28. The energy of the 2*p* to 1*s* Lyman transition is

$$E = (-13.60570\text{ eV})\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.20428\text{ eV}$$

and its wavelength (in the absence of fine structure) is

$$\lambda = \frac{hc}{E} = \frac{1239.842\text{ eV}\cdot\text{nm}}{10.20428\text{ eV}} = 121.5022\text{ eV}$$

With the fine structure energy splitting of 4.5×10^{-5} eV, the wavelength splitting is

$$\Delta\lambda = \frac{\lambda^2}{hc}\Delta E = \frac{(121.5\text{ nm})^2}{1240\text{ eV}\cdot\text{nm}}(4.5\times10^{-5}\text{ eV}) = 0.00054\text{ nm}$$

The fine structure splits one level up by $0.5\Delta E$ and the other down by the same amount, so the wavelengths are

$$\lambda + \tfrac{1}{2}\Delta\lambda = 121.5024\text{ nm} \qquad \text{and} \qquad \lambda - \tfrac{1}{2}\Delta\lambda = 121.5019\text{ nm}$$

1. (a) For a 2p electron, $n = 2$, $l = 1$, $m_l = 0,\pm1$ and $m_s = \pm½$, so the possible sets of quantum numbers (n,l,m_l,m_s) are:

$$(2,1,+1,+½), (2,1,+1,–½), (2,1,0,+ ½), (2,1,0,–½), (2,1,–1,+½), (2,1,–1,–½)$$

(b) There are 6 possible sets of quantum numbers for each electron, so the total number of possibilities for 2 electrons is $6 \times 6 = 36$.

(c) The Pauli principle prevents the two sets from being identical. There will be 6 combinations in which the two sets are identical; eliminating these combinations leaves 30 allowed combinations.

(d) Because the n values are different, the Pauli principle does not restrict the number of combinations, to there will be 36 possible combinations.

2. (a) The two electrons in the 1s level have m_s of +1/2 and –1/2, so they do not contribute to the total m_s, and the same is true for the two electrons in the 2s level. In the 2p level, there are three different possible values of m_l, and for each of those values we can assign a set of quantum numbers with $m_s = +1/2$, so the maximum possible value of the total m_s is +3/2.

(b) $(n, l, m_l, m_s) = (2, 1, +1, +1/2), (2, 1, 0, +1/2), (2, 1, -1, +1/2)$

(c) There is only one possible value of the total m_l in the states that maximize m_s, and from the states listed in (b) that value is $+1 + 0 + (-1) = 0$.

(d) We could maximize the total m_l by giving the first 2p electron $m_l = +1$, and the second electron can also have $m_l = +1$ if we give these two electrons opposite values of m_s. The third electron cannot have $m_l = +1$, so we must assign it $m_l = 0$ and the maximum total m_l is +2.

4. (a) In beryllium ($1s^2 2s^2$) the smallest energy jump is from $2s$ to $2p$.
(b) In neon ($1s^2 2s^2 2p^6$) the smallest energy jump is from $2p$ to $3s$.
(c) From Figure 8.1 we see that the $2s \rightarrow 2p$ energy difference is smaller than the $2p \rightarrow 3s$ difference, so the minimum absorption energy would be smaller for beryllium.

11. Singly ionized lithium has two electrons. When one of those is excited to a higher level, it is screened by the one electron remaining in the 1*s* level so $Z_{\text{eff}} = 3 - 1 = 2$. The expected energy when the outer electron is excited to the 2*p* level is

$$E_n = (-13.6\text{ eV})\frac{Z_{\text{eff}}^2}{n^2} = (-13.6\text{ eV})\frac{2^2}{2^2} = -13.6\text{ eV}$$

which agrees very well with the measured value of −13.4 eV. When the outer electron is in the 3*d* level, its expected energy is

$$E_n = (-13.6\text{ eV})\frac{Z_{\text{eff}}^2}{n^2} = (-13.6\text{ eV})\frac{2^2}{3^2} = -6.0\text{ eV}$$

in excellent agreement with the measured value.

8. (a) $[He]2s^2 2p^1$ (b) $[He]2s^2 2p^4$ (c) $[Ne]3s^2 3p^6$ (d) $[Ar]4s^2 3d^{10}$

16. Solving Equation 8.4 for Z with $\Delta E = \dfrac{hc}{\lambda} = \dfrac{1240 \text{ eV} \cdot \text{nm}}{0.1940 \text{ nm}} = 6392 \text{ eV}$, we obtain

$$Z = 1 + \sqrt{\frac{\Delta E}{10.2 \text{ eV}}} = 1 + \sqrt{\frac{6392 \text{ eV}}{10.2 \text{ eV}}} = 26$$

so the element is iron.

17. Ca (Z = 20): $\Delta E = (10.2 \text{ eV})(Z-1)^2 = (10.2 \text{ eV})(19)^2 = 3.68 \text{ keV}$

Zr (Z = 40): $\Delta E = (10.2 \text{ eV})(Z-1)^2 = (10.2 \text{ eV})(39)^2 = 15.5 \text{ keV}$

Hg (Z = 80): $\Delta E = (10.2 \text{ eV})(Z-1)^2 = (10.2 \text{ eV})(79)^2 = 63.7 \text{ keV}$

The values computed from Moseley's law are smaller than the measured values, and the discrepancy increases as Z increases.

27. (a) For the 3*s* outer electron of sodium, inserting $E_3 = -5.14$ eV into Equation 8.1 gives

$$Z_{\text{eff}} = n\sqrt{\frac{E_n}{-13.6\text{ eV}}} = 3\sqrt{\frac{-5.14\text{ eV}}{-13.6\text{ eV}}} = 1.84$$

The simple screening model predicts $Z_{\text{eff}} = 1$, so clearly the 3*s* electron is slightly penetrating the inner orbits and so is less screened by the inner electrons.

(b) For the 4*f* state,

$$Z_{\text{eff}} = n\sqrt{\frac{E_n}{-13.6\text{ eV}}} = 4\sqrt{\frac{-0.85\text{ eV}}{-13.6\text{ eV}}} = 1.00$$

so the screening is complete, with the 11 positive charges in the nucleus screened by the 10 electrons in the $n = 1$ and $n = 2$ shells.

30. The wavelength difference is $\Delta\lambda = 0.59$ nm. By taking differentials of $E = hc/\lambda$, we can find the corresponding energy difference:

$$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{(590 \text{ nm})^2}(0.59 \text{ nm}) = 2.1\times10^{-3} \text{ eV}$$

This energy difference comes from the interaction of a magnetic field B with a magnetic moment that we assume is of the order of 1 μ_B. The energy difference between the cases with the magnetic moment parallel to B and antiparallel to B is (see Figure 7.25) $\Delta E = 2\mu_B B$, so

$$B = \frac{\Delta E}{2\mu_B} = \frac{2.1\times10^{-3} \text{ eV}}{2(5.8\times10^{-5} \text{ eV/T})} = 18 \text{ T}$$

This is quite a large magnetic field, of the order of the largest that can be produced in the laboratory with superconducting electromagnets.