Week 4

1. (a) ~~Given that~~ Since the four-current of a point particle is a four-vector, it must be possible to write ~~it~~ in covariant form. Show that it can be written as $j^\mu(x) = e\int d\tau \, \frac{dy^\mu(\tau)}{d\tau} \delta^{(4)}(x - y(\tau))$, where $y^\mu(\tau)$ is the worldline of the particle and $\delta^{(4)}(x) = \delta(x^0) \cdots \delta(x^3)$.

i.e, Show that j^μ transforms as $j^{\mu\prime}(x) = \Lambda^\mu{}_\nu \, j^\nu(\Lambda^{-1}x)$ under Lorentz transf.'s and that in the rest frame of the particle $j^\mu = (e\delta^{(3)}(\vec{x}-\vec{y}), \vec{0})$.

(b) Using the expression from part (a), show that the current is conserved, i.e. $\partial_\mu j^\mu = 0$.

2. A sphere of dielectric permittivity ε and radius a moves with velocity $\vec{v} = v\hat{z}$ in a uniform magnetic field $\vec{B} = -B_0\hat{y}$.

(a) Transforming to the rest frame of the sphere, find its induced polarization to first order in v/c.

(b) ~~Find~~ Transforming back to the lab frame, find the electric field to first order in v/c.

3. Consider an infinite sheet of charge density σ in the $z=0$ plane. Initially stationary, at time $t=0$ it starts to move in the y-direction with speed v.

(a) Find its current density $\vec{J}(z,t)$

(b) Use the Green's function to calculate $A_y(z,t)$. Calculate $B_x(z,t)$ and $E_y(z,t)$.