EMHD explanation of ionization wave propagation in strong magnetic field region

Haotian Mao[1]

[1]Department of Mechanical and Aerospace Engineering, University of California San Diego, La Jolla, California 92093, USA

E-mail: h6mao@eng.ucsd.edu

Phys 218B Project, February 2020

Abstract. Previous work has shown that a long-lasting relativistic ionization wave can be launched into surrounding gas by the sheath field of a plasma filament with high energy density electrons. Using 1D particle-in-cell (PIC) simulation, we observed that this ionization wave can penetrate in strong magnetic field ($\sim$ megagauss) before it is stopped. We use electron magnetohydrodynamic (EMHD) to discuss the mechanism of how magnetic field effects the propagation of ionization wave.

1. Introduction

Electron transport in magnetic fields is one of the fundamental physics problem. Understanding the underlying mechanism is quite beneficial for diverse application scenarios. For instance, the wiggling motion of electron bunch inside an external magnetic field is a promising method to obtain the controllable collimated light sources. The bending of highly energetic electrons induced by the ultra strong magnetic field nearby the neutron star (or magnetar) is a potential option to explain the concept of gamma-ray burst in astrophysics. Recently, encouraged by the optical laser technology, the feedback played by the electrons driven by short intense laser pulse is capable of realizing an amplified quasi-static magnetic field, which is orders of magnitude higher than the initial static magnetic seed. Moreover, the energy deposits of the fast electrons facilitated by the plasma generated or external imposed magnetic fields plasma self-generated or external imposed magnetic fields is in great significance for the inertial confinement fusion.

Recently work has presented experimental result of a novel transport mechanism that is dominant because of the high energy-density of electrons produced by high intensity laser pulses interacting with clustering gas jets. The observed object in the experiments was a radially expanding ionization wave at the interface between the plasma and the surrounding neutral gas. This ionization wave traveled with a velocity of a fair fraction of speed of light over the first few picoseconds. Such a high velocity wave could neither be the result of simple electron transport not hydrodynamic expansion. Previous simulation result has shown that it is a collisionless mechanism with rapid ionization by sheath electric electric field that develops from hot electron population accumulated in the laser irradiation of the target gas clusters.

However, the fast ionization wave propagation and the accompanied hot electron transport in strongly magnetized high energy-density (HED) plasma remains unexplored. The key barrier for carrying out the above research objectives in magnetized HED plasma is the requirement to generate an external field strong enough to alter the electron dynamics. Fortunately, recent progress in compact pulsed power technology provides a viable solution by enabling generation of a megagauss magnetic field on a microsecond time scale, which significantly exceeds characteristic time scales in laser-generated plasma. Meanwhile, the characteristic volume where the generated field is uniform can also greatly exceed the dimensions of these plasma. Therefore, such magnetic fields can effectively serve as a static uniform field for nanosecond time scale to investigate magnetized laser-generated.

In combination with 1D Particle-in-cell (PIC) simulation, we demonstrate how the strong ($\sim$ megagauss) externally applied magnetic field influences the hot electron transport using electron magnetohydrodynamic (EMHD). EMHD is a limiting case of multicomponent MHD in which the motion of the ions can be neglected. In the case we are studying, the hot electrons from the plasma is the main object we are studying as the electrons from ionization possess lower energy after crossing the ionization barrier. Ionization wave has a small length scale ($\sim \mu m$) and a short characteristic time scale ($\sim$ fs) and we focus on how magnetic field affect the ionization wave propagation.

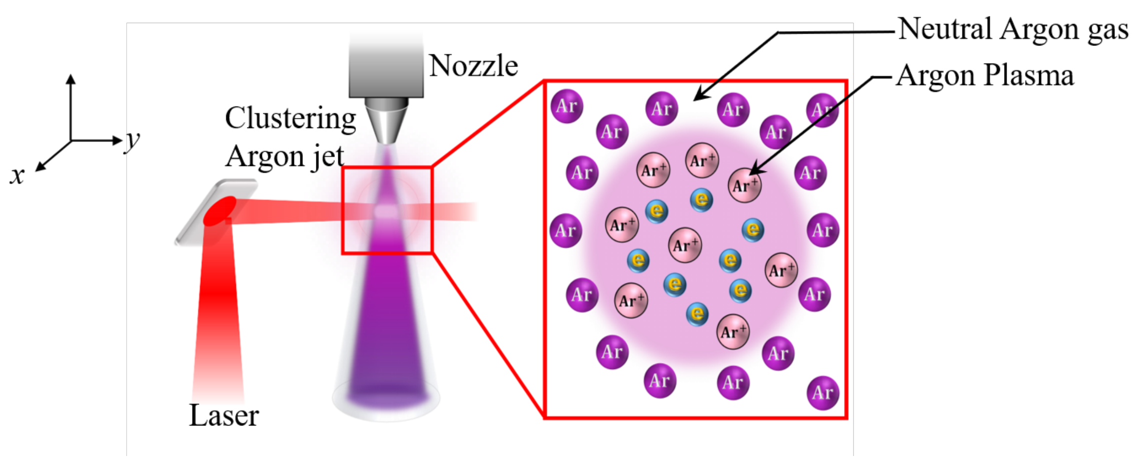

Figure 1: Experimental Set up for ionization wave study. Petawatt laser shooting into clustering gas jet can create highly ionized plasma.

2. Ionization wave in non-magnetized plasma

To first understand the mechanism of ionization wave propagation, we apply 1D EPOCH particle-in-cell (PIC) simulation to study ionization wave propagation. A fully-ionized plasma filament ($n_i = n_e = 3.3 \times 10^{19}\ cm^{-3}$) is initially placed in the region of $x \in$ [-100 μm, 100 μm] with a resolution of 100 cells per micron. The plasma is embedded in neutral hydrogen gas environment ($n_0 = 3.3 \times 10^{19}\ cm^{-3}$). As it expands, the sheath electric field at the edge of the expanding plasma filament will ionize the surrounding gas, creating a propagating ionization wave. We hereby call the electrons born by ionization 'generated electrons' in order to distinguish from the 'original electrons' set up in the simulation. We use a water-bag distribution with cutoffs at $p_e = \pm m_e c$ for original electrons.

In the snapshot of the right edge of the expanding plasma slab [Fig. 2], we can see from the electric field profile that sheath field is followed by a trapping field and the two has created a potential well. This field structure remains unperturbed throughout propagation while traveling like a soliton with a relativistic speed ($\sim$ 0.3 c) for tens of picosecond or more. The phenomenon raises questions about the mechanism behind this long-lasting propagation.

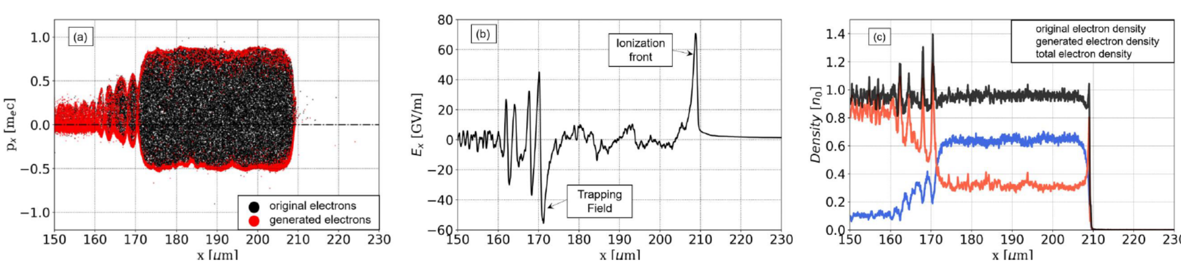

Figure 2: Snapshot of electron phase-space, electric field at $t = 2\ ps$ in 1D particle-in-cell simulation

During the propagation, original electrons create the sheath field as they carry the majority of kinetic energy inside the plasma ($p_{xo} \sim 0.1\ m_e c$). The generated electrons created at the sheath, after crossing the ionization energy barrier, possess negligible energy compared to the original ones ($p_{xg} \sim 0.01\ m_e c$). Therefore, the generated electrons will be accelerated away from the wave front by the sheath field. Their density will drop below ion density, creating a positive charge region at the wave front. Consequently, this will generate the counteracting field and create a potential well moving with the front. The original electrons, on the other hand, has a comparable speed to the ionization wave, which makes them hard to leave the potential well created by the wave-pocket structure. As a result, they will be trapped and carried by the soliton.

In this approach, energy density of the original electrons at the front remain unchanged throughout propagation rather than dropping as $1/l$ as plasma expands. This will help to maintain a strong sheath field amplitude. Therefore, the sheath can further ionize the surrounding gas and sustain. The non-stopping propagation raises concern as it could be a potential problem in achieving desirable plasma condition. Due to this, we propose to introducing a magnetic field into the regime. The magnetic field can rotate the original electron momentum, which might create the possibility of reducing the sheath field and altering the wave propagation.[1]

3. Ionization wave in magnetized plasma

When we are introducing a magnetic field into the ionization wave propagation, akin to laser penetration stopped by critical density in plasma, one might expect that there will be a criterion for magnetic field in order to confine the ionization wave propagation. To confirm the assumption, we will look at the simulation which has the same set up as in the last chapter except that we introduce a magnetic field region beyond 200 μm. The magnetic field is oriented in z-direction. We want to find out the criterion for the magnetic field to stop the ionization wave.

Magnetic field can alter the electron dynamics by changing the Larmor radius. Therefore, one might expect the Larmor radius be larger than the soliton width L so that the electron will leave the soliton structure. This will corresponds to a magnetic field of $B_{ext} \sim 60$ T. Others might think the magnetic field will reduce the Larmor radius as to $\rho_e < \lambda_{De}$, which will affect sheath field formation. This will require a minimal magnetic field of $B_{ext} \sim 500$ T. In order to find out the magnetic field criterion , we set a sharp-rise external magnetic field like a step function with amplitude of 500 T beyond 200 μm. Notice that the magnetic field introduced here acts on particles yet doesn't involve in Maxwell solvers in the code. In other words, this magnetic field introduced here won't generate additional E_y into the domain so that we can study the effect of magnetic field only. To distinguish it from the magnetic field response from the plasma, we call it the 'external field' in this paper. The result shows the ionization wave cannot even enters the magnetized region. The soliton structure is completely destroyed. This can be interpreted by the fact the length criterion $\rho_e \sim \lambda_{De}$ corresponds to the ratio of thermal pressure and magnetic pressure, which is known as the β factor.

$$\frac{E_{sheath}}{B} \sim \frac{\rho_e}{\lambda_{De}} \sim \sqrt{\beta} \tag{1}$$

For the case we are studying, $B_{ext} \sim 500$ T means the length scale satisfies $\rho_e \sim \lambda_{De}$ which corresponds to $\beta \sim 1$. In this case, the thermal energy inside the soliton cannot overcome the magnetic pressure at magnetic boundary. Therefore, having $\beta < 1$ is the sufficient condition to stop the ionization wave propagation. However, we still need to find out that is $\beta < 1$ also the necessary condition to stop ionization wave.

To answer the question, we lower the external magnetic field amplitude to 200 T and we let the same soliton propagate through this magnetize region. The soliton will have $\beta > 1$ at the magnetic field boundary and we want to see if ionization wave can penetrate inside the magnetic field region. As we can see from the result in Fig. ?(?), the ionization wave is able to pass through the external magnetic field region while generating a counteracting B-field which cancels external background field behind it. The diamagnetism shows that the criterion we expect as $\rho_e > L$ becomes invalid as there is no magnetic field behind the ionization wave path, which is a crucial phenomenon to keep original hot electrons from further momentum rotation.

We hereby interprets the expulsion of magnetic field using EMHD. As it is previously shown by Alfvén, the effect of a strong magnetic field generated by the particle current results in return motion of the electrons, which induces the return current. Therefore, it is impossible for a fast alternation of B-field in hot plasma as

$$\nabla \times \mathbf{B} = \frac{\mathbf{4}\pi}{\mathbf{c}}(\mathbf{j} + \mathbf{j}') \tag{2}$$

where $\mathbf{j}$ is hot electron current and $\mathbf{j}'$ is return current. The magnetic field is described by the equation

$$-\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} \tag{3}$$

where the electric field is determined by the Ohmic law

$$\mathbf{j} = \sigma \mathbf{E} \tag{4}$$

The electron temperature by the magnetic field expulsion is determined by the Ohmic dissipation

$$\frac{n}{\alpha - 1}\frac{dT}{ds} = \frac{\mathbf{j^2}}{\sigma(T)} \tag{5}$$

where α is effective adiabatic power. In 1D case, the magnetic field expulsion propagates in x-direction with the magnetic field oriented in y-direction we are looking for a solution related to $x - ut$. Then these equation can be transformed as

$$j + j' = -(\frac{c}{4\pi}\frac{d^2A}{ds^2}) \tag{6}$$

$$\frac{n}{\alpha - 1}\frac{dT}{ds} = -(\frac{j}{c})(\frac{dA}{ds}) \tag{7}$$

$$j = \sigma(\frac{u}{c})\frac{dA}{ds} \tag{8}$$

where A is the y-component vector potential. In kinetic approximation, the diamagnetic current of hot electron current j' is equal to

$$j' = -2en_0'c\frac{(eA/c)\{P_0^2 - (eA/c)^2\}^{1/2}}{m_e^2c^2\{\gamma_m(\gamma_m^2 - 1) + ln(\gamma_m + (\gamma_m^2 - 1))\}} \tag{9}$$

where $P_0 = (\epsilon_{max}^2/c^2 - m_e^2c^2)^{1/2} \equiv m_ec(\gamma_m^2 - 1)^{1/2}$ and the vector potential $A_0 = cP_0/e$ which corresponds to the hot electron able to turn to the opposite direction of motion. Substitute the expression of j' into the above equations, we can obtain the function of electron temperature as A and dA/ds. We substitute T in $\sigma(T) = \sigma_0 T^k$ into it and we can obtain the equation with the form:

$$\frac{dg}{da} = -\lambda b g^k \tag{10}$$

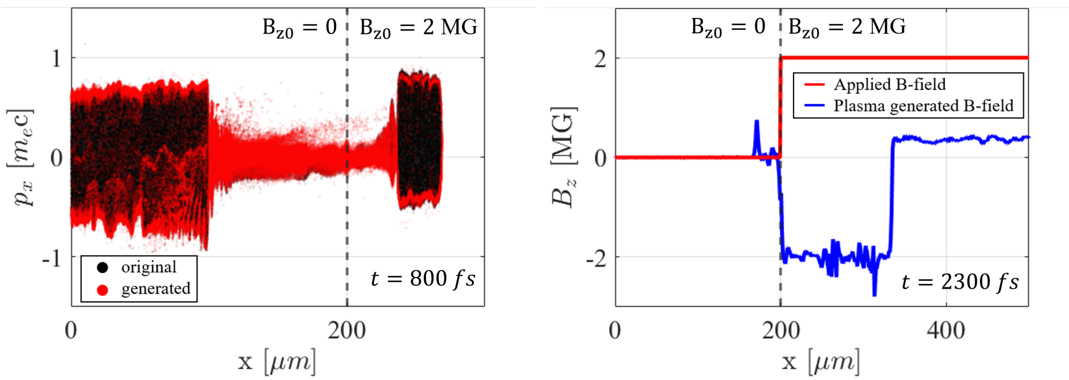

Figure 3: Snapshot of electron phase space and magnetic field profile in 1D Particle-in-cell simulation

where $g(a,b) = \tau_0 + \frac{1}{3}\beta(b^2 - b_0^2) + \frac{1}{3}(1-a^2)^{3/2}$, $a = eA/P_0c$, $b = B/B_*$, $b_0 = B_0/B_*$, $\tau_0 = T_0/T_*$, $\sigma_* = \sigma_0 T_*^k$, $\lambda = (u/c)(\sigma_* B_*/j_*)$, $\beta = 8\pi p_0'/B_*^2$, and p_0' us the hot electron pressure for $x = -\infty$. B_*, T_* and j_* are some characteristic values. For a given λ this will round up as a magnetic expulsion wave solution.As we see in the B_z profile all the magnetic field behind the ionization wave will goes to zero while the B-field outside is enhanced.[2][Fig. 3]

If we lower the magnetic field amplitude to 200 T, the soliton will have $\beta > 1$ as it reaches the magnetic field boundary. Thus, ionization wave will be capable of penetrate through the magnetic field boundary. Therefore, we establish $\beta > 1$ to be the criterion for the ionization wave to penetrate in the magnetized region.

As a result, one might that expect we need a external magnetic field larger than 500 T as any magnetic field below fails to satisfy the $\beta > 1$ criterion and cannot stop the ionization wave. However, this is contradictory to the fact that the ionization wave will be stopped at a weaker magnetic field criterion after traveling some extra distance in the magnetized region. In order to reconcile this phenomenon, we investigate the ionization wave travel in weaker field. To make sure the ionization wave can pass through, we set a sharp-rise external magnetic field of 200 T beyond 200 μm (The sharp transition will not substantially disturbed the soliton). In the simulation, we let the same ionization wave enter the magnetized region and see if it can be stopped. The result shows that the ionization wave will be eventually stopped before propagate an extra 300 μm in the magnetized region. This indicates that even weaker field is capable of stopping the ionization wave. The question we need to ask is : how is the ionization wave stopped in the weak magnetized region?

Recalling that we thought that the weaker field fails to satisfy the $\beta \sim 1$ criterion, we made the false assumption that the thermal pressure of the soliton will not change. The assumption is true for the soliton traveling only in non-magnetized region. However, once the ionization wave is able to enter the magnetized region, the energy density inside the soliton will start to decrease as the ionization wave propagates.

Although the soliton structure is able to confine the hot original electrons, the

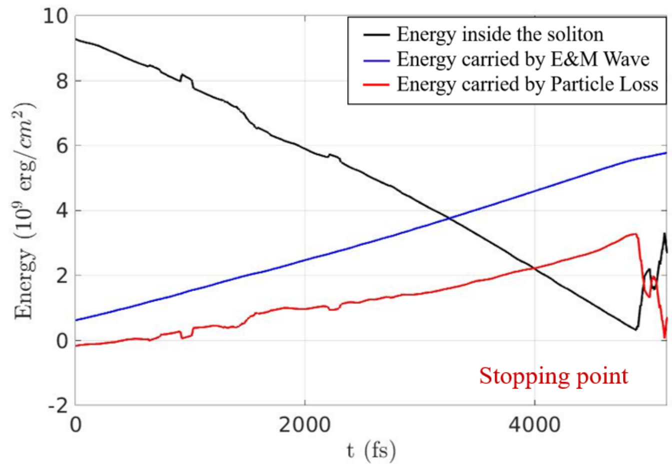

Figure 4: The beta evolution of the edge of ionization wave in 1D particle-in-cell simulation

magnetic field can perturb the mechanism by rotating original electron momentum to y-direction. This will introduce the energy loss by converting electron kinetic energy to an emission of electromagnetic wave propagating in x-direction. In addition, original electrons are left behind from fast-traveling soliton. As a result, a part of energy is also carried away by particle transport from the soliton. Therefore, the energy inside the soliton will be decreasing which corresponds to the dropping of β factor. The β factor evolution [Fig. 5] shows that the ionization wave also stops as β drops to a scale of $\beta \sim 1$. This confirms the idea that $\beta > 1$ is the criterion for ionization wave to propagate. Thus, the stopping point at $\beta \sim 1$ will correspond to a weaker magnetic field criterion.

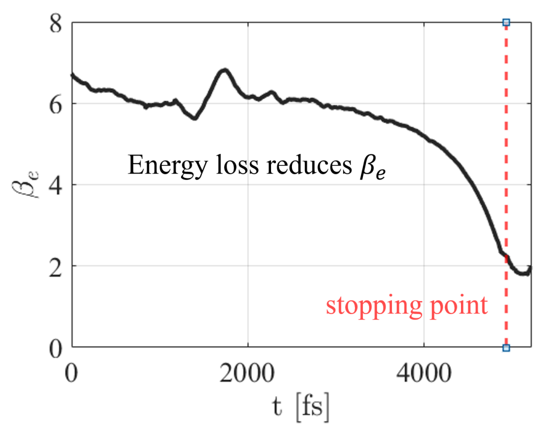

Figure 5: The beta evolution of the edge of ionization wave in 1D particle-in-cell simulation

Another way of interpreting the ionization wave stopping in the magnetized region is that the β factor corresponds to the sheath field amplitude. Since B scale as the magnetic field which is constant, the sheath field will drop as β reaches to 1. At the stopping point where $\beta \sim 1$, the sheath field amplitude will be decreased as the original electrons are confined by Larmor radius and cannot go further than Debye length. The decrease of the sheath field leads to the low production of generated electrons which is responsible for the formation the trapping field. As a result, the potential well at the wave front can no longer confine the hot original electrons at the wave front, which leads to the destroy the soliton structure. In the non-magnetized case, the soliton structure helps to facilitate the ionization wave by keeping energy density inside the soliton constant. Now, the magnetic field will destroy this soliton structure and leads the ionization wave to a equilibrium of thermal pressure and magnetic pressure

References

[1] M.McCormick, Arefiev et al, Obeservation of Self Sustaining Relativistic Ionization Wave Launched by a Sheath Field, *Phys. Rev. Lett.112,045002 (2014)*

[2] A.V.Gordeev, Electron magnetohydrodynamic, *Phys. Reports 243 (1994) 215-315*

[3] N. O. Weiss, The Expulsion of Magnetic Flux by Eddies, *Proceedings of the Royal Society A, 293 (1966) 310-328*