3. $$\Delta t = t_{\text{up}} + t_{\text{down}} - 2t_{\text{across}} = \frac{2L}{c}\left[\frac{1}{1-u^2/c^2} - \frac{1}{\sqrt{1-u^2/c^2}}\right]$$

Assuming $u \ll c$,

$$\frac{1}{1-u^2/c^2} \cong 1 + \frac{u^2}{c^2} \quad \text{and} \quad \frac{1}{\sqrt{1-u^2/c^2}} \cong 1 + \frac{1}{2}\frac{u^2}{c^2}$$

$$\Delta t \cong \frac{2L}{c}\left[1 + \frac{u^2}{c^2} - \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right)\right] = \frac{Lu^2}{c^3}$$

$$u = \sqrt{\frac{c^3 \Delta t}{L}} = \sqrt{\frac{(3\times 10^8\ \text{m/s})^3 (2\times 10^{-15}\ \text{s})}{11\,\text{m}}} = 7\times 10^4\ \text{m/s}$$

5. With $L = \frac{1}{2}L_0$, the length contraction formula gives $\frac{1}{2}L_0 = L_0\sqrt{1-u^2/c^2}$, so

$$u = \sqrt{3/4}\,c = 2.6\times10^8 \text{ m/s}$$

6. The astronaut must travel 600 light-years at a speed close to the speed of light and must age only 12 years. To an Earth-bound observer, the trip takes about Δt = 600 years, but this is a dilated time interval; in the astronaut's frame of reference, the elapsed time is the proper time interval Δt_0 of 12 years. Thus, with $\Delta t = \Delta t_0 / \sqrt{1-u^2/c^2}$,

$$600 \text{ years} = \frac{12 \text{ years}}{\sqrt{1-u^2/c^2}} \qquad \text{or} \qquad 1-\frac{u^2}{c^2} = \left(\frac{12}{600}\right)^2$$

$$u = \sqrt{1-(12/600)^2}\,c = 0.9998c$$

7. (a) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{120.0 \text{ ns}}{\sqrt{1-(0.950)^2}} = 384 \text{ ns}$$

(b) $$d = v\,\Delta t = 0.950(3.00\times10^8 \text{ m/s})(384\times10^{-9} \text{ s}) = 109 \text{ m}$$

(c) $$d_0 = v\,\Delta t_0 = 0.950(3.00\times10^8 \text{ m/s})(120.0\times10^{-9} \text{ s}) = 34.2 \text{ m}$$

10. Let ship A represent observer O, and let observer O' be on Earth. Then $v' = 0.831c$ and $u = -0.743c$, and so

$$v = \frac{v' + u}{1 + v'u/c^2} = \frac{0.831c + 0.743c}{1 + (0.831)(0.743)} = 0.973c$$

If now ship B represents observer O, then $v' = -0.743c$ and $u = -0.831c$.

$$v = \frac{v' + u}{1 + v'u/c^2} = \frac{-0.743c - 0.831c}{1 + (-0.743)(-0.831)} = -0.973c$$

11. Let O' be the observer on the space station, and let O be the observer on ship B. Then $v' = 0.811c$ and $u = -0.665c$.

$$v = \frac{v' + u}{1 + v'u/c^2} = \frac{0.811c - 0.665c}{1 + (0.811)(-0.665)} = 0.317c$$

13. With $f' = f\sqrt{(1-u/c)/(1+u/c)}$ and $\lambda = c/f$, we obtain

$$\frac{1-u/c}{1+u/c} = \left(\frac{f'}{f}\right)^2 = \left(\frac{\lambda}{\lambda'}\right)^2 = \left(\frac{650\text{ nm}}{550\text{ nm}}\right)^2 = 1.397$$

Solving, $u/c = 0.166$ or $u = 5.0 \times 10^7$ m/s.

17. For the light beam, observer O measures $v_x = 0$, $v_y = c$. Observer O' measures

$$v'_x = \frac{v_x - u}{1 - uv_x / c^2} = 0 - u = -u \quad \text{and} \quad v'_y = \frac{v_y\sqrt{1-u^2/c^2}}{1 - uv_x / c^2} = c\sqrt{1-u^2/c^2}$$

According to O', the speed of the light beam is

$$v' = \sqrt{(v'_x)^2 + (v'_y)^2} = \sqrt{u^2 + c^2(1-u^2/c^2)} = c$$

18. O measures times t_1 and t_2 for the beginning and end of the interval, while O' measures t'_1 and t'_2. Using Equation 2.23*d*,

$$t'_1 = \frac{t_1 - ux/c^2}{\sqrt{1-u^2/c^2}} \qquad \text{and} \qquad t'_2 = \frac{t_2 - ux/c^2}{\sqrt{1-u^2/c^2}}$$

The same coordinate x appears in both expressions, because the bulb is at rest according to O (so Δt is the proper time interval). Subtracting these two equations, we obtain

$$t'_2 - t'_1 = \frac{t_2 - t_1}{\sqrt{1-u^2/c^2}} \qquad \text{or} \qquad \Delta t' = \frac{\Delta t}{\sqrt{1-u^2/c^2}}$$

27. (a) To an Earth-bound observer Alice's round trip takes 20 years each way (20 years × 0.6c = 12 light-years) for a total time of 40 years. Bob's travel time is 15 years each way (15 years × 0.8c = 12 light-years) for a total travel time of 30 years. With Bob's 10-year delay in departing, the two arrive on Earth simultaneously.

(b) To Alice, the distance to the star is contracted to

$$L = L_0\sqrt{1 - v^2 / c^2} = 12 \text{ light-years}\sqrt{1-(0.6)^2} = 9.6 \text{ light-years}$$

So in Alice's frame of reference the trip takes a time of (9.6 light years)/0.6c = 16 years each way. To Bob, the distance to the star is

$$L = L_0\sqrt{1 - v^2 / c^2} = 12 \text{ light-years}\sqrt{1-(0.8)^2} = 7.2 \text{ light-years}$$

and in Bob's frame the travel time is (7.2 light-years)/0.8c = 9 years each way. Relative to Alice's original departure time, Alice has aged 32 years while Bob has aged 10 + 18 = 28 years. So Bob is younger by 4 years.

28. (a) Suppose Agnes travels at speed v. Then in her reference frame the distance to the star is shortened to $L = L_0\sqrt{1-v^2/c^2}$, so the time for her one-way trip is L/v and thus

$$\frac{16 \text{ light-years}\sqrt{1-v^2/c^2}}{v} = 10 \text{ y} \qquad \text{or} \qquad \sqrt{\frac{c^2}{v^2}-1} = \frac{10}{16}$$

Solving, we find v = 0.848c.

(b) According to Bert, Agnes traveled on a journey of 32 light-years at a speed of 0.848c which corresponds to a time of (32 light-years)/0.848c = 37.7 years.