PHYSICS 210B : NONEQUILIBRIUM STATISTICAL PHYSICS
HW ASSIGNMENT #2

(1) Consider a monatomic ideal gas in the presence of a temperature gradient $\boldsymbol{\nabla} T$. Answer the following questions within the framework of the relaxation time approximation to the Boltzmann equation.

(a) Compute the particle current $\boldsymbol{j}$ and show that it vanishes.

(b) Compute the 'energy squared' current,

$$\boldsymbol{j}_{\varepsilon^2} = \int \! d^3\!p \, \varepsilon^2 \boldsymbol{v} \, f(\boldsymbol{r}, \boldsymbol{p}, t) \quad .$$

(c) Suppose the gas is diatomic, so $c_p = \frac{7}{2} k_{\mathrm{B}}$. Show explicitly that the particle current $\boldsymbol{j}$ is zero. *Hint: To do this, you will have to understand the derivation of eqn. 8.85 in the Lecture Notes and how this changes when the gas is diatomic. You may assume* $\mathcal{Q}_{\alpha\beta} = \boldsymbol{F} = 0$.

(2) Consider a classical gas of charged particles in the presence of a magnetic field $\boldsymbol{B}$. The Boltzmann equation is then given by

$$\frac{\varepsilon - h}{k_{\mathrm{B}} T^2} \, f^0 \, \boldsymbol{v} \cdot \boldsymbol{\nabla} T - \frac{e}{mc} \, \boldsymbol{v} \times \boldsymbol{B} \cdot \frac{\partial \, \delta f}{d\boldsymbol{v}} = \left(\frac{\partial f}{\partial t} \right)_{\mathrm{coll}} \quad .$$

Consider the case where $T = T(x)$ and $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. Making the relaxation time approximation, show that a solution to the above equation exists in the form $\delta f = \boldsymbol{v} \cdot \boldsymbol{A}(\varepsilon)$, where $\boldsymbol{A}(\varepsilon)$ is a vector-valued function of $\varepsilon(\boldsymbol{v}) = \frac{1}{2} m \boldsymbol{v}^2$ which lies in the (x, y) plane. Find the energy current $\boldsymbol{j}_\varepsilon$. Interpret your result physically.

(3) A photon gas in equilibrium is described by the distribution function

$$f^0(\boldsymbol{p}) = \frac{2}{e^{cp/k_{\mathrm{B}} T} - 1} \quad ,$$

where the factor of 2 comes from summing over the two independent polarization states.

(a) Consider a photon gas (in three dimensions) slightly out of equilibrium, but in steady state under the influence of a temperature gradient $\boldsymbol{\nabla} T$. Write $f = f^0 + \delta f$ and write the Boltzmann equation in the relaxation time approximation. Remember that $\varepsilon(\boldsymbol{p}) = cp$ and $\boldsymbol{v} = \frac{\partial \varepsilon}{\partial \boldsymbol{p}} = c\hat{\boldsymbol{p}}$, so the speed is always c.

(b) What is the formal expression for the energy current, expressed as an integral of something times the distribution f?

(c) Compute the thermal conductivity κ. It is OK for your expression to involve *dimensionless* integrals.

(4) Suppose the relaxation time is energy-dependent, with $\tau(\varepsilon) = \tau_0 \, e^{-\varepsilon/\varepsilon_0}$. Compute the particle current $\boldsymbol{j}$ and energy current $\boldsymbol{j}_\varepsilon$ flowing in response to a temperature gradient $\boldsymbol{\nabla}T$.

(5) Use the linearized Boltzmann equation to compute the bulk viscosity ζ of an ideal gas.

(a) Consider first the case of a monatomic ideal gas. Show that $\zeta = 0$ within this approximation. Will your result change if the scattering time is energy-dependent?

(b) Compute ζ for a diatomic ideal gas.

(6) Consider a two-dimensional gas of particles with dispersion $\varepsilon(\boldsymbol{k}) = J\boldsymbol{k}^2$, where $\boldsymbol{k}$ is the wavevector. The particles obey photon statistics, so $\mu = 0$ and the equilibrium distribution is given by

$$f^0(\boldsymbol{k}) = \frac{1}{e^{\varepsilon(\boldsymbol{k})/k_\mathrm{B}T} - 1} \quad .$$

(a) Writing $f = f^0 + \delta f$, solve for $\delta f(\boldsymbol{k})$ using the steady state Boltzmann equation in the relaxation time approximation,

$$\boldsymbol{v} \cdot \frac{\partial f^0}{\partial \boldsymbol{r}} = -\frac{\delta f}{\tau} \quad .$$

Work to lowest order in $\boldsymbol{\nabla}T$. Remember that $\boldsymbol{v} = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial \boldsymbol{k}}$ is the velocity.

(b) Show that $\boldsymbol{j} = -\lambda \, \boldsymbol{\nabla}T$, and find an expression for λ. Represent any integrals you cannot evaluate as dimensionless expressions.

(c) Show that $\boldsymbol{j}_\varepsilon = -\kappa \, \boldsymbol{\nabla}T$, and find an expression for κ. Represent any integrals you cannot evaluate as dimensionless expressions.