Simple Ideas in Non-Ideal MHD I

→ Freezing-in law:

$$\frac{d}{dt}\underline{B} = \underline{B}\cdot\nabla\underline{v} + \eta\nabla^2\underline{B}$$

↑ breaking ⇒ small scale → singularity / turbulence

→

→ current sheet
singular layers

→ sites of reconnection → boundary layer problem.

→ topology change

→ so

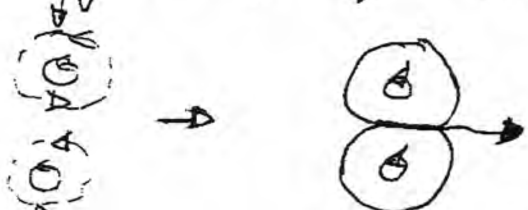
- Sweet-Parker Reconnection theory
- Re-visiting magnetic helicity; Taylor Theory → more coming
- anomalous resistivity (again)
- flux expulsion

next

→ Breakdown of Flux Freezing – Magnetic Reconnection ?!

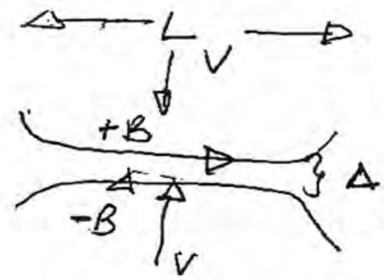
Simple Example : Sweet – Parker Problem
(re-visit later)

→ consider two cylinders of plasma, carrying current ⊥ plane, brought together



⇒ consider layer

2 plasma slabs brought together at V



$\Delta < L$

What Happens ?!
Stationary Solution Possible ?

$\nabla\cdot\underline{V} = 0 \qquad \frac{d}{dt}\underline{B} = \underline{B}\cdot\nabla\underline{V} + \eta\nabla^2\underline{B}$

$S_{ij}^{(0)} = \begin{pmatrix} 0, & 0 \\ 0, & -V\delta(y) \end{pmatrix}$

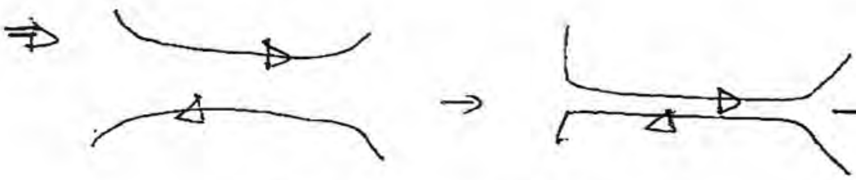
S – rate-of-strain tensor

→ singularity !

tip-off of small scale generation in $\underline{B}$
⇒ resistive diffusion, breaking of freezing-in.

i.e. for stationary solution,

$$-\frac{\underline{B}\cdot\underline{\nabla}\,\underline{V}}{\eta} = \nabla^2\underline{B} \quad , \quad \underline{\nabla}\cdot\underline{V} = 0$$

⇒ 

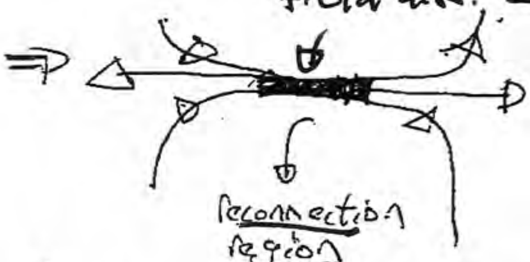
→ Δ small

$\nabla(B^2/8\pi)$ increasing

⇓

fluid expelled

field diss. ⇒ current sheet

⇒ 

out flow along B_0

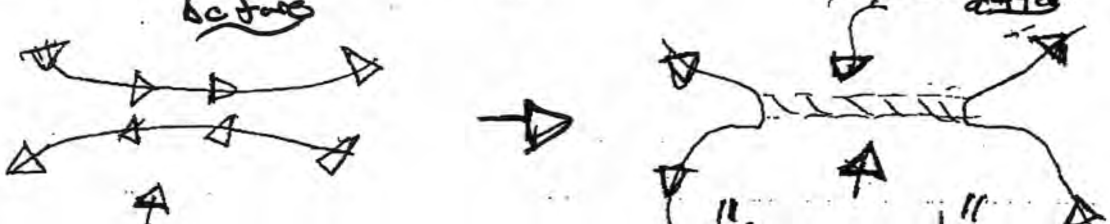
expect opposed fields "annihilate"

(large, resistive dissipation)

N.B. { A particular V value is required for stationarity. Layer

N.B. → why "reconnection"?

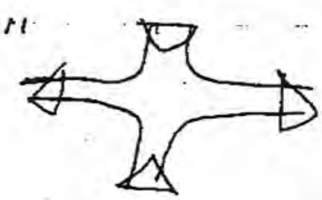
before / after



initial magnetic configuration

"reconnected" configuration

→ Flow is "stagnation point"



aka' shock

→ How Calculate? → Match In-Flow → Out-Flow
(S.-P. is a great Back-of-Envelope...)

Conserved: ① – mass ($\underline{\nabla}\cdot\underline{V} = 0$)

② – momentum in $\hat{x}$ direction (symmetry)

③ – energy balance →

→ rate of field 'delivery' to reconnection region

MUST BALANCE

→ rate of Ohmic dissipation $E\cdot J \sim \eta J^2$

① extent in $\hat{x}$, extent in $\hat{y}$

$$\rho_0 V L = \rho_0 V_0 \Delta$$

inflow, outflow

Mass balance

$\nabla\cdot\underline{V} = 0$

$$VL = V_0 \Delta$$

$$V = V_{out}\,\Delta / L$$

② $$\rho_0\left(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}\right) = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{\underline{B}\cdot\nabla\underline{B}}{4\pi}$$

$$\underline{V}\cdot\underline{\nabla}\,\underline{V} = -\underline{\nabla}\left(\frac{V^2}{2}\right) + \underline{V}\times\underline{\omega}$$

symmetry: $$0 = \underline{\nabla}\left(P + \frac{B^2}{8\pi} + \frac{\rho_0 V^2}{2}\right)$$

modified Bernoulli Eqn.

a   b

a → $V = 0$, B finite

b → $V = V_{out}$, $B \to 0$

$$\left(\frac{B^2}{8\pi} \ll P\right)$$

So $\quad p + \frac{B^2}{8\pi} + \frac{\rho_0 V^2}{2} = \text{const.}$

$$p + \frac{B^2}{8\pi} = p + \frac{\rho_0 V_{out}^2}{2}$$

$$V_{out}^2 = B^2/4\pi\rho_0 = V_A^2$$

↳ Alfven speed

$$\left[\begin{array}{l} V_{out} = V_A \\ V = V_A \frac{\Delta}{L} \end{array}\right.$$

specifies "speed V", in terms Δ.

③ 

Energy balance

$$\Rightarrow \left[\begin{pmatrix}\text{Rate of Magnetic} \\ \text{Energy Inflow}\end{pmatrix} = \begin{pmatrix}\text{Rate of Ohmic} \\ \text{Dissipation, net}\end{pmatrix} \right.$$

$$P_{OH} = \frac{J^2}{\sigma}\Delta L \quad \text{so} \quad \dot{E}_{OH} = \frac{J^2}{\sigma} L\Delta$$

$$= \left(\frac{c}{2\pi}\right)^2 \frac{B^2}{\Delta^2} \frac{L\Delta}{\sigma}$$

$$\nabla \times B = \frac{4\pi}{c} J$$

$$2B = \frac{4\pi}{c} J\Delta$$

$$P_{in} = 2\left(B^2/8\pi\right) VL = \dot{E}_{in}$$

balance $\Rightarrow \cancel{2}\left(\frac{\cancel{B^2}}{\cancel{8\pi}}\right) V\cancel{L} = \frac{c^2}{\cancel{4\pi}\,\cancel{\sigma}} \frac{\cancel{B^2}}{\Delta^2} \cancel{L}\,\Delta$

$$V = \left(\frac{c^2}{\pi\sigma}\right) / \Delta \sim \frac{\eta}{\Delta}$$

$$\frac{c^2}{4\pi\sigma} \equiv \eta \left(\sim \frac{L^2}{T}\right)$$

$$\frac{V}{L} \sim \left(\frac{\eta}{L^2}\right)^{1/2} \frac{V_A^{3/2}}{\Delta} \sqrt{\frac{L}{V_A}}$$

$$\sim \left(\frac{\eta}{L^2}\right)^{1/2} \sqrt{\frac{L}{V_A}}\,\frac{V_A}{L}$$

$$\sim S^{-1/2}\,\frac{V_A}{L}$$

$$V \sim \frac{V_A\,\eta^{1/2}}{L^{1/2}}$$

$$1/\tau \sim \frac{V}{L}$$

$$V = V_A\,\Delta/L$$

$$V = \eta/\Delta$$

$$\Rightarrow \boxed{\frac{\Delta}{L} = \left(\frac{\eta L}{L^2 V_A}\right)^{1/2} = \left(1/R_m\right)^{1/2}}$$

and $\boxed{V = V_A/\sqrt{R_m}}$

$R_m = \frac{VL}{\eta}$ ≡ Magnetic Reynolds # (here with $V = V_A$)

⇒ Punch Line: (for large R_m)

① – layer is thin $\quad \frac{\Delta}{L} \sim 1/\sqrt{R_m}$

– speed is faster than η/L, slower than V_A $\quad V \sim V_A/\sqrt{R_m}$

② Flow pattern is a la' stagnation ⇒ ejection from reconnection layer at V_A

Moral of this story: $\quad \frac{1}{\tau_R} \sim \frac{V}{L} \sim \frac{(V_A)^{1/2}\,\eta^{1/2}}{L^{3/2}}$

→ freezing-in violated when flows bring opposing $\underline{B}$ into contact

⇒

→ generates singularities ⇒ thin current layers, which alter initial magnetic topology
⇒ "magnetic reconnection", "tearing", etc.

Note asymmetry; L, Δ $\qquad$ slow

→ Sweet-Parker is too ~~slow~~

i.e. $\frac{V}{L} \sim 1/\tau_R \sim \underbrace{\frac{V_A}{L}}_{\text{ideal}} S^{-1/2}$

→ Alternatives

- shorten asymmetry
  i.e. $\Delta \to L$

  ⇒ hyperdiffusion, electron viscosity
  electron inertia

  i.e. $\frac{\Delta}{L} \sim 1/R_{m_{eff}}^{1/4}$

  $\sim \frac{c^2}{\omega_{pe}^2} \mu_H$

- anomalous resistivity → P_{ei} wave
  → low collisionality

- small scale Hall whistler

- waves

- shocks

$$B \cdot \left[\underline{E} + \frac{\underline{v} \times \underline{B}}{c} = \eta \underline{J} \right]$$ ②

<u>DKE</u>

$$\underline{e} \quad \frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} - \frac{c}{B_0} \nabla \phi \times \hat{z} \cdot \nabla f$$

$$- \frac{|e|}{m_e} E_{\parallel} \frac{\partial f}{\partial v} = C(f)$$

$$\frac{|e|}{m_e} E_{\parallel} = \overset{①}{\langle v\, C(f) \rangle} + \overset{②}{\frac{\partial J_{\parallel}}{\partial t}}$$

$$\int v \cdot \left(-\nu \delta (f - f_0) \right)$$

$$- \underset{③}{\frac{c}{B_0} \nabla \phi \times \hat{z} \cdot \nabla J_{\parallel}} \quad \underset{④}{- \frac{\partial}{\partial z} \langle v^2 f \rangle}$$

① → resistivity
coll n.

② → electron inertia $\sim \frac{c^2}{\omega_{pe}^2}$

③ → electron visc.
CDIA

④ → driftwave / pressure

ϕ → waves

i.e. $\frac{\tilde{n}_e}{n} \sim \frac{|e|\hat{\phi}}{T} + \cdots$

Anomalous Resistivity

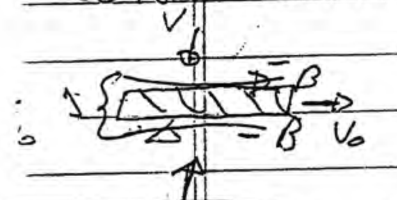
42. ~~Applications III~~ 22.

Anomalous Resistivity → $\left\{ \begin{array}{l} \text{Application} \\ \text{of} \\ \text{QLT} \end{array} \right.$ $\left\{ \begin{array}{l} \text{Follows / expands} \\ \text{on Galeev/Sagdeev} \\ \text{Rev. Pl Phys 7.} \end{array} \right.$

→ an instructive and important example of quasilinear theory is anomalous resistivity

→ here, try approach classic current-driven ion acoustic instability (CDIA) model of anomalous resistivity via coupled micro-macro dynamics W.

– consider Sweet-Parker model, i.e.



$$V L = V_{out} \Delta$$

$$V_{out} = V_A$$

$$2 V \frac{B^2}{8\pi} L = \eta \bar{J}^2 L \Delta$$

cf. 218B notes

$\langle E \rangle = \langle \frac{v B}{c} \rangle \approx 0$
Cuts parcel

$$\frac{\Delta}{L} = \frac{v}{V_A} \Rightarrow \boxed{\Delta^2 = \frac{(L)(\eta)}{V_A}} \Rightarrow \text{layer width}$$

$$\frac{\Delta}{L} \sim 1/\sqrt{R_m}$$

What happens as η decreased ?

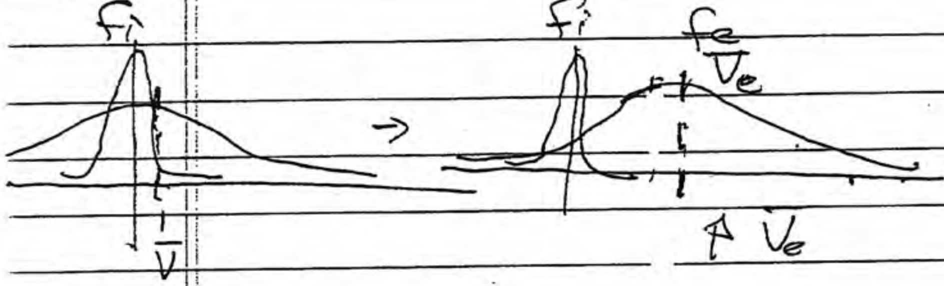
$$-\frac{c}{4\pi\Delta} B = J = n q \bar{V}_e$$

electron drift speed

so $\bar{V}_e = cB/4\pi n q \Delta = \frac{d_{skin\,e}^2}{\Delta} \Omega_e$

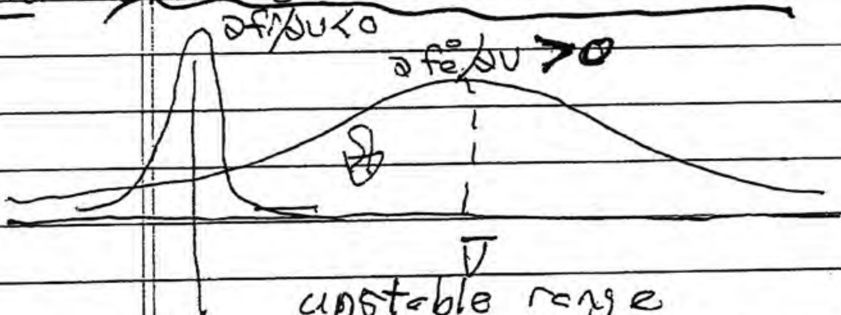
Now $\bar{V}_e \sim B/\Delta n \Rightarrow \bar{V}_e \uparrow$ as
- $\Delta \downarrow \Rightarrow$ narrow layer
- $n_b \Rightarrow$ few charge carriers
- $B \uparrow \Rightarrow$ stronger field (drive)

$\Delta \uparrow \Rightarrow$



⇒ decreasing Δ raises $\bar{V}$
→ up-shifts fe centroid relative to fi
⇒ destabilizes CDIA !

i.e. classic scenario of CDIA



CDIA as origin of anomalous resistivity

∴ expect CDIA will:
→ exchange momentum between electrons and waves
so → slow down electrons, reduce $\bar{V}_e$
→ act as "anomalous turbulent" resistivity

44.

i.e. $\left\{ \begin{array}{l} \Delta^2 = \frac{L}{V_A}\left(\eta + \eta_A(\overline{V})\right) \\ \quad \hookrightarrow \text{anomalous resistivity} \\ \overline{V} = \frac{cB}{4\pi}/nq\Delta \quad \text{threshold} \end{array} \right.$

How calculate:

(a) Brute Force

- confining oneself to 1D model, ignoring layer structure, have:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{\varepsilon}{m}E\frac{\partial f}{\partial V} = -C(f)$$

here $x \rightarrow$ vertical
$v \rightarrow$ vertical velocity
vertical $\rightarrow$ $\perp$ to layer

$m_e V \int \Rightarrow$

$$\frac{\partial \langle P_e \rangle}{\partial t} - q\left\langle E \int dv f \right\rangle = -\nu_{e,i} n_0 m_e \overline{V}_e$$

collisional loss to ions

$$\frac{\partial \langle P_e \rangle}{\partial t} - q n_0 \langle E \rangle - q\langle \tilde{E}\tilde{n} \rangle = -\nu_{e,i} n_0 m_e \overline{V}_e$$

so

$$\langle E \rangle + \frac{\langle \tilde{E}\tilde{n} \rangle}{n_0} - \frac{1}{n_0 q}\frac{\partial \langle P_e \rangle}{\partial t} = +\frac{\nu_{ei} m_e n_0 q \overline{V}_e}{n_0 q^2}$$

at ⓐ stationary state,

$$\langle E \rangle + \langle \tilde{E} \frac{\tilde{n}}{n_0} \rangle = \eta \langle J \rangle$$

driving field $\sim \langle VB \rangle$

electron acceleration by turbulence → QL

⇒ "anomalous resistivity"

collisional resistivity

to calculate:

$$\langle \tilde{E} \frac{\tilde{n}}{n_0} \rangle = \sum_k +ik \hat{\phi}_{-k} \frac{\tilde{n}^e_k}{n_0}$$

$$= \int dv \sum_k ik \hat{\phi}_{-k} \tilde{f}^e_k$$

electron density perturbation

$f^e_k \rightarrow f^{e\,L}_k$

- quasilinear calculation
- stationarity ⇒ resonant transport

ⓑ Conservation Argument

- as in ⓐ anticipate stationarity ⇒ resonant quasilinear evolution

- recall,

$$\frac{\partial}{\partial t}\left(\mathcal{E}^{RP} + \mathcal{E}^{wave}_{\omega N}\right) = 0$$

$$\frac{\partial}{\partial t}\left(P^{RP} + P^{wave}_{kN}\right) = 0$$

$$\mathcal{E}^{w}_{k} = \omega_k \left.\frac{\partial \epsilon}{\partial \omega}\right|_k \frac{|E_k|^2}{8\pi} \equiv \omega_k \underbrace{N_k}_{\#\ \text{quanta}}$$

$$P^{w}_{k} = \frac{k}{\omega} \mathcal{E}^{W}_{k} = k N_k$$

as water (CDIA) electrostatic, can ignore field momentum

so, for resonant electrons: kN

$$\frac{\partial}{\partial t} P^{RP}_e = -\frac{\partial P^w}{\partial t} = -\sum_k (2\gamma^e_k) \frac{k}{\omega_k} \mathcal{E}^{w}_{k}$$

$\gamma^e_k \equiv$ electron (resonant) growth rate

but

$\frac{\partial}{\partial t} P^{RP}_{electron}$ → slowing down

→ macro-representation as effective collision frequency

so $\frac{\partial P^{RP}_{electron}}{\partial t} = -n m_e \nu_{eff} \overline{V}$

slowing down by resonant scattering (resonant particle interaction)

effective collision frequency

$$\boxed{n m_e \nu_{eff} \overline{V} = \sum_k (2\gamma_k^e) \frac{k}{\omega_k} \mathcal{E}_k^{\omega}}$$

- defines $\overline{V}$
- for macro-micro link

$$\overline{V} = \frac{cB}{4\pi n q \Delta}$$

* – n.b. if 2D, 3D theory, i.e. ⊥ dynamics ⇒ non-resonant scattering ⇔ wave driven momentum flux i.e. $\Pi_{\perp,\parallel}$ → ⊥ radiation ∥ momentum. Relation to whistler interpretation of Bellan ?! There need include wave radiation in energy balance.

so now have

$$\boxed{\begin{array}{l} n\, m\, \nu_{eff}(B_0, \Delta)\bar{V} = \sum_k (\rho\, \gamma_k^e) \frac{k}{\omega_k} \mathcal{E}_k^w \\ \\ \Delta^? = \frac{k}{V_A}\left(1 + \frac{c^3}{\omega_{pe}^2} \nu_{eff}\right) \end{array}} \quad (1)$$

$\Rightarrow$ need γ_k^e, $\mathcal{E}_k^w$ and $\langle f^e \rangle$ evolution

at simplest level, proceed via linear/quasilinear theory in 1D

- at more advanced level:
  - consider 1D phase space structures
    - $\rightarrow$ electron/ion clumps, momentum exchange
    - $\rightarrow$ electron scattering off ion hole
  - consider 3D $J_{\parallel}$ driven instability with electron viscosity

Now, proceed in usual fashion:

$\gamma_k^e \rightarrow$ linear theory

$\mathcal{E}_k^w \rightarrow$ nonlinear saturation

$\langle f_e \rangle \rightarrow$ QL equation — flattening

49.

For linear theory of CDIA ;

$$\nabla^2 \hat{\phi} = -4\pi n_0 |e| \left(\frac{\hat{n}_i}{n_0} - \frac{\hat{n}_e}{n_0} \right)$$

$$\hat{n}_i / n_0 = \frac{k^2 c_s^2}{\omega^2} \frac{|e|\phi}{T}$$

$$\frac{\hat{n}_e}{n_0} = \frac{|e|\hat{\phi}}{T} \left[1 - i\Gamma(k) \right]$$

$\sim \quad \Gamma(k)$,

$$\frac{\partial \tilde{f}}{\partial t} + v \frac{\partial \tilde{f}}{\partial x} = -\frac{|e|}{m_e} \tilde{E} \frac{\partial \langle f \rangle}{\partial v}$$

$$\hat{f} = \frac{|e|\hat{\phi}}{T} \langle f \rangle + g$$

$$\frac{\partial \tilde{g}}{\partial t} + v \frac{\partial \tilde{g}}{\partial x} = -v \frac{\partial}{\partial x} \left(\frac{|e|\hat{\phi} \langle f \rangle}{T} \right) \pm \frac{|e|}{m_e} \frac{\partial \tilde{\phi}}{\partial x} \frac{\partial \langle f \rangle}{\partial v}$$

$$- \frac{\partial}{\partial t} \left(\frac{|e|\hat{\phi} \langle f \rangle}{T_e} \right)$$

$$= v \frac{\partial \tilde{\phi}}{\partial x} \frac{|e|}{T} \langle f \rangle + \frac{|e|}{m_e} \frac{\partial \phi}{\partial x} - \frac{(v - \bar{V})}{T/m_e} \langle f \rangle - \frac{\partial}{\partial t} \frac{|e|\hat{\phi} \langle f \rangle}{T_e}$$

$$= -\frac{\partial}{\partial t} \frac{|e|\hat{\phi} \langle f \rangle}{T} + \bar{V} \frac{\partial}{\partial x} \frac{|e|\hat{\phi} \langle f \rangle}{T}$$

$$\Rightarrow g_k = \frac{i(\omega - k\bar{v})}{-i(\omega - kv)} \frac{|e|\hat{\phi}_k}{T} \langle f \rangle$$

$$= -\left(\frac{\omega - k\bar{v}}{\omega - kv}\right) \frac{|e|\hat{\phi}_k}{T} \langle f \rangle$$

$$\omega_k^{2\,(0)} = \frac{k^2 c_s^2}{1 + k^2 \lambda_D^2}$$

$$-i\,r(k) = \int dv \frac{+(\omega - k\bar{v})}{(\omega - kv)} \langle f \rangle$$

$$\bar{f} = \frac{1}{\sqrt{\pi}} \exp\left[\frac{-(\omega/k - \bar{v}_i)^2}{v_{Th}^2}\right]$$

$$= -(\omega - k\bar{v}) \frac{(+i\pi)}{|k| v_{Th}} \bar{f}\Big|_{\omega/k v_{Th}}$$

$$1 + k^2 \lambda_D^2 = \frac{k^2 c_s^2}{\omega^2} + \frac{(\omega - k\bar{v})}{|k| v_{Th}} (i\pi)\, \bar{f}\Big|_{\omega/k v_{Th}}$$

$$\omega \to \omega + \delta\omega$$

$$0 = -\frac{2\delta\omega}{\omega} + \frac{(\omega - k\bar{v})(-i\pi)}{|k| v_{Th}} \bar{f}\Big|_{\omega/k v_{Th}}$$

$$\frac{\delta\omega}{\omega} = -\frac{i\pi}{2} \frac{(\omega - k\bar{v})}{|k| v_{Th}} \bar{f}\Big|_{\omega/k v_{Th}}$$

$\delta\omega \Rightarrow i\gamma_k$ growth rate

(2) $$\boxed{\gamma_k \sim \frac{-\pi}{2} \omega_k \frac{(\omega - k\bar{v})}{|k| v_{Th}} \bar{f}\Big|_{\omega/k v_{Th}}} \Rightarrow$$ $\gamma > 0$ for $\bar{v}_i > c_s$
$\Rightarrow$ critical velocity

for $\langle f \rangle$ evolution,

$$\frac{\partial \langle f \rangle}{\partial t} = +\frac{\partial}{\partial v} \sum_k \frac{+|e|}{m_e} \tilde{E}_{-k} \tilde{g}_k$$

$$= +\frac{\partial}{\partial v} \sum_k \frac{|e|}{m_e} \tilde{E}_{-k} \left(\frac{-(\omega - k\bar{v})}{(\omega - kv)} \frac{|e|\vec{\phi}_k}{T} \langle f \rangle \right)$$

$$= \frac{\partial}{\partial v} \sum_k \frac{T_e |e|}{m_e} \frac{+ik\vec{\phi}_k}{T_e} \left(-(\omega - kv) i\pi\delta(\omega - kv) \right) \frac{|e|\vec{\phi}_k}{T} \langle f \rangle$$

$$= \frac{\partial}{\partial v} \sum_k (-v_{th}^2) \left| \frac{|e|\vec{\phi}_k}{T} \right|^2 k(\omega - k\bar{v})\, \pi\delta(\omega - kv) \langle f \rangle$$

$$\boxed{\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} \sum_k (-v_{Th}^2) \left| \frac{|e|\vec{\phi}_k}{T} \right|^2 k(\omega - k\bar{v})\, \pi\delta(\omega - kv) \langle f \rangle} \quad (3)$$

— mean evolution

Note:

— really only assumed $\langle f \rangle = \langle f \rangle \left((v - \bar{v})^2 / 2v_{Th}^2 \right)$

so $\frac{\partial \langle f \rangle}{\partial v} = \left(\frac{v - \bar{v}}{v_{Th}^2} \right) \langle f \rangle$

and

$$\langle f \rangle' = -\langle f \rangle$$

→ minimal assumption on structure

- can write as $\bar{v}$ evolution

$$\bar{v} = \int dv\, v \langle f \rangle \Big/ \int dv \langle f \rangle$$

$$\Rightarrow \boxed{\frac{\partial \bar{v}}{\partial t} = + \int dv \sum_k v_{Th}^2 \left| \frac{e\hat{\phi}_k}{T} \right|^2 k^2 \left(\frac{\omega}{k} - \bar{v} \right) \pi \delta(\omega - kv) \langle f \rangle}$$

$\omega/k < \bar{v} \Rightarrow \partial \bar{v}/\partial t < 0$

$\quad > \bar{v} \Rightarrow \partial \bar{v}/\partial t > 0$

- Remains to determine fluctuation intensity level

Generically, can write:

$$\frac{\partial}{\partial t} \mathcal{E}_k^{w} = \underset{(a)}{\gamma_k \mathcal{E}_k^{w}} - \underset{(b)}{\left(\sum_{k'} \omega_k C_1(k, k') \frac{\mathcal{E}^{w}_{k'}}{nT} \right)} \mathcal{E}^{w}_k$$

$$- \underset{(c)}{\left(\sum_{k', k''} \omega_k C_2(k, k', k'') \frac{\mathcal{E}^{w}_{k'}}{nT} \frac{\mathcal{E}^{w}_{k''}}{nT} \right)} \mathcal{E}_k$$

spectral equation constituents:

(a) - linear growth

(b) - quadratic nonlinearity $\rightarrow$ 3 wave coupling / NL ion-wave interaction

(c) - cubic "NL" $\rightarrow$ wave coupling

Now, for ion-acoustic wave:

- 3 wave coupling effects negligible
  $\Rightarrow$ can't satisfy resonance
- NL wave-particle effects weak $\Rightarrow$
  intrinsically
$\Rightarrow$ consider 4 wave process

$$\boxed{\frac{\partial \mathcal{E}_k^\omega}{\partial t} = \left[\gamma_k - \omega_k B(\omega, k)\left(\frac{\mathcal{E}_k^\omega}{nT}\right)^2\right]\mathcal{E}_k^\omega} \qquad (5)$$

- 'cartoon' NL saturation equation

Now, (1)-(5) $\Rightarrow$ $\begin{cases}\text{coupled, (2)-stationary} \\ \text{micro-macro system}\end{cases}$

$\Rightarrow$ describe anomalous resistivity dynamics and its effect on reconnection

$\Rightarrow$ coupled solution corresponds to solution of the problem

5Ⅰ4.

$$\textcircled{1}\begin{cases} n m_e \nu_{eff}(B,\Delta)\,\overline{V} = \sum_k 2\gamma_k^e \frac{k}{\omega_k} \mathcal{E}_k^W \\ \Delta^2 = \frac{L}{V_A}\left(M + \frac{c^2}{\omega_{pe}^2}\nu_{eff}\right), \quad \overline{V} = cB/4\pi n q \Delta \end{cases}$$

$$\textcircled{2}\; \gamma_k^e = -\frac{\pi}{2}\,\omega_k\,\frac{(\omega - k\overline{V})}{|k|v_{th}}\,\overline{F}\Big|_{\omega/k v_{th}}$$

$$\textcircled{4}\; \frac{\partial \overline{V}}{\partial t} = \int dv\left(\sum_k v_{th}^2 \left|\frac{e\tilde{\phi}_k}{T}\right|^2 k^2 \left(\frac{\omega}{k} - \overline{v}\right)\pi\delta(\omega - kv)\langle F\rangle\right)$$

$$\textcircled{5}\; \frac{\partial \mathcal{E}_k^W}{\partial t} = \left[\gamma_k - \omega_k\,B(\omega,k)\left(\frac{\mathcal{E}_k^W}{nT}\right)^2\right]\mathcal{E}_k^W$$

Now, stationarity $\Rightarrow$

$$\mathcal{E}_k^W = nT\left(\gamma_k/\omega_k B\right)^{1/2}$$

$$\gamma_k = +\frac{\pi}{2}\left(\frac{\overline{V} - c_s}{|k v_{th}|}\right) k\omega_k\,\overline{F}\Big|_{\omega/k v_{Th}}$$

So, for scalings:

55.

$$\nu_{eff} = \frac{1}{n m \bar{V}} \sum_k 2 \gamma_k^e \frac{k}{\omega_k} \mathcal{E}_k^w$$

$$\sim \frac{1}{n m \bar{V}} \frac{(\bar{V}-c_s)}{|k| v_{th}} k \omega_k \bar{f}\Big|_{\frac{\omega_k}{k v_{th}}} \frac{k}{\omega_k}(nT)\left(\frac{\gamma_k}{\omega_k B}\right)$$

$$\sim \frac{(1-c_s/\bar{V})}{n m} \frac{k^2 nT}{|k| v_{th}} \left(\frac{\gamma_k}{\omega_k B}\right) f\Big|_{\frac{\omega_k}{k v_{th}}}$$

$$\sim (1-c_s/\bar{V})\bar{f}\Big|_{\omega/|k|v_{th}} \left(\frac{k^2 v_{th}}{|k|}\right)\left(\frac{\gamma_k}{\omega_k B}\right)$$

$$\sim (1-c_s/\bar{V})\left(\frac{k\omega_k}{|k v_{th}|}\frac{(\bar{V}-c_s)}{\omega_k B}\bar{f}\right)^{1/2} \bar{f}\,\frac{k^2 v_{th}}{|k|}$$

$$\sim \left[(\bar{V}-c_s)k\right]^{3/2} \bar{f}^{3/2} \frac{k(v_{th}/\bar{V})}{|k|\,|k v_{th}|^{1/2}}$$

$$\boxed{\nu_{eff} \sim \frac{\left[(\bar{V}-c_s)k\right]^{3/2}}{|k v_{th}|^{1/2}} \frac{v_{th}}{\bar{V}} f^{3/2}\left(\frac{k}{|k|}\right)}$$

Turbulent collision frequency

so have

$$\Delta^2 = \frac{L}{V_A}\left(\eta + \frac{c^2}{\omega_{pe}^2}\nu_{eff}\right)$$

$$\bar{V} = cB_0/4\pi n_0 q\Delta$$

$$\nu_{eff} = \frac{\left[(\bar{V}-C_s)k\right]^{3/2}}{|kv_{Th}|^{1/2}}\frac{v_{Th}}{\bar{V}}\bar{F}^{3/2}\left(\frac{k}{|k|}\right)$$

with:

$$\frac{\gamma_k}{\omega_k} = \frac{\pi}{2}(\bar{V}-C_s)\frac{k}{|k|v_{Th}}\bar{F}$$

$$\bar{F} = \frac{1}{\sqrt{\pi}}\exp\left[-(\omega/k-\bar{V})^2/2v_{Th}^2\right]$$

→ characterize micro-macro coupling with anomalous resistivity

→ now can envision situation
- finite current, $\Delta \sim (L\eta/V_A)^{1/2}$
  $\nu_{eff} = 0$

so if:

- decrease η ⇒ Δ decreases
- Δ decreases ⇒ $\bar{V}$ increases

- $\bar{V}$ increases $\Rightarrow$ $\gamma_{\parallel} > 0$
- $\gamma_{\parallel} > 0 \Rightarrow \begin{cases} \epsilon^{\omega}_{\parallel} > 0 \\ \nu_{eff} > 0 \end{cases}$
- $\nu_{eff} > 0 \Rightarrow \begin{cases} \Delta \text{ increases} \\ \bar{V} \text{ decreases} \end{cases}$

$\Rightarrow$ decreasing ν so Δ decreases triggers feedback so Δ increases $\Rightarrow$ self-regulation / $\oplus$ feedback

$\therefore$ in this model, can expect:

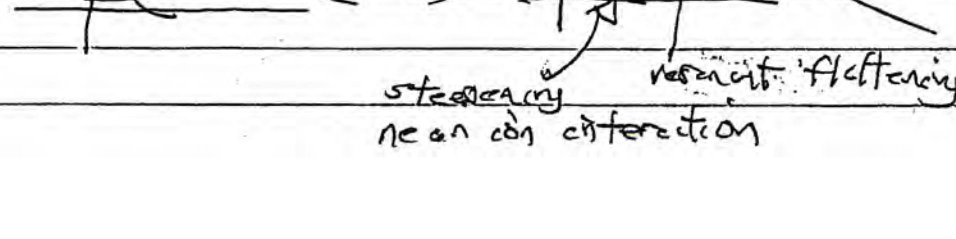
- at low $\nu_{collisional}$, so $\bar{V}\big|_c \sim cB_0 / 4\pi n_0 e \Delta_c \sim C_s$

$\Rightarrow$ CDIA "hovers" near marginal stability

- for stronger drive (i.e. $B_\theta \uparrow$)

$\rightarrow$ ion interaction important
$\rightarrow$ strong ion distortion possibly significant
$\rightarrow$ granulation formation important
$\rightarrow$ distortion of electron distribution function need be considered

i.e.



58.

⇒ Useful extensions:

- 1D avalanche model → avalanching ⇒ jitter effects on Δ, $\overline{V}$
- nonlinear noise effects via fluctuations
- 2D, 3D ⇒ wave radiation, esp. wave momentum flux ⊥ layer.

* – granulation effects ⇒ strong distortion hole (cf. later in course)

Comment:

This simple problem is surprisingly poorly understood. Excellent example of:

→ micro – macro feedback
→ self-regulation
→ marginal stability