Final Exam

To have your exam graded you must sign and submit the following “CSE Integrity of Scholarship Agreement” acknowledging you have read it and pledging you will abide by it:

Students are expected to complete the course in compliance with the instructor’s standards. No student shall engage in any activity that involves attempting to receive a grade by means other than honest effort, for example:

1. No student shall knowingly procure, provide, or accept any materials that contain questions or answers to any examination or assignment to be given at a subsequent time.
2. No student shall complete, in part or in total, any examination or assignment for another person.
3. No student shall knowingly allow any examination or assignment to be completed, in part or in total, for himself or herself by another person.
4. No student shall plagiarize or copy the work of another person and submit it as his or her own work.
5. No student shall employ aids excluded by the instructor in undertaking course work.
6. No student shall alter graded class assignments or examinations and then resubmit them for regrading.
7. No student shall submit substantially the same material in more than one course without prior authorization. A student acting in the capacity of an instructional assistant (IA), including but not limited to teaching assistants, readers, and tutors, has a special responsibility to safeguard the integrity of scholarship. In these roles the student functions as an apprentice instructor, under the tutelage of the responsible instructor. An IA shall equitably grade student work in the manner agreed upon with the course instructor. An IA shall not make any unauthorized material related to tests, exams, homeworks, etc. available to any student.

   The following are additional examples not listed in the General Catalog specific to programming classes:

8. No student shall provide their assignments, in part or in total, to any other student in current or future classes of this course. No student shall procure or accept assignments from any other student from current or prior classes of this course.
9. All programming code and documentation submitted for evaluation or existing inside the students computer accounts must be the students original work or material specifically authorized by the instructor. The course accounts are authorized for course work only.
10. Collaborating with other students to develop, complete or correct course work is limited to activities explicitly authorized by the instructor. Use of your own previous course work or other students course work, in part or in total, to develop, complete or correct course work, including documentation, is unauthorized. Use of texting or messaging services or Internet sites like Pastebin or GitHub or similar systems to share or publish course files in part or in total is unauthorized. Unless otherwise explicitly authorized, each student is completely responsible to keep their code, homeworks, design files and other course work off of Internet sites.
11. For all group assignments, each member of the group is responsible for the academic integrity of the entire submission.

Each student is responsible for knowing and abiding by UCSD’s Policies on Integrity of Scholarship and Student Conduct. Any student violating these policies will earn an ’F’ in the course and will be reported to the University for the violation.

Authorized course assistance is available in person and electronically from the course instructor and instructional assistants.

By signing this form, I understand and will abide by the above policies and by the spirit of the above policies. I will seek authorized assistance when I need help. I will keep a copy of this agreement for future reference.

Sign and date here:

Final Exam

1. The vector potential of a magnetic dipole $\vec{\mu}$ at the origin of the inertial frame K' is
$$A^{0\prime} = 0, \qquad \vec{A}'(x^0, \vec{x}) = \frac{\vec{\mu} \times \vec{x}}{|\vec{x}|^3}. \tag{1}$$
The frame K' moves with constant velocity $c\vec{\beta}$ in the frame K. Determine the electric and magnetic fields observed in frame K.

2. Consider the dynamics of a real scalar field $\phi(x)$ determined by the Lagrangian density
$$\mathcal{L} = \tfrac{1}{2}\eta^{\mu\nu}\partial_\mu\phi(x)\partial_\nu\phi(x) - \tfrac{1}{2}\mu^2\phi^2(x) + j(x)\phi(x).$$
Here $j(x)$ is the "source" for the equation of motion of the scalar field.
   (a) Find the equation of motion of the field ϕ.
   (b) From here on set $\mu = 0$. Find a retarded Green's function to solve the equation of motion, giving $\phi(x)$ in terms of $j(x)$.
   (c) Model a source for a pointlike particle with worldline $y^\mu(\lambda)$, where λ is an arbitrary parameter, by
$$j(x^0, \vec{x}) = q\sqrt{1-\beta^2}\,\delta^{(3)}\big(\vec{x} - \vec{y}(x^0)\big). \tag{2}$$
   Here q is an arbitrary constant characterizing the strength of the source; we have used x^0 for the parameter of the trajectory and $c\vec{\beta}$ is the particle's velocity.
      i. Show $j(x)$ defined by this expression is a scalar under Lorentz transformations.
      ii. Show $j(x)$ thus defined is re-parametrization invariant (*i.e.*, invariant under $\lambda \to \lambda' = \lambda'(\lambda)$.
   (d) Give $\phi(x^0, \vec{x})$ that results from the particle source in Eq. 2.

   The steps above are preparation for the central question. Give an expression for the angular distribution of power radiated, and for the total power radiated, by the particle in arbitrary motion $\vec{y}(t)$, and, for the case of circular motion (radius R and angular frequency ω) give the energy radiated per revolution.

3. An infinitely long coaxial cable of inner radius $\rho = a$ and outer radius $\rho = b$, and empty in the region $a < \rho < b$, carries steady currents along the length of the cable in both inner and outer conductors, both with same magnitude I but opposite directions. The resistance per unit length is λ for both conductors. The first part of this problem requires understanding of basic electromagnetism, a prerequisite for our course.
   (a) Determine the electric and magnetic fields in the region between the conductors. WIth cylindrical (polar) coordinates, choosing the axis of the cable on the z-axis, assume $E_\rho(\rho, \phi, 0) = 0$. (*Hint:* Determine the electric potential first. What is the potential drop per unit length along each cable?).
   (b) By considering the energy/momentum density of the field determine the integrated energy flux from the region between conductors into the conductors. Compare this with the Ohmic energy dissipated in the conductors.

4. A magnetic dipole moment $\vec{\mu}$ lies at the center of two concentric spherical metal shells of radii $a < b$. The inner and outer spheres carry charge $+q$ and $-q$, respectively, and the space between is empty. Find the electromagnetic field's angular momentum in this setup. (*Hint:* The field of a magnetic dipole can be obtained from Eq. (1) above.)

5. Consider a perfectly conducting semi-infinite cylinder of circular cross section with radius a. The cylinder is capped at one end by a perfectly conducting disc. Give an explicit expression for the transverse magnetic modes that can be excited inside this cylinder, in a coordinate system with the positive z axis running along the axis of the cylinder, $z = 0$ giving the location of the disc capping the cylinder, assuming that the component of the electric field along the cylindrical axis vanishes at $t = 0$. What is the cut-off frequency for $a = 7.0\,\text{m}$?

6. Two equal and opposite charges $\pm q$ move on antipodal points of a circle of radius a with angular velocity ω. Determine the angular distribution of the power radiated as wellas the total power radiated. Describe the polarization of the radiation.