→ Unidirectional NL Alfven Wave

- key to Alfvenic turbulence is counter-streaming population $\quad (\underline{\nabla}\cdot\underline{V}=0)$

$$\frac{d}{dt}Z_\pm = \underline{Z}_\mp\cdot\underline{\nabla}\,\underline{Z}_\pm + \dots$$

if unidirectional,

$$\frac{d}{dt}Z_\pm = 0 \quad \rightarrow \text{ invariant packet}$$

⇒

- begs for compressibility

"weak" ↔ slow

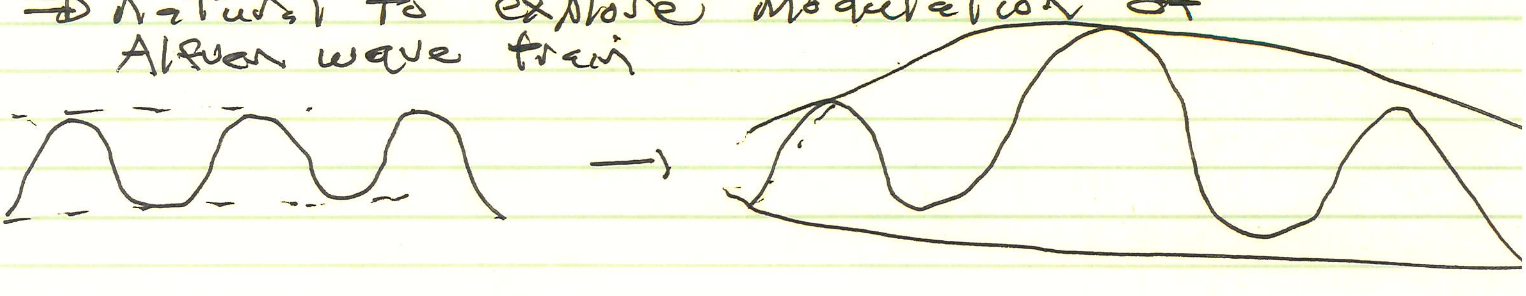
"strong" ↔ fast

Consider parallel compression → $\nabla_\parallel \tilde{V}_\parallel \neq 0$

→ acoustic coupling

Now $\quad \omega = k_\parallel V_A = \dfrac{k_\parallel B_0}{(4\pi\rho)^{1/2}}$, $\quad \rho \rightarrow \rho_0 + \tilde{\rho}$

⇒ natural to explore modulation of Alfven wave train

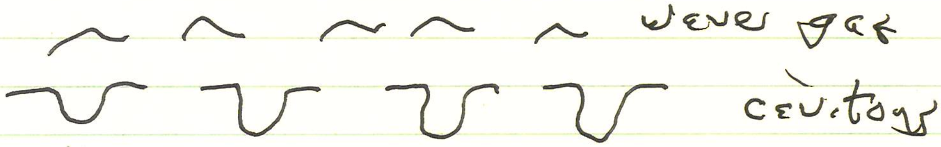


→ begs physical idea, as in Langmuir

i.e. recall,

$$\omega^2 = \omega_{pe}^2 + \alpha k^2 v_{The}^2$$

→ NLS / Zakharov Eqns.



Langmuir Turbulence I

→ Disparate Scale Interaction
(see notes)

→ Deriving Zakharov Eqns.
critical

~ ~ ~ ~ ~ wave gas Langmuir (energy)
waves

$\mho$ $\mho$ $\mho$ $\mho$ density
perturbations

Coupling ?
Fate.

Observe: → $\omega^2 = \omega_p^2 (1 + \alpha k^2 \lambda_{De}^2)$ Langmuir

$\omega = k c_s$ (Density)
$\rightarrow 0$

so basic interaction must be
$L + L \rightarrow \delta n$

i.e. interaction of Langmuir wave energy gas with low frequency perturbations.

⇒ As energy field ↔ δn evolution, must be envelope.

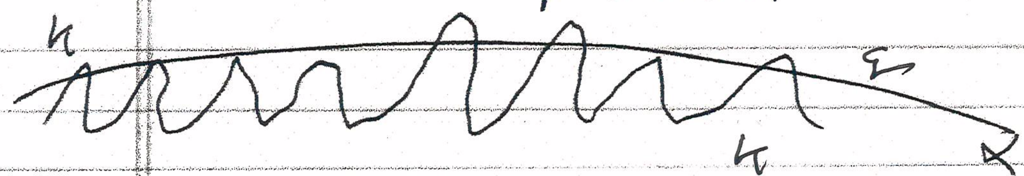
Now: $\omega^2 = \omega_{pe}^2(1+\alpha k^2\lambda_{De}^2)$

$$(\omega_0 + i\gamma)^2 = \omega_{pe_0}^2(1 \pm \delta n/n_0)\,(1+\alpha(k+q)^2\lambda_{De}^2)$$

slow evolution

modulation by density perturbation

envelope wave #



⇒

$$\omega_0'^2 + 2i\gamma\omega_0 + \cancel{(i\gamma)^2} = \omega_{pe_0}'^2 \pm \frac{\delta n}{n_0}\omega_{pe_0}^2$$
$$+ \alpha k'^2 v_{Th}^2 + \alpha v_{Th}^2(2\underline{k}\cdot\underline{q})$$
$$+ \alpha q^2 v_{Th}^2$$

$$\boxed{2i\gamma\omega_0 = \cancel{\pm}\frac{\delta n}{n_0}\omega_{pe_0}^2 + \alpha(2\underline{k}\cdot\underline{q})v_{Th}^2 + \alpha q^2 v_{Th}^2}$$

$\omega_0 \cong \omega_{pe}$

$$i\gamma\omega_0 = \frac{\omega_{pe_0}^2}{2}\frac{\delta n}{n_0} + \alpha(\underline{k}\cdot\underline{q})v_{Th}^2 + \frac{\alpha}{2}q^2 v_{Th}^2$$

$$\boxed{i\gamma \cancel{=} \frac{\omega_{p_0}}{2}\frac{\delta n}{n_0} + \frac{\alpha k v_{Th}^2\cdot q}{\omega_0} + \alpha\frac{q^2 v_{Th}^2}{\omega_0}}$$

$$\omega = \omega_{pe}\left(1+\alpha k^2 \lambda_D^2\right)^{1/2}$$

$$\frac{d\omega}{dk} = \omega_p\left(\alpha \underline{k}\, \lambda_D^2\right) \Big/ \cancel{\left(1+\cdots\right)^{1/2}}$$

$$= \alpha \underline{k}\, \frac{v_{T_e}^2}{\omega_{p0}} = v_{gr}$$

so $\displaystyle \alpha \frac{\underline{k}\, v_{T_e}^2}{\omega_0} \cdot \underline{\nabla} = \underline{\nabla} \cdot \underline{v}_{gr}$

Now $\displaystyle i\gamma = i\frac{\partial}{\partial t}$

so if shift to co-moving frame

i.e. $\underline{x} \to \underline{x} - \underline{v}_{gr}\, t$

then can eliminate $\underline{\nabla}\cdot\underline{v}_{gr}$ term.

$\therefore$

$$i\gamma = \frac{\omega_{pe}}{2}\frac{\delta n}{n_0} + \alpha \underline{\nabla}^2 \frac{v_{The}^2}{\omega_0}$$

$\Rightarrow$ if $\underline{E} = \mathcal{E}(x,t)\, e^{i(k\cdot x - \omega t)}$

$\uparrow$ envelope $\quad \uparrow$ carrier

$$i\frac{\partial \mathcal{E}}{\partial t} = \frac{\omega_{pe0}}{2}\frac{\delta n}{n}\mathcal{E} - \frac{\alpha v_{The}^2}{\omega_0}\nabla^2\mathcal{E}$$

refraction   diffraction

Can re-write:

$$\boxed{i\omega_0\frac{\partial \mathcal{E}}{\partial t} = \frac{\omega_{pe0}^2}{2}\frac{\delta n}{n_0}\mathcal{E} - \alpha v_{The}^2\nabla^2\mathcal{E}}$$

Now, for δn

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\underline{v}) = 0$$

$$n m_i \frac{d\underline{v}}{dt} = -\underline{\nabla} p$$

$$p = p_{Th} + p_{rad} \cong c_s^2 m_i \delta n + \frac{|\mathcal{E}|^2}{8\pi}$$

Ponderomotive Pressure

$$\partial_t \delta n + n_0 \underline{\nabla}\cdot\tilde{\underline{v}} = 0$$

$$n m_i \frac{\partial \underline{v}}{\partial t} = -\underline{\nabla}\left(c_s^2 m_i \delta n + \frac{|E|^2}{8\pi}\right)$$

$$\frac{\partial \underline{v}}{\partial t} = -\underline{\nabla}\left(c_s^2 \frac{\delta n}{n_0} + \frac{|E|^2}{8\pi n m_i}\right)$$

so

$$\partial_t(\underline{\nabla}\cdot\tilde{\underline{v}}) = -\nabla^2\left(c_s^2 \frac{\delta n}{n_0} + \frac{|E|^2}{8\pi n m_i}\right)$$

$$\partial_t^2(\delta n/n_0) + \partial_t(\underline{\nabla}\cdot\underline{v}) = 0$$

so

$$\boxed{\partial_t^2 \frac{\delta n}{n_0} - c_s^2 \underline{\nabla}^2 \frac{\delta n}{n_0} = \nabla^2\left(\frac{|E|^2}{8\pi n m_i}\right)}$$

so have 2 - eqns.

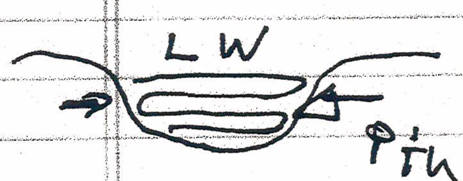
$$\boxed{\begin{aligned} i\omega_0 \frac{\partial \mathcal{E}}{\partial t} &= \frac{\omega_{pe0}^2}{2}\frac{\delta n}{n_0}\mathcal{E} - \alpha v_{The}^2 \nabla^2 \mathcal{E} \\ \partial_t^2 \frac{\delta n}{n_0} - c_s^2 \nabla^2 \frac{\delta n}{n_0} &= \nabla^2\left(\frac{|\mathcal{E}|^2}{8\pi n m_0}\right) \end{aligned}}$$

For $T \gg L/c_s$

$$-c_s^2 \nabla^2 \frac{\delta n}{n_0} = \nabla^2\left(\frac{|\mathcal{E}|^2}{8\pi n m_0}\right)$$

$$\therefore \frac{\delta n}{n_0} = \overset{\text{cavity}}{\overset{\downarrow}{-}}\frac{|\mathcal{E}|^2}{8\pi n T} \qquad \underline{\text{caviton}}$$

State of (thermal + Ponderomotive) Press ≈ 0



so plug into $\mathcal{E}$ eqn,

$$\boxed{i\omega_0 \frac{\partial \mathcal{E}}{\partial t} = -\frac{\omega_{p0}^2}{2}\left(\frac{|\mathcal{E}|^2}{8\pi n T}\right)\mathcal{E} - \alpha\, v_{the}^2 \nabla^2 \mathcal{E}}$$

i.e.

refraction $\downarrow$ potential $\downarrow$

$$i\omega_0 \frac{\partial \mathcal{E}}{\partial t} = -\alpha^2 v_{th}^2 \nabla^2 \mathcal{E} - \frac{\omega_{p0}^2}{2}\left(\frac{|\mathcal{E}|^2}{8\pi n T}\right)\mathcal{E}$$

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi + V\psi$$

$\Rightarrow$ NLS ! $\quad V < 0$ (attractive)

Also similar to self-focusing problem.

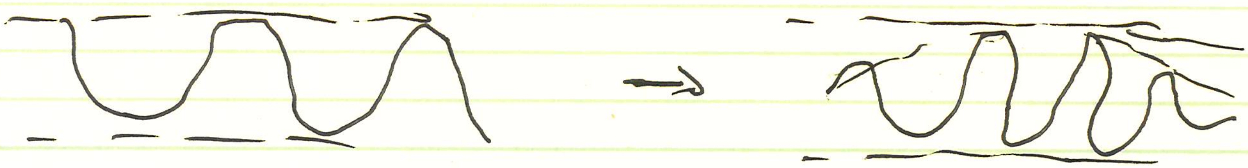
$\Rightarrow$ Now, can simplify description to

$\rightarrow$ acoustic wave

+

$\rightarrow$ adiabatic (Action) eqn.

so



Now, like NLS, need envelope equation.

$\rightarrow$ scale separation
- fast $\rightarrow$ carrier
- slow $\rightarrow$ envelope

$$\omega = k_{\parallel} V_A = \frac{k_{\parallel} B}{(\rho_0 + \tilde{\rho})^{1/2}\sqrt{4\pi}}$$

$$= k_{\parallel} V_{A_0} - \frac{k_{\parallel} V_{A_0}}{2} \frac{\tilde{\rho}}{\rho_0}$$

$$\omega = \omega^{(0)} + i\, \partial_t^{\text{slow}} -$$

$$k_{\parallel} = k_{\parallel}^{(0)} + i \left(\frac{\partial}{\partial z}\right)_{\text{slow}}$$

z, t slow

$$i \frac{\partial}{\partial t} \delta B = -i \frac{\partial}{\partial z} \frac{V_A}{2} \frac{\tilde{\rho}}{\rho_0} \delta B$$

$$\partial_t \delta B = -\partial_z \frac{V_A}{2} \frac{\tilde{\rho}}{\rho_0} \delta B$$

For $\tilde{\rho}$:

$$\partial_t \tilde{\rho} = -\rho_0\, \partial_z \tilde{V}_{\parallel}$$

$$= -\rho_0\, \nabla_{\parallel} \tilde{V}_{\parallel}$$

Now,

$$\rho \frac{\partial v}{\partial t} = -\nabla p - \nabla \frac{B^2}{8\pi} + \hat{B}\cdot\nabla\hat{B}$$

$$\quad - \rho\nabla \frac{V^2}{2} + \rho\, \underline{v} \times \underline{\omega}$$

$\hat{z}$.

$$\partial_t \tilde{V}_\parallel = -\frac{\nabla_\parallel \tilde{p}}{\rho} - \nabla_\parallel\left(\frac{B^2}{8\pi\rho} + V^2\right)$$

AW

$$\partial_t \tilde{V}_\parallel = -\frac{\nabla_\parallel \tilde{p}}{\rho} - \nabla_\parallel\left(\frac{|\tilde{B}|^2}{4\pi\rho}\right)$$

$$\partial_t^2 \frac{\tilde{\rho}}{\rho_0} = -\rho_0 \nabla_\parallel\left(-\frac{\nabla_\parallel \tilde{p}}{\rho_0} - \nabla_\parallel \frac{|\tilde{B}|^2}{4\pi\rho_0}\right)$$

$$= c_s^2 \nabla_\parallel^2 \frac{\tilde{\rho}}{\rho} + \rho_0 \nabla_\parallel^2 \frac{|\tilde{B}|^2}{4\pi\rho_0}$$

$$\rho = \rho(z - V_A t)$$

driven by wave

$$(V_A^2 - c_s^2)\nabla_\parallel^2 \tilde{\rho} = \nabla_\parallel^2 \frac{|\tilde{B}|^2}{4\pi}$$

$$(V_A^2 - C_s^2)\, \partial_{\parallel}^2 \frac{\tilde{\rho}}{\rho_0} = \cancel{\rho_0}\, \partial_{\parallel}^2 \frac{|\tilde{B}|^2}{4\pi \rho_0}$$

$\beta \neq 1$

$$\frac{\tilde{\rho}}{\rho_0} = \frac{|\tilde{B}|^2}{B_0^2} \frac{V_A^2}{V_A^2 - C_s^2}$$

$$= \left|\frac{\tilde{B}}{B_0}\right|^2 \frac{1}{1-\beta} \qquad \beta = \frac{C_s^2}{V_A^2}$$

$$\partial_t \delta B = -\frac{\partial}{\partial z} \frac{V_A}{2} \left|\frac{\delta B}{B_0}\right|^2 \frac{1}{(1-\beta)} \delta B$$

$$\boxed{\partial_t \frac{\delta B}{B_0} + V_A \partial_z \left\{ \frac{1}{2} \left|\frac{\delta B}{B_0}\right|^2 \frac{1}{1-\beta} \frac{\delta B}{B_0} \right\} = 0}$$

steepening

DNLS.

steepening halted by:

- dissipation $\quad \eta\, \partial_z^2 \delta B$
- dispersion $\quad d\, d_i^2\, \Omega_0\, \partial_z^2\, \delta B$

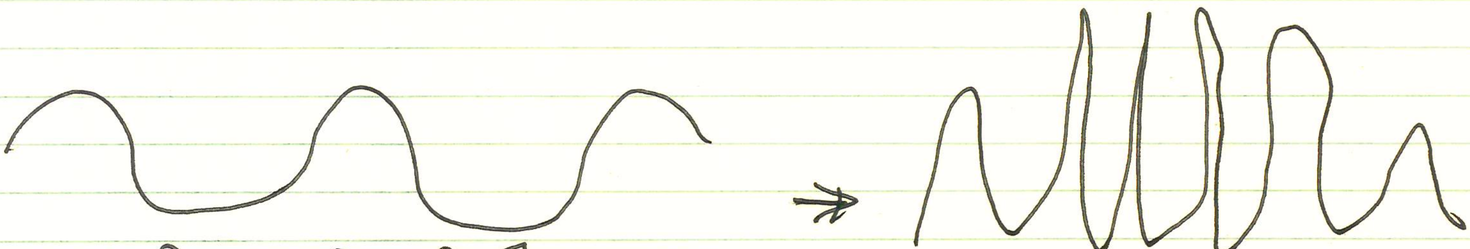
inertial scale

quasi-parallel "collisionless Alfvenic shock"

coherent coupling to kinetic scales

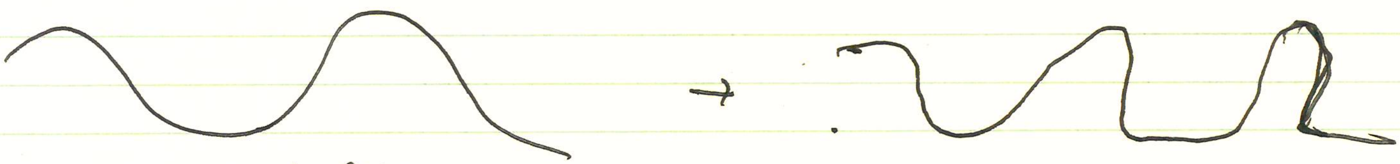
– evolve to RD / DD in MHD shocks.

Note that wave train compresses/steepens along propagation direction, like slinky.



$B = B(z)\,\hat{x}$

vs



$\rho = \rho(z)$

$\underline{v} = v(z)\,\hat{z}$

shock train

→ KNLS
KDNLS
$V_\parallel$ couples to Landau damping.

cf. Medvedev, P.D, et al.

N.B.: $\beta \to 1$

can't sleeve Acoustic to Alfven.

Need treat Alfven – Acoustic coupling.

i.e. Decay instability.