1. (a) Conservation of momentum gives $p_{x,\text{initial}} = p_{x,\text{final}}$, or

$$m_{\text{H}} v_{\text{H,initial}} + m_{\text{He}} v_{\text{He,initial}} = m_{\text{H}} v_{\text{H,final}} + m_{\text{He}} v_{\text{He,final}}$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$\begin{aligned} v_{\text{He,final}} &= \frac{m_{\text{H}}(v_{\text{H,initial}} - v_{\text{H,final}})}{m_{\text{He}}} \\ &= \frac{(1.674\times10^{-27}\ \text{kg})[1.1250\times10^{7}\ \text{m/s} - (-6.724\times10^{6}\ \text{m/s})]}{6.646\times10^{-27}\ \text{kg}} = 4.527\times10^{6}\ \text{m/s} \end{aligned}$$

(b) Kinetic energy is the only form of energy we need to consider in this elastic collision. Conservation of energy then gives $K_{\text{initial}} = K_{\text{final}}$, or

$$\tfrac{1}{2} m_{\text{H}} v_{\text{H,initial}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,initial}}^2 = \tfrac{1}{2} m_{\text{H}} v_{\text{H,final}}^2 + \tfrac{1}{2} m_{\text{He}} v_{\text{He,final}}^2$$

Solving for $v_{\text{He,final}}$ with $v_{\text{He,initial}} = 0$, we obtain

$$\begin{aligned} v_{\text{He,final}} &= \sqrt{\frac{m_{\text{H}}(v_{\text{H,initial}}^2 - v_{\text{H,final}}^2)}{m_{\text{He}}}} \\ &= \sqrt{\frac{(1.674\times10^{-27}\ \text{kg})[(1.1250\times10^{7}\ \text{m/s})^2 - (-6.724\times10^{6}\ \text{m/s})^2]}{6.646\times10^{-27}\ \text{kg}}} = 4.527\times10^{6}\ \text{m/s} \end{aligned}$$

5. (a) The kinetic energy of the electrons is

$$K_\mathrm{i} = \tfrac{1}{2} m v_\mathrm{i}^2 = \tfrac{1}{2}(9.11\times10^{-31}\ \mathrm{kg})(1.76\times10^{6}\ \mathrm{m/s}) = 14.11\times10^{-19}\ \mathrm{J}$$

In passing through a potential difference of $\Delta V = V_\mathrm{f} - V_\mathrm{i} = +4.15$ volts, the potential energy of the electrons changes by

$$\Delta U = q\Delta V = (-1.602\times10^{-19}\ \mathrm{C})(+4.15\ \mathrm{V}) = -6.65\times10^{-19}\ \mathrm{J}$$

Conservation of energy gives $K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$, so

$$K_\mathrm{f} = K_\mathrm{i} + (U_\mathrm{i} - U_\mathrm{f}) = K_\mathrm{i} - \Delta U = 14.11\times10^{-19}\ \mathrm{J} + 6.65\times10^{-19}\ \mathrm{J} = 20.76\times10^{-19}\ \mathrm{J}$$

$$v_\mathrm{f} = \sqrt{\frac{2K_\mathrm{f}}{m}} = \sqrt{\frac{2(20.76\times10^{-19}\ \mathrm{J})}{9.11\times10^{-31}\ \mathrm{kg}}} = 2.13\times10^{6}\ \mathrm{m/s}$$

(b) In this case $\Delta V = -4.15$ volts, so $\Delta U = +6.65\times10^{-19}$ J and thus

$$K_\mathrm{f} = K_\mathrm{i} - \Delta U = 14.11\times10^{-19}\ \mathrm{J} - 6.65\times10^{-19}\ \mathrm{J} = 7.46\times10^{-19}\ \mathrm{J}$$

$$v_\mathrm{f} = \sqrt{\frac{2K_\mathrm{f}}{m}} = \sqrt{\frac{2(7.46\times10^{-19}\ \mathrm{J})}{9.11\times10^{-31}\ \mathrm{kg}}} = 1.28\times10^{6}\ \mathrm{m/s}$$

12. The combined particle, with mass $m' = m_1 + m_2 = 3m$, moves with speed v' at an angle θ with respect to the x axis. Conservation of momentum then gives:

$$p_{x,\text{initial}} = p_{x,\text{final}}: \quad m_1 v_1 = m'v'\cos\theta \qquad \text{or} \qquad v = 3v'\cos\theta$$

$$p_{y,\text{initial}} = p_{y,\text{final}}: \quad m_2 v_2 = m'v'\sin\theta \qquad \text{or} \qquad \tfrac{4}{3}v = 3v'\sin\theta$$

We can first solve for θ by dividing these two equations to eliminate the unknown v':

$$\tan\theta = \tfrac{4}{3} \qquad \text{or} \qquad \theta = 53.1°$$

Now we can substitute this result into either of the momentum equations to find

$$v' = 5v/9$$

The kinetic energy lost is the difference between the initial and final kinetic energies:

$$K_{\text{initial}} - K_{\text{final}} = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2 - \tfrac{1}{2}m'v'^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)(\tfrac{2}{3}v)^2 - \tfrac{1}{2}(3m)(\tfrac{5}{9}v)^2 = \tfrac{26}{27}(\tfrac{1}{2}mv^2)$$

The total initial kinetic energy is $\tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)(\tfrac{2}{3}v)^2 = \tfrac{17}{9}(\tfrac{1}{2}mv^2)$. The loss in kinetic energy is then $\tfrac{26}{51} = 51\%$ of the initial kinetic energy.

15. (a) With $K = \frac{3}{2}kT$,

$$\Delta K = \tfrac{3}{2}k\Delta T = \tfrac{3}{2}(1.38\times10^{-23}\ \text{J/K})(80\ \text{K}) = 1.66\times10^{-21}\ \text{J} = 0.0104\ \text{eV}$$

(b) With $U = mgh$,

$$h = \frac{U}{mg} = \frac{1.66\times10^{-21}\ \text{J}}{(40.0\ \text{u})(1.66\times10^{-27}\ \text{kg/u})(9.80\ \text{m/s}^2)} = 2550\ \text{m}$$

16. We take dE to be the width of this small interval: $dE = 0.04kT - 0.02kT = 0.02kT$, and we evaluate the distribution function at an energy equal to the midpoint of the interval ($E = 0.03kT$):

$$\frac{dN}{N} = \frac{N(E)\,dE}{N} = \frac{2}{\sqrt{\pi}}\frac{1}{(kT)^{3/2}}(0.03kT)^{1/2}e^{-(0.03kT)/kT}(0.02kT) = 3.79\times10^{-3}$$