825\text{ u}}\right) = R_\infty(1.000544)$$

and in “heavy” hydrogen or deuterium:

$$R_{\mathrm{D}} = R_\infty\left(1+\frac{m}{M_{\mathrm{D}}}\right) = R_\infty\left(1+\frac{5.48580\times10^{-4}\ \mathrm{u}}{2.104102\ \mathrm{u}}\right) = R_\infty(1.000272)$$

From Equation 6.33 the difference in wavelengths for the first line of the Balmer series (n = 3 to n = 2) is

$$\lambda_{\mathrm{D}} - \lambda_{\mathrm{H}} = \left(\frac{1}{R_{\mathrm{D}}} - \frac{1}{R_{\mathrm{H}}}\right)\left(\frac{3^2 2^2}{3^2 - 2^2}\right) = \frac{7.2}{1.09737\times10^{7}\ \mathrm{m}^{-1}}\left(\frac{1}{1.000272} - \frac{1}{1.000544}\right) = 0.178\ \mathrm{nm}$$

This small wavelength difference led to the discovery of deuterium in 1931.

46. (a) From Eq. 6.28 for the allowed radii:

$$n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{10^{-6}\ \text{m}}{5.29\times10^{-11}\ \text{m}}} = 137$$

(b)

$$E_n = \frac{-13.6\ \text{eV}}{n^2} = \frac{-13.6\ \text{eV}}{(137)^2} = -7.25\times10^{-4}\ \text{eV}$$

so the ionization energy would be 0.725 meV.

(c)

$$\Delta E = (-13.6\ \text{eV})\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = (-13.6\ \text{eV})\left(\frac{1}{137^2} - \frac{1}{136^2}\right) = 1.07\times10^{-5}\ \text{eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\ \text{eV}\cdot\text{nm}}{1.07\times10^{-4}\ \text{eV}} = 0.116\ \text{m}$$

This is in the microwave region of the electromagnetic spectrum.

48. The Rutherford formula will fail when the distance of closest approach is less than the combined radii of the alpha particle and aluminum nucleus. Rewriting Eq. 6.19 for the distance of closest approach in a head-on collision,

$$K = \frac{e^2}{4\pi\varepsilon_0}\frac{zZ}{d} = (1.44\ \text{MeV}\cdot\text{fm})\frac{(2)(13)}{1.9\ \text{fm} + 3.6\ \text{fm}} = 6.8\ \text{MeV}$$

Alpha particles of energy greater than 6.8 MeV will penetrate the aluminum nucleus can cause a breakdown in the Rutherford scattering formula. Such energies from radioactive decay were available in Rutherford's time.