Week 7

1. Another symmetry of EM we have not discussed is scale invariance. In this problem, we consider the consequences of this symmetry.

(a) Show that the action for the EM field $S = \int d^4x \left(\frac{-1}{16\pi} F_{\mu\nu} F^{\mu\nu} \right)$ is invariant under the following scale transformation: $x_\mu \to \lambda x_\mu$, $A_\mu(x) \to \lambda^{-1} A_\mu(\lambda^{-1} x)$.

(b) Find how A_μ transforms under an infinitesimal scale transformation $\lambda = 1+\epsilon$.

(c) Find the Noether current for a scale trans. and show that it can be written as $j_\mu = T^c_{\mu\nu} x^\nu - \frac{1}{4\pi} F_{\mu\nu} A^\nu$, where $T^c_{\mu\nu}$ is the canonical stress tensor. (i.e. ~~the~~ stress tensor derived from the Noether procedure directly).

(d) Check explicitly that j_μ is conserved, i.e. $\partial_\mu j^\mu = 0$. Show that this is equivalent to the statement that the symmetric stress tensor is traceless, $T^\mu{}_\mu = 0$. The tracelessness of $T_{\mu\nu}$ is a consequence of the scale invariance of EM, or equivalently the masslessness of the photon.

(e) Show from $T^\mu{}_\mu = 0$ that the ~~pressure~~ energy density ~~p~~ U of the E.M. field is related to its pressure p by $U = 3p$. This is the equation of state for a photon gas. From thermodynamic considerations it implies the Stefan-Boltzmann law $U \sim T^4$.

2. If we add a mass term μ for the photon we get the Proca Lagrangian, which is $\mathcal{L} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + \frac{\mu^2}{8\pi} A_\mu A^\mu$. ~~Compta~~ Compute the canonical stress tensor for this Lagrangian. Symmetrize this to obtain the symmetric stress tensor.