First Midterm Exam:

1. An electric field of magnitude E points in the direction of the x-axis in a certain inertial frame K''. No magnetic field is detected in that frame. Frame K'' moves with speed v_2 along the direction of the y-axis as seen from frame K'. Frame K' is in turn moving along the x-axis with speed v_1 as seen in frame K. Determine the magnetic field in frame K.

2. The action integral S describing the motion of a point particle of mass m in the field of a 2-index symmetric tensor potential $\phi_{\alpha\beta}(x)$ is given by

$$S = \int d\lambda \left[-mc\sqrt{u_\mu u^\mu} - \phi_{\alpha\beta}\frac{u^\alpha u^\beta}{\sqrt{u_\mu u^\mu}} \right],$$

where the potential is evaluated on the trajectory $x^\mu = x^\mu(\lambda)$ and $u^\mu = \frac{dx^\mu}{d\lambda}$ is the 4-velocity.

   (a) Show that S is invariant under reparametrizations, $\lambda \to \lambda' = f(\lambda)$.

   (b) Find the equation of motion.

   (c) Assuming $\phi_{\alpha\beta}/mc \ll 1$ use the equation of motion to determine $\frac{d\mathcal{U}_\alpha}{d\lambda}$ where we have defined $\mathcal{U}_\alpha \equiv u_\alpha/\sqrt{u_\mu u^\mu}$. Put your result in the form

$$\frac{d\mathcal{U}_\alpha}{d\lambda} = t_{\alpha\mu\nu}u^\mu u^\nu$$

   with the tensor $t_{\alpha\mu\nu}$, a function of $\phi_{\alpha\beta}$ and u^α, satisfying $t_{\alpha\mu\nu} = -t_{\mu\alpha\nu}$, and use this to show that $u^\alpha d\mathcal{U}_\alpha/d\lambda = 0$.

   (d) Using time for the parameter for the trajectory, determine the canonical momentum $\vec{P}$.

3. An infinitely long straight wire of negligible cross sectional area is at rest and has a uniform linear charge density λ in the inertial frame K'. The wire and the frame K' move with speed v in the direction of the wire as observed from frame K. Determine the electric and magnetic fields observed in frame K.