Hw 3:

1. This is a standard problem and is solved in most textbooks, but it is worth your while doing from scratch. As you know, the electric charge of a stationary charge q at the origin is $E^i = qx^i/r^3$, where $r^2 \equiv \vec{x} \cdot \vec{x}$.

   (a) Determine the electric and magnetic fields of a charge q moving at constant speed v along a line parallel the x^3-axis (*i.e.* z-axis), going through the x^1-axis (x-axis) a distance b away from the origin at time $t = 0$.

   (b) Make some graphical representations of your results (plots of magnitudes, sketches of directions of fields).

   (c) Discuss the super-relativistic limit $v \to c$.

2. What is the electrostatic potential and electric field of a uniformly charged straight wire of length $2L$ for points on the midplane of the wire? (By points on the midplane we mean a plane perpendicular to the wire that bisects it). *Hint:* Solve the Poisson equation using the method of Green's function.

3. The world line of a point particle of charge q is $y^\mu(\lambda)$. The corresponding charge density is $\rho(x^0, \vec{x}) = q\delta^{(3)}(\vec{x} - \vec{y}(\lambda))$. You are free to choose the parameter λ to best suit your needs.

   (a) Determine the corresponding current density $\vec{\jmath}(x^0, \vec{x})$.

   (b) For the special case $y^\mu = (x^0, \vec{v}x^0)$ with $\vec{v}$ a constant velocity, check explicitly that ρ and $\vec{\jmath}$ transform as components of the 4-vector current j^μ under boosts Λ in the x^1 direction, that is, from the explicit form of j^μ and of a boost it follows that $\jmath'^\mu(x) = \Lambda^\mu{}_\nu \jmath^\nu(\Lambda^{-1}x)$. It should not be difficult for you to extend this exercise to the arbitrary case $y^\mu = (x^0, \vec{y}(x^0))$ (although not required, give it a try!).

   (c) Verify that these satisfy the continuity equation.

4. Using the retarded Green function, determine the electric and magnetic fields of a point particle of charge q moving in a straight line at constant speed.

   (a) To this effect, first compute the 4-vector potential due to this point charge (in Lorentz gauge).

   (b) Use this result to compute $\vec{E}$ and $\vec{B}$.

   (c) Compare your result with that of problem 1