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SIMULACE GALAKTICKÉ DYNAMIKY A JEJICH VYUŽITÍ VE VÝUCE FYZIKY

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ON SIMULATIONS OF GALAXY DYNAMICS AND THEIR APPLICATION TO PHYSICS EDUCATION

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in partial fulfillment of the requirement
for the degree of doctor in
“Theory of Education in Physics”.

Supervisor: RNDr. Miroslav Randa, Ph.D.
Date of submission: April 30, 2007

Declaration

Herewith, I, Jakub Schwarzmeier, declare that I composed this work alone except when I clearly indicated otherwise. I have used only literature listed in the bibliography section.

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ABSTRACT

This work describes the self-teaching educational-research project involving many-body computer simulations with the objective of studying galaxy dynamics. This self-teaching project guides a student through numerical models and computer simulations of galaxy dynamics in detail. It shows the numerical construction of near-equilibrium galaxy models and how these artificial galaxies are evolved. Evolution is based on the Barnes-Hut algorithm and space division with a three-dimensional Hilbert's curve generated by a geometry-based technique. The educational-research part of this project shows how to simulate the evolution of the Milky Way galaxy, the Large and Small Magellanic Clouds and all of the 19 known satellite galaxies of the Milky Way, including recently discovered ones. A future evolution of the Local Group is simulated in the collision of two disk galaxies representing Andromeda galaxy (M31) and the Milky Way galaxy; Galaxy harassment is also briefly explored. Modified Newtonian Dynamics simulation as a possible rival of dark matter is described. Models were evolved for up to 8.1 billion years.

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"I never teach my pupils. I only attempt to provide the conditions in which they can learn."
— Albert Einstein (1879–1955) —

1

INTRODUCTION

1.1 Motivation

A view onto a midnight blue of sky with shining stars is free for everyone. The ancient Egyptian conception of heavens as a woman with the stars painted on her body and supported by a man is unforgettable. Ancient peoples were just as curious as we are today. Their wondering about day and night, stars, and the Sun and the Moon led to the observation that the heavenly bodies appear to move in a regular manner – this was the dawn of science.

For as long as I can remember, stars did not impress me under the clear night sky, but under the dome of planetarium. Early in elementary school, during a visit to the local planetarium, where a sunny day changed into the deepest nightly darkness, I saw the clear starry night sky projected and moving on the planetarium's dome. Astronomy was fascinating, however accessible only to some exceptionally bright people. My first conscious connection with the night sky was a pure simulation.

Soon after that, I became much more interested in more mundane computer programming. Black computing boxes with flashing red lights created by the company with the closefitting name of Thinking Machines Corporation fascinated me. These machines were mainly used to study the principles concerning natural phenomena. Places filled with mystic atmosphere that is violated only by a silent noise of circulating air. As in peaceful meditation places in temples, that are disturbed only by a quiet sound of spoken mantras and accessible only to several chosen ones, who may discover secret knowledge beyond the realm of direct experience.

Nowadays, we have free access to the pictures of the universe in all its glory through the Internet, taken by billion-dollar telescopes that orbit the Earth. Several years ago, the images of the great impressive ensembles of stars, rotating as brilliant cosmic whirlpools, stunned me. Such objects "awake an intense desire to learn something of the laws which give order to these wonderful systems," as wrote in 1845 Ireland's Earl of Rosse[1], who first recognized a spiral structure in the "Whirlpool Nebula" now known as the Whirlpool galaxy Messier 51 (M51).

To study these "wonderful systems", I created my own galaxy simulator. Now, I want to guide others, who may share the same enthusiasm through an easy, yet comprehensive way to get some insight into these exciting systems of stars and lead them to create their own simulations. I am glad that I have had an opportunity and a chance to explore the natural processes governing the movement of heavenly bodies. The simulations of these beautiful systems are challenging, but accessible to everyone.

1.2 History and state-of-the-art

Before the invention of reading and writing, people were taught through the direct and informal education of their parents, elders, and priests. They learned how to survive against natural forces, animals, and other humans. By using a language, people learned to create and use symbols or words to express their ideas. Still, their thinking was limited to the knowledge given to them by their teachers and a limited amount of pre-selected books.

Ongoing technological development is providing means for new methods of education. Students can freely choose what to study without any limitations. Study materials are available not only for general areas of science, but also for specialized fields due to self-teaching educational-research projects. Unfortunately, the educational texts joining education with state-of-the-art research are very young and there is just a small amount of them. Piet Hut and Junichiro Makino[2] are developing similar ideas of educational-research project focused on the simulations of stellar clusters. The self-teaching educational-research project for the simulations of galaxies did not exist until now. It is produced here for the first time.

1.3 Thesis objectives

This work aims to create the self-teaching educational-research project involving many-body computer simulations with the objective of studying galaxy dynamics.

The main goal of this work is to create the self-teaching educational-research project, which will guide a student through the numerical models and computer simulations of galaxy dynamics. It will show in detail a numerical construction of galaxy models and how these artificial galaxies may be evolved with the computer simulation.

Output at the technical level will be a study material for the education of galaxy dynamics showing the development of many body computer simulations step-by-step. Output at the pedagogical level will be the project in computational astronomy with a self-teaching approach.

[1] William Parsons, third Earl of Rosse (1800–1867)

[2] Hut P., Makino J. (2006): *The Art of Computational Science*, Volume 1, <http://www.ArtCompSci.org/>, accessed on December 22, 2006.

Output at the level of general interest will be animations suited for classic school education and the popularization of astronomy.

Dissertation itself is composed of two parts. Chapters 1 and 2 handle the theoretical pedagogical framework of the thesis and chapters 3–8 contain technical material with my original contribution to the simulations of galaxy dynamics.

The final idea is to lead students into the understanding of the principles behind the many body simulations for galaxy dynamics – reading the technical part of the thesis, experimenting with galaxies, letting them collide, taking a look at the source code, modifying it and developing new modules for it.

The aim of this thesis is to develop numerical model of galaxy dynamics, which permits future maintenance and modification by non-expert programmers. I do not assume that students reading the technical part of the thesis are mathematical or programming experts[3]. These skills will be developed during the creation of models.

1.4 Experimental methodology

I have used computer modeling and simulations throughout the dissertation. Many-body simulations were evolved for up to hundreds of thousands timesteps at different resolutions that allowed me to study galaxy dynamics accurately with modest computational resources. All galaxies were modeled self-consistently as fully three-dimensional collisionless many-body systems. The galaxies were evolved with an algorithm containing no geometrical or spatial limitations.

1.5 Information sources and literature

My diploma and rigorous theses were the starting-point for this thesis. I had to read a lot of new articles and research papers ranging from galaxy dynamics, physics, and astronomy, through computer simulations and numerical methods, to pedagogical and psychological articles.

I used almost entirely digital sources available through the Internet provided with full-text search; I utilized the search engine Google[4]. The majority of papers and articles cited in this work along with many others and references therein were retrieved from astrophysics pre-print archives (astro-ph)[5] operated by the Cornell University and from the NASA Astrophysics Data System (ADS)[6].

[3] To meet these requirements, I abandoned my original super-massively parallel simulation software AMON-1, which was much more complicated in the sense of computational distribution to PC cluster and multi-threading in each of cluster's PC.

[4] http://www.google.com/

[5] http://arxiv.org/archive/astro-ph

[6] http://adsabs.harvard.edu/

"Teaching is not filling a pail, it is lighting a flame."
— attributed to Heraclitus of Ephesus (535–475 BCE) —

2

NEW PEDAGOGY

In this chapter, we will show the importance of the self-teaching educational-research projects as an extension of compulsory education and a psychological motivation underlying the understanding of galaxy dynamics.

2.1 Attractive science education in basic curriculum

Technology, the use of scientific knowledge for practical purposes, has a profound impact on the way we live and on the quality of our lives. A powerful method of science and research endeavor brings us every aspect of our comfort. People should know merits behind their everyday lives brought by the science. A science teacher that serves as a local expert in some specialized field of the science should teach others about these advantages.

Until the 18th century, great scientific discoveries were not explained nor popularized. Scientific knowledge was gathered for its own sake and it had a few practical applications. Scientific knowledge should be disseminated to a wider audience of people, even if they are not directly involved in scientific research. Scientific work should be made clear from its foundations. In an open and democratic society, science should be accessible to everyone and not become an exclusive domain of specialists. All people are able to understand science; scientific knowledge should be available to all of them. When new findings are not transferred to people, they lose their significance; instead, a mix of pseudoscience emerges.

The role of a traditional teacher in every educational system is to transform the majority of students from a state of pure desire to receive good grades and succeed (secondary motivation) into a state of desire for the knowledge itself (primary motivation). From my experience as both a student and as a teacher, I believe that teachers at lower educational levels should always serve physics in the basic curriculum in an easy and interesting way, together with a classical lecturing.

Astronomy is probably the most visually exciting science and it can capture the attention of those students, who would otherwise hesitate to choose a physics course (seminar, field of study, etc.). The universe has interesting topics to study for nearly all students without a difference in age or abilities. The universe is the source of inspiration, unusual images and information, which can capture the students' attention and awake other questions and curiosity. However, studying the universe is more than looking to exciting pictures of space with a great aesthetic experience. It also answers the most fundamental questions for which every human being need to know an answer. From this point of view, the knowledge of general physics might be more interesting. Students can be motivated by the space science[7].

Very little of this curiosity of physics is present in the traditional physics course. Students usually associate learning physics with a rote memorization of laws[8].

2.2 Self-teaching[9] with educational-research projects

Primarily motivated students can easily start their own education. *Many people think of education as something that occurs in a school or classroom. However, knowledge-eager students can gain additional skills behind the walls of schools. This self-teaching approach in the "New Pedagogy" is based on motivated people studying outside of general compulsory education.* For example, a study conducted in the United Kingdom revealed that one in six people undertakes a learning project outside of formal education system[10]. Students should have a chance to acquire other knowledge based on their interests, which are not the interests of their teachers through the self-teaching approach – from an arbitrary area of art or science. This approach is the part of lifelong education. Anyone who does not engage in the self-education, voluntarily or not, lags behind the demands of the time[11].

The self-teaching project requires an active approach from the student. Students are learning when they are active and remember information they understand. "Learning is not a spectator sport."[12] Students are not learning only compiled knowledge, but they are *constructing* and

[7] Dunkin S. K., Ball A. J., Taylor E. A. (1997): *Using space science and technology as an educational tool: two different approaches*, Adv. Space Res. 20, 1379.

[8] Redish E. F., Wilson J. M. (1993): *Student Programming in the Introductory Physics Course: M.U.P.P.E.T.*, American Journal of Physics 61, 222; American Association of Physics Teachers.

[9] Also referred as self-directed learning or self-education.

[10] Brockett R. G. and Hiemstra R. (1991): *Self-direction in Learning: Perspectives in Theory, Research, and Practice*, Routledge, London, UK.

[11] Ruvinsky L. I. (1986): *Activeness and self-education* (J. Sayer, Trans.), Progress Publishers, Moscow, as cited in Hiemstra R. (1994): *Self-directed learning*, in The International Encyclopedia of Education (second edition), Husen T. and Postlethwaite T. N. (Eds.), Pergamon Press, Oxford.

[12] Svoboda J., Obst O., Prášilová M. (1999): *Řízení kvality*, Charles University, Prague – in Czech, <http://www.msmt.cz/Files/DOC/OlomMETPRIR.DOC>, accessed on January 3, 2007.

updating a memory map of abilities through their own activity and effort. Students are subsequently able to apply the acquired knowledge in other situations. Students remember competences they gathered through their own endeavors and efforts. Students should look for information on the internet and classify it independently. Students should learn to read technical writings of others. In educational-research projects, students are developing a whole spectrum of cognitive abilities – thinking and reasoning, memory and learning, attention, perception, judgment, imagining and problem solving.

Every student as a human being is *different*, with different abilities, interests, needs, different learning curve and speed. The self-teaching approach has many *humanistic* effects leading to the student's *individual* personal development. Self-teaching gives to the student a greater degree of self-fulfillment, the liberty of action and the power of control. The student then has a positive enjoyment from an education. This will eventually start positive student attitudes towards the science and high technology. A free choice raises a motivation and the education is more meaningful. Education is spontaneous and naturally rises from individual abilities, interests and needs. Such activity, arising from personal interest leads to a concentrated work and self-nurture. Output of such a creative education is a product, which can serve as the learning material for others – the student is in the role of the teacher of others. On the other side, the student is completely responsible for his or her actions and asserts.

A teacher usually plays a leading role and determines the speed of education. I am convinced that it is insignificant to go sit on a lecture and write down derivations lasting several pages. A better scenario is that a tutor should give to the student a complete derivation with all related thoughts. The student should be provided with educational materials showing problems from various viewpoints. The student then walks through the educational materials by the self-teaching approach through a trial and error. For this purpose, a recorded form of language is better than a spoken language, because students can jump over things known to them, and return back and read over and over the things that they do not understand.

Educational-research projects from various areas of the science are on the Internet and there should be more of them. I would like to encourage others to make their software and thoughts freely available so that everyone can learn and appreciate them. Accompany your scientific software with documentation and publish it on your webpage so that it will serve for the greatest possible use to the public. These projects contribute to the globalization and democratization of education and research.

Piet Hut and Junichiro Makino started the initiative Open Knowledge[13] based on educational-research project. Basic underlying goals of this initiative are namely

- self-contained description: a high-school student should be able to start at page 1, and work her or his way through the educational series,

[13] Hut P. (2006): *Virtual Laboratories*, Progress of Theoretical Physics Supplement 164, 38.

- provide all the details needed when starting from scratch,
- walk through the actual process of learning through trial and error,
- audience: anyone interested.

I am convinced that education will evolve closer to an ideal model of total differentiation or individualized learning together with forms of social and interpersonal education. Apart from the latter, thanks to the development of computer and information technology, there is a glimmer of light for the individualized education with self-teaching educational-research projects and e-learning programs made-to-measure to student's needs right now.

2.3 Research method as the form of education

Teaching methodologies can be arranged on the basis of relative amounts of the teacher's and student's contribution to the education. A similar division depends on how much emphasis the teacher puts on learning and how much is placed on student's personal individual cultivation. At one end of the spectrum the teacher is the controller of the class and the facilitator of knowledge. At the other end is a free discovery method, which is characterized by students exploring subjects of their own interest in ways most comfortable to them.

The research method of education requires individual problem solving of students for an integrated problem assignment. Teacher's activity is suppressed in this form of education. Teacher's task is to identify and select right problems that evoke a student's complex creative behavior, and let them select their own decision procedure. Teacher's role is no longer central; the teacher becomes an adviser. Teacher's duty is to stimulate and cooperate with the student, not just examine the student's knowledge.

The research method of education is a method of active learning that develops complex intellectual abilities in connection with a work on a complex and uneasy project. Active education-research demands from the student thinking not only about technical matters of the project, but also about activities encompassing this project, such as a stress, time schedule, relaxation, sport and free time usage.

The research method of education requires classical forms of perception and reproduction, which are directly incorporated inside it. The research method, however, also requires the discovery and fixation of complex cognitive operations and the interiorization of algorithms to solve problems. The research method is more demanding than a formal learning process in the education system and involves various activities and resources.

The self-teaching in research method also has *social aspects*. The self-teaching does not mean that all learning will take a place in isolation from others. Although students work in part independently, they must always *cooperate* in larger educational-research projects. Students must participate in study groups and the overall success of the project depends on each member of the group. Students must develop communication skills and use a global consciousness to solve problems with others on the Earth via the Internet.

I am convinced that students must be exposed to research level problems at an early stage of their education in order to sustain a continuous advancement of technology and science in long terms. On the other hand, education is a complex system concerning very complex people. The research method of education is not suitable for every educational situation or every student on every school. It depends on the teacher how wisely she or he will choose the methods of education.

2.4 Student's motivation

Why should students want to start learning about galaxy dynamics simulations?

There are psychological aspects motivating our will to understand the universe (nature) around us. For people, it is not sufficient to accept natural phenomena as they are. Our brain needs to understand causes of phenomena, what is their deeper nature[14]. When there is no scientific clarification, a human mind is looking for an alternative mythological explanation. As the only known species of billions that ever lived on the Earth, modern humans were posing questions concerning the nature of the world around them since they had ability to ask questions more than 100,000 years ago. We are an integral part of cosmos, so we want to deeply understand the patterns of its behavior.

Other motivation rises from the desire to be able to control state-of-the-art technological devices – the simulations of galaxy dynamics are very demanding in terms of computational power. From the dawn of computer technology, leading role for pushing its limits was in the hands of physicists and astronomers. In 1990s, several Gordon-Bell prizes won *N*-body simulations for the most powerful numerical calculations (e.g. Warren et al., 1997). The GRAPE-4 computer[15], which exceeded as the first computer on the world 1 TFLOPS ($1 \cdot 10^{12}$ calculations with "real" numbers every second, literally "floating-point operations per second") performance limit in 1995 was dedicated to *N*-body simulations of star clusters. Motivation is still going on as the GRAPE-DR is expected to be the first computer breaching 1 PFLOPS ($1 \cdot 10^{15}$ FLOPS) barrier in 2008. It will be used for *N*-body/SPH (Smoothed Particle Hydrodynamics) simulation of the Milky Way galaxy.

Students may be also motivated to excel in some field of the science. In many countries and schools, where the teaching of physics and astronomy is limited to theoretical equations and some old instruments, the usage of computers would be a great improvement. *N*-body simulations have led to a significant progress in the galaxy dynamics understanding. Once the student learns the basics of *N*-body simulations, he or she may begin to improve it by adding other physical

[14] Manhart K. (2005): *Likely Story,* Scientific American Mind 16, 58.

[15] Developed by Junichiro Makino, one of the Open Knowledge concept creators.

phenomena while creating an astrophysical laboratory, obtaining completely new results and become researcher in a highly attractive and developing field of astrophysics or cosmology.

Motivation for choosing the field of computer simulations in physics can also stem from a practical aspect of employment. Education should prepare the student for a future occupation. Computational physics and programming is good for many jobs both in industrial and academic sectors.

The student can also visit many *N*-body schools arranged all over the world, e.g. MODEST (MOdeling DEnse STellar Systems)[16], Cambridge *N*-body School[17], Computing Our Universe![18] or Late Summer School on *N*-body Simulation[19].

2.5 Computer models and simulations in education

A virtual nature, virtual universe or virtual reality is essential for the science education. The virtual reality mimics the real world and students can safely perform experiments on it. Students can perform *thought experiments* otherwise impossible to do in reality. Moreover, a computation is becoming as important as a theory and experiment. In the past, the natural sciences were characterized by interplay between an experiment and theory. This has gradually changed and instead the theory, experiment and simulation are three equally essential elements of natural sciences today. Yet, the classical education is still lagging behind the teaching simulations of nature in a whole way.

Physics as a school subject should reflect the methods of physics as science. Computers are inherent tools in physics from the basic scientific research to commercial and industrial applications. If computer modeling and simulations are important in physics science, they should play a comparative role in the physics education. Through computer simulations, students are able to explore new phenomena that were not accessible previously. Modeling and simulations expose students to contemporary modern physics.

Modeling and simulations are very effective for vivid education[20]. Students manage the simulation and are free to vary inputs to obtain outputs according to their choice, to form hypotheses and to test them, asking questions such as “how?” and “what if?” Numerical models and simulations give to students a deeper understanding of the physics they have learned in

[16] http://www.manybody.org/

[17] http://www.cambody.org/

[18] Held by Lawrence Berkeley National Laboratory, USA.

[19] Held by Center for Computational Astrophysics (CfCA), National Astronomical Observatory of Japan (NAO).

[20] Jan Ámos Komenský (1592–1670), known as John Amos Comenius, was the first one, who created illustrated textbook for children. We understand a contemporary usage of multimedia and interactivity in education as the continuity of his approach.

classes. They can confront analytically solvable models acquired during classes and compare them with cases that are more realistic. Alternatively, they can just play with a working model.

From the theoretical point of view, numerical models and simulations are the best tool to understand physics involved in galaxies. Modeling and simulations allow the students to understand basic galaxy dynamics concepts that need a high degree of abstraction. Fantasy and creativity are important qualities of students that create computer models and simulations. The student integrates theory and experimentation in computer simulations, and then modifies computer models and tunes numerical solutions.

Computer experiment connects a model provided by a theory with calculation carried out by a machine simulating the real experiment. Numerical computational techniques can be used to improve our understanding of nature. Students can learn about the science through an experience.

Models of physics and computer simulations can also be used in mathematics, so that the student can identify the usability of mathematics in physics applications. It is easier to understand, solve and receive the solution of mathematical problem when it has a physical background and relations. I recognized that some students of mathematics or physics are a bit fearful of programming computer models. On the other hand, students of computer science are usually intimidated by the mathematics.

This is the next dimension for which current pedagogical theories call: an interdisciplinarity in education – a connection between many fields of science: computational physics, which includes theoretical physics as the main driver, computer science with numerical analysis and computer languages as an expression for mathematical and computational representation of physics and observational (experimental) astronomy. Moreover, this can be expanded into other highly attractive fields like biology, chemistry or biochemistry.

Physics as the connecting link between natural sciences can be very useful. Research conducted among students of different branches of science showed that undergraduate physics students display more understanding of physical models versus reality than did a graduate students of biology and chemistry faculties[21]. *N*-body simulation as the versatile method in computational physics is well suited for the science education with computers[22].

This thesis is giving strong emphasis on using numerical models of nature. Students are sometimes confused with the relation between "laws of physics" and reality, identifying specific physical model or theory with the reality. In the virtual nature, student is able to change "laws" governing the behavior of their universe and has a power to create her or his own universes.

[21] Thoermer C. and Sodian B. (2002): *Science undergraduates' and graduates' epistemologies of science: the notion of interpretive frameworks*, New Ideas in Psychology 20, 263.

[22] Johnson V. and Teuben P. (2003): *Teaching Scientific Computing with N*-body *Simulations,* Astronomical Data Analysis Software and Systems XII, ASP Conference Series 295, 370.

Through this experience, the student will learn that contemporary models are imperfect models and just try to imitate reality.

Computational physics gives to the student many competences[23]:

1. The ability to express the laws of nature in the form of equations and to manipulate them in a variety of situations: analytic skill,
2. the ability to express these laws in the form of quantifiable entities,
3. the ability to understand natural scales and the estimation of scales,
4. the ability to have approximation skills,
5. numerical skills,
6. intuition and large problem skills.

When students learn a computer how to solve the physical problem in the form of computer program, they will have a perfect understanding about how to solve the problem without the computer. More important, the student will develop the ability to think in a critical sense, because the student will not be a mere user.

2.6 Application

Therefore, I propose a four-level educational architecture (Figure 2–1), which is divided into four levels. The first level is for casual students who are interested in nothing more than in animations that are suitable for public presentations. Students in the second level will use an existing simulation program, change input parameters and look for results. Third level students will be more interested and will read technical information written in the second part of this thesis to get a better insight. The four-level architecture culminates with students reading, programming, analyzing and expanding galaxy dynamics simulations, and with a deep understanding of numerical simulations.

Students in the first level are only occasional people caught by a nice animation. Since galaxies evolve very slowly in comparison with one lifetime, it is hard to see any changes in real galaxies. Galaxy simulation allows to actually see a model galaxy evolving with student's own eyes, something that is not possible in the real universe, but what is actually happening. The second level is for users who are interested only in using galaxy models and simulations, executing the software applications. Having some deeper doubts, the user might want to know how galaxy dynamics simulation works: its concepts, routines and different approaches.

The level 3 user will be interested in reading the technical part of this work. It focuses on the theory behind the galaxy dynamics, and on algorithms used in the simulation software. In each chapter, there are explanations of the algorithms. In addition, the user can access the project's web site, where all materials including codes and animations regarding this work are listed and

[23] Sheth C. (2002): *Computational physics programme in research and teaching – an African experience*, Computer Physics Communications 147, 590.

can be downloaded. If the student is willing to contribute, she or he will also look at commented source code, to better understand how the software works.

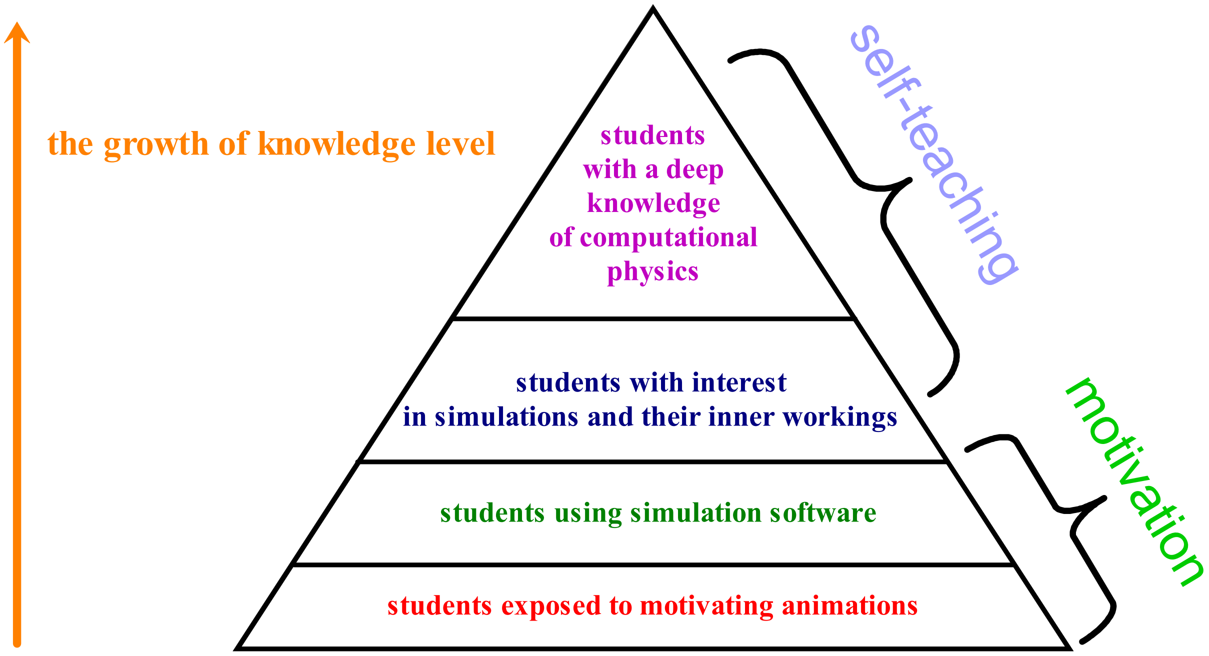


Figure 2–1: The four-level educational pyramidal architecture that is grounded on the solid foundations of primary motivated students. Upper levels contain students with the high interest and understanding of numerical simulations.

At this point, the student will be able to develop a new module (which could be an assignment, for instance) or upgrade the old ones. This is the fourth level. People who get into this level become developers that extend the code and make up the part of the galaxy dynamics worldwide team. It is our desire that all users reach this level, but no one is obliged to do so. The final goal of this four-level user approach is to provide to the student with a means of learning simulations of galaxy dynamics in a whole way. The student will be able to read about it, understand its principles and further expand it.

Now numerical galaxy dynamics should not be a mystery to the student, and the gap between the concepts being taught in classes and the state at the research level will be minimized.

2.7 Concluding remarks

In Chapter 2, we have shown that physics education is an important and crucial element for human society. Students should be more motivated by their teachers with less importance on learning and more emphasis on differentiation, individualization and self-teaching. It is for this purpose that the formation of self-teaching projects is suggested. Together with advancement in science and technology, an early connection of education and research should be made. Self-teaching educational-research projects created by specialists in their fields should be made freely available on the Internet as a service to society. A research method of education can develop student's abilities in a complex way. Computer models and simulations of nature's behavior are acknowledged as useful, providing connections between various fields of science education. A scheme incorporating these approaches is suggested in the "four-level educational architecture". Surely, education is a complex system and this concept may not be valid for every student.

"Could we ourselves be in such a computer simulation? Could what we think of as the universe be some sort of vault of heaven rather than the real thing?"
— Martin Rees, Astronomer Royal —

3 MODELS AND SIMULATIONS

In this chapter, we will sketch our basic understanding of nature, laws of physics, models, simulations and confusion among them.

3.1 Nature, laws of physics and models

The main method used throughout this work is a computer simulation of nature. This method is based on believe that nature is understandable and has a mathematical underpinning.

Natural philosophers or physicists discovered that nature behaves according to regular patterns. They discovered that the behavior of nature is mathematically describable. Marquis Pierre Simon Laplace[24] thought that if we knew where all bodies in the universe are at a particular time and knew what their velocities are then by applying the discovered laws of physics – such as Newton's laws – we could find out where they would be located at any time in the future. He imagined the universe as a clock mechanism so that everything is predictable. Such a finitely complex universe could be modeled exactly, if we knew initial positions and velocities of all particles during the "Big Bang". In addition, we would get through such a simulation not just an approximation, but the exact reproduction of the evolution of such universe.

Is a description of phenomena in nature by the laws of physics exact? Science is in the state of permanent change and development. A scientific community continuously updates the laws of physics that describe patterns of nature. For example, Albert Einstein showed that Newton's laws of motion do not apply to objects traveling at speeds close to the speed of light. However, Einstein himself knew that his work will be challenged one day – and it has already happened in astronomical and cosmological observations. We are facing the problem of apparently striking lack of mass-energy in the universe, which we must find in order to vindicate current laws of

[24] Pierre Simon Laplace (1749–1827)

physics. Probably there is some dark matter and dark energy of yet unknown kind, but maybe our current laws of physics do not apply on the scales of universe.

Laws of physics were created by men and are not a precise description of nature. Laws of physics are only an incomplete and limited description of nature. Laws of physics do not govern nature's behavior as the notion can suggest.

Greek astronomers were the first who devised and tested mathematical models of heavens, how the universe is working. Such a first known scientific modeler was Eudoxus[25] who tried to model the motion of planets with the set of uniformly rotating crystal spheres that held the planets, while the Earth was at the center. Since Eudoxus, people are improving models to be in always better agreement with an observable reality.

Again, the model always represents only a certain idealization. Thus, we should be cautious to identify any real physical phenomenon with the corresponding mathematical model. The model is an imperfect image of nature's behavior. Through, some physicists motivated by an enormous success of physics believe that we are capable to fully understand nature[26].

Models are sometimes intentionally simplified when we are confident that not all known facts are essential for our purpose of study (e.g. we can neglect people living on the Earth when we want to study the evolution of our galaxy even if we are able to model human social interactions).

3.2 Computer models and simulations

Computer modeling and simulations play an essential role in today physics. For a long time persisting division of natural sciences to the theoretical and experimental research is now out of play. With the advancement of computers, numerical[27] physics has become an increasingly important branch of physics, distinct from both experimental and theoretical physics, but borrowing parts of both (Nelson, 2000). Computational physics is a way of doing physics research, next to the experiment and theory.

Some characteristic problems accompany galaxy studies. There were theoretical attempts to create the universe in a laboratory[28]. However, even if it would be successful, this universe would be hard to study, because it would start expansion in its own space. Computers are used to create mathematical models of such complex phenomena and to explore them. Computer models and simulations are employed when it is impossible to perform the real world experiment. What we have learned from observations about the patterns of nature is programmed into a computer and

[25] Eudoxus of Cnidus (408–355 BCE)

[26] Theory of Everything (TOE) should explain why the laws of physics are what they are. Steven Weinberg has argued that such theory would be *logically isolated* — that is, it could not be modified without being destroyed.

[27] Numerical refers to the study of finding approximate solutions of equations as opposed to symbol manipulation like we would do with algebra.

[28] Alan H. Guth speaking in BBC's TV program "Parallel Universes".

through the process of numerical simulation is evolved; new results about nature's behavior are obtained.

Simulation programs serve as an essential tool for contemporary astrophysics. It is impossible to imagine today's astrophysics without computer simulations. Even a personal computer technology is on such a level that it can perform astrophysical simulations. These machines are able to mimic or simulate the evolution of the universe lasting billions of Earth years in short times of several hours or days of computer time.

There is no need for expensive material equipment. Moreover, the model is safe and indestructible. There is a multiplicity of situations to be explored and this can be done at ours discretion. In contrast to the real models, parameters can be easily changed. An enormous range of both length and time scales can be covered in simulations[29]. The complexity reduced to simple parts makes these computer simulations so close to reality[30].

Before using numerical models, we should always master underlying physics and explore analytical solutions as much as possible. A numerical computation is coming in cases where analytical solution cannot be found[31] or cannot be found by elementary means. The numerical computation can answer problems that are not solvable to exact solution. We are trying to turn an analytical problem into a numerical problem. However, the numerical problem has its own challenges so we end up with a different set of challenges and limitations.

Laws of physics are usually expressed by differential equations[32]. A s*imulation* is a process numerically solving a set of differential equations. These differential equations are trying to express the behavior of nature. A numerical formulation of differential equations forms a computer model, which in turn determines the behavior of the computer simulation. The calculation with the computer model (simulation) consists of simultaneously solving a set of equations describing the physical processes involved in the model. This is executed repeatedly.

3.3 Modeling and simulating galaxy dynamics

Galaxy dynamics is one of the liveliest subfields of astrophysics. To understand the past, present and future of galaxies, looking at them is not sufficient. Why are not all galaxies the same? What causes spiral arms? Questions about how evolutionary changes occurred and about relationships between different events have become increasingly important. Since the galactic history occurred

[29] However, numerical computations are not miraculous. Even "largest" numerical simulation performed by computer, could be realized with just a pencil, paper and human brain. Nevertheless, usually you cannot realize such computation in reasonable time.

[30] Since 1960s, some researchers are also trying to understand the behavior of systems as whole (e.g. complexity theory).

[31] There are also approximate analytical methods that extend set of solvable problems (e.g. perturbation theory).

[32] Not always. E.g. in the chaos theory are sometimes used iterative relations.

only once and doing the real experiment is impossible, computer simulations play an important role. Simulations can capture important elements of a process and can suggest avenues to explore further in the field.

However, it is hopeless to create models of galaxies only while sitting in offices with computer simulations. Simulations must be compared to real galaxies so that we can be sure that our results are not just nice pictures. It is impossible to observe directly the evolution of concrete single galaxy with a respect to the size of a temporal dimension. It is not possible to observe the galaxy forming or evolve; the processes are far too slow. Researchers may only observe many different galaxies, each caught at a different phase of its evolutionary history. Nevertheless, we are able to observe different galaxies in different phases of their evolution and study their interactions by observing distant galaxies.

Telescopes are probing the very earliest galaxies and computer simulations can extend our knowledge behind the real world observations. When we notice a pattern in nature, we are curious about it and attempt to investigate it. The numerical tool is often the only one available to the researcher studying the long-term evolution of galaxies. When we perform a computer simulation of galaxies, we hope to learn why real galaxies have the features we observe.

"All motion is an illusion."
— Zeno of Elea (5th century BCE) —

4 GRAVITATIONAL SIMULATION

How separate stars in a galaxy interact? In this chapter, I will show how to simulate the effect of a gravitational field and of Newton's laws of motion to move stars (simplified to mass bodies) around. The method is referred to as many-body problem, particle simulation, *N*-body simulation, *N*-particle simulation or *N*-body problem[33].

4.1 *N*-body problem

Dynamics is interested in more than a pure description of motion. Dynamics is a study of how a system of bodies evolves over time under the presence of force. Bodies are represented by mass particles (or mass points) that are under the influence of force. The *N*-body simulation is used to determine the evolution of *N*-body system.

The *N*-body simulations can be used in theoretical studies of the large-scale structure of the universe, galaxies, star clusters, solar systems, stellar dynamics, climate modeling, gases, liquids and plasmas. On smaller scales, quantum effects become important. *N*-body simulations are applied in a wide range of areas outside physics such as *N*-body simulations of large biomolecules, for example realistic protein unfolding or even simulations of deoxyribonucleic acid (DNA). The *N*-body simulation has proven to be one of the most versatile methods.

The *N*-body problem is defined as follows. We have initial conditions i.e. initial positions and initial velocities of all bodies in the system. Interactions (forces) between all bodies in the system have to be evaluated to receive new positions and new velocities. This evaluation is performed repeatedly so that we are getting information about the time evolution of the system.

Mathematically is the *N*-body problem formulated by the system of ordinary differential equations (ODE) coming from Newton's laws of motion expressed as

[33] *N* being the total number of particles present in the system.

$$\frac{\mathrm{d}\vec{r}_i}{\mathrm{d}t} = \vec{v}_i \tag{4.1}$$

$$m_i \cdot \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = \vec{F}_i, \tag{4.2}$$

where m_i, $\vec{r}_i$ and $\vec{v}_i$ are the mass, position and velocity of the *i*-th particle, respectively, and $i = 1, 2, \ldots, N$. The force $\overrightarrow{F_i}$ is usually the sum of external forces. When dealing with molecules, Lennard-Jones force, van der Waals force or Coulomb's electrostatic force is used. When dealing with systems of stars or stellar systems, we will use Newton's gravitational force[34].

4.2 Gravitational *N*-body problem

Galaxies are controlled by the force of gravity that acts between all bodies except of self-interaction. First, we will look onto a straightforward application of Newton's law of gravity and then on a complex hierarchical approximation.

Gravity interaction between all pairs of bodies (except of self-interaction) will introduce a sum to the right side of Equation (4.2) and will become

$$m_i \cdot \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \vec{F}_{ij}\,. \tag{4.3}$$

A force $\vec{F}_{ij}$ given by Newton's universal law of gravitaty, is given by

$$\vec{F}_{ij} = G \cdot \frac{m_i \cdot m_j}{r_{ij}^3} \cdot \vec{r}_{ij}, \tag{4.4}$$

where G is Newton's gravitational constant, $j = 1, 2, \ldots, N$, $\vec{r}_{ij} = \vec{r}_j - \vec{r}_i$ and $r_{ij} = \left|\vec{r}_j - \vec{r}_i\right|$.

After plugging a term for the velocity $\vec{v}_i$ (Eq. 4.1) to the left side of Eq. (4.2), Equation 4.4 is simplified to a form, where the acceleration $\frac{\mathrm{d}^2\vec{r}_i}{\mathrm{d}t^2}$ is independent on the mass of selected body *i*

[34] In this approximation, stellar systems are not relativistic (stars are not moving at speeds close to the speed of light) and not extraordinary massive, so that Einstein's theory of General Relativity is reduced to Newtonian limit.

$$\cancel{m_i} \cdot \frac{\mathrm{d}^2\vec{r}_i}{\mathrm{d}t^2} = G \cdot \cancel{m_i} \cdot \sum_{\substack{j=1 \\ j\neq i}} \frac{m_j}{r_{ij}^3} \cdot \vec{r}_{ij} \,. \tag{4.5}$$

A dynamical evolution of the system composed of N bodies under the influence of gravity is not solvable to exact solution for more than two bodies[35]. Therefore, a numerical approach is required to study this problem.

4.3 Numerical techniques for solving ODEs

The new position and new velocity of i-th body must be calculated from the known acceleration. Equation (4.5) contains second derivatives and it is therefore second-order differential equation. However, all higher-order differential equations can be transformed into a set of first-order differential equations (e.g. Press et al., 2002). In our case, Equation (4.5) can be rewritten as two first-order differential equations by introducing a new variable $\vec{v}_i$ as follows

$$\begin{aligned} \frac{\mathrm{d}\vec{r}_i}{\mathrm{d}t} &= \vec{v}_i \\ \frac{\mathrm{d}\vec{v}_i}{\mathrm{d}t} &= G \cdot \sum_{\substack{j=1 \\ j\neq i}} \frac{m_j}{r_{ij}^3} \cdot \vec{r}_{ij}. \end{aligned} \tag{4.6}$$

As can be seen, the variable $\vec{v}_i$ is the velocity. We already had both equations (4.6) in (4.1) and (4.3). However, many problems in computational physics are described by a second order differential equation and it is useful to know, how they can be semi-automatically reduced to the set of first-order differential equations. This transformation corresponds in theoretical mechanics to the Hamiltonian form of equations of motion.

4.3.1 Euler's method

We describe laws of physics with continuous variables and differential equations like Eq. (4.6). However, we must somehow convert these mathematical equations into a numerical form suitable for a limited arithmetic of a digital computer. We have to solve differential equations by the numerical approximation. Let us have some continuous function $x(t)$ in one-dimension and its numerical representation as shown on Figure 4–1.

[35] Analytic solutions are known only to special cases of three-body-problem.

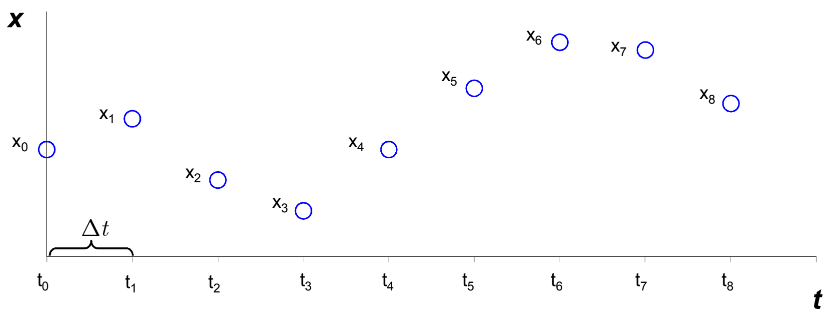


Figure 4–1: A function $x(t)$ tabulated at discrete time instants $t_0, t_1, t_2, \ldots$ as values $x_0, x_1, x_2, \ldots$ with timestep Δt.

Starting from an initial discrete value x_0, we can calculate a following function value x_1 using the equation for the derivative of a function. The derivative $x'(t)$ of the function $x(t)$ at a given point t is equal to the slope of the function $x(t)$ that is tangent to the function at that given point. It is formally defined as

$$x'(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}. \tag{4.7}$$

Here we will approximate the limit by taking some small, but finite Δt rather than mathematically letting it go to zero. Through this very small interval (timestep) Δt, a good approximation to the underlying differential equation is achieved. The derivative at the starting point of each interval is linearly extrapolated to find the next function value. We assume that the derivative remains constant in the interval between t_0 and t_1. Therefore, in order to solve ODE and find the value x_1 at the point t_1, we need to know the initial value x_0 at the point t_0. This problem involving ODE is therefore specified not only by its equations, but also by the initial value and therefore is called *the initial value problem*[36]. Consequently, an approximate solution to the differential equation is given by the initial value and the scheme

$$x_{t+1} = x_t + \Delta t \cdot F(x_i, t_i), \tag{4.8}$$

[36] Other ODEs may be *two-point boundary value problems*, where boundary conditions are specified at more than one point.

where $F(x,t) = \dfrac{\mathrm{d}x}{\mathrm{d}t}$. Second term on the right side of Equation (4.8) $\Delta t \cdot F(x_i, t_i)$ is the first term of series in the Taylor expansion of $x(t)$ about t_i. This scheme for numerically approximating the solution of ODE invented Leonhard Euler[37].

We must now apply Euler's scheme to equations (4.6) and finally compute the new position and new velocity of the *i*-th body. When we consider the change of d to Δ we can express the increments of the position and velocity as

$$\Delta \vec{r}_i = \vec{v}_i \cdot \Delta t\,, \tag{4.9}$$

$$\Delta \vec{v}_i = G \cdot \Delta t \cdot \sum_{\substack{j=1 \\ j \neq i}} \frac{m_j}{r_{ij}^3} \cdot \vec{r}_{ij}\,. \tag{4.10}$$

With certain initial conditions this will ultimately change to

$$\begin{aligned} \vec{v}_i^{(t+1)} &= \vec{v}_i^{(t)} + G \cdot \Delta t \cdot \sum_{\substack{j=1 \\ j \neq i}} \frac{m_j}{r_{ij}^3} \cdot \vec{r}_{ij} \\ \vec{r}_i^{(t+1)} &= \vec{r}_i^{(t)} + \vec{v}_i^{(t+1)} \cdot \Delta t, \end{aligned} \tag{4.11}$$

where $\vec{v}_i^{(t+1)}$, $\vec{r}_i^{(t+1)}$ are new values of velocity and position of *i*-th body and $\vec{v}_i^{(t)}$, $\vec{r}_i^{(t)}$ are previous values of velocity and position of *i*-th body, respectively.

4.3.2 Midpoint method

Although Euler's method is cheap to compute, it is not very desirable for practical purposes, due to its inaccuracy when compared to other methods. These methods use more points and higher derivatives to get a better estimate of the derivation in the point of interest, which is paid off by the requirement of more computational power.

Nevertheless, we can improve Euler's method, so that it is much more accurate and that its computational cost is only slightly higher. This method uses derivatives computed at the midpoint of each step. In addition, it avoids the need for computing second derivative. Moreover, the midpoint method is very well suited for second-order differential systems of type $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} = G(x,t)$ by design and is therefore suitable for *N*-body problem solutions.

The extra actions taken to prepare the system for midpoint method are the prediction of the position of body $\vec{r}_i$ at the middle of the timestep Δt according to

[37] Leonhard Euler (1707-1783)

$$\vec{r}_i^{\left(t+\frac{1}{2}\right)} = \vec{r}_i^{(t)} + \vec{v}_i^{(t)} \cdot \frac{\Delta t}{2} \tag{4.12}$$

and the acceleration based on this positions at $r_{ij} = r_{ij}^{\left(t+\frac{1}{2}\right)}$ computed as

$$\vec{a}_i^{\left(t+\frac{1}{2}\right)} = G \cdot \sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{m_j}{r_{ij}^3} \cdot \vec{r}_{ij} \,. \tag{4.13}$$

Then the *i*-th particle is advanced according to

$$\vec{v}_i^{(t+1)} = \vec{v}_i^{(t)} + \vec{a}_i^{\left(t+\frac{1}{2}\right)} \cdot \Delta t \,, \tag{4.14}$$

$$\vec{r}_i^{(t+1)} = \vec{r}_i^{(t)} + \left(\vec{v}_i^{(t)} + \vec{v}_i^{(t+1)}\right) \cdot \frac{\Delta t}{2} \,. \tag{4.15}$$

Velocities are one-half of the timestep out of synchrony with positions.

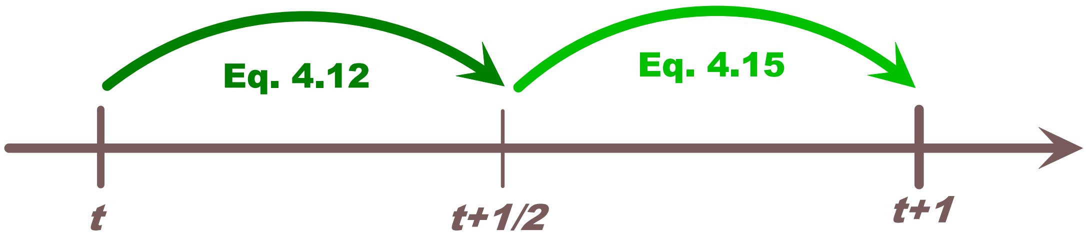


Figure 4–2: The evaluation of position in the midpoint method is looking like a jumping frog. Therefore, this method is also called the leapfrog.

4.4 Methods for solving galactic *N*-body problem

To solve the *N*–body problem for large *N*, we must look at the efficiency of a computer solution to be sure that simulation time will not be prohibitive. An algorithm's efficiency can be evaluated through the number of computations that must be carried out in order to solve a given problem. Usually, it is impossible to exactly predict the number of operations performed by a computer, because it depends on exactly what a machine or language implementation is being used. Therefore we generally use the "big O" notation to generalize a computational complexity ("the speed of a program"). The computational complexity usually depends on the size *n* of the input data $O(f(n))$.

Ideally, a dynamical evolution of a galaxy would be modeled with a direct *N*-body simulation. In the framework of the direct *N*-body simulation, each body interacts with $(N-1)$ other bodies.

The interactions must be carried out for each of the N bodies. To compute the accelerations (4.13) for each of the N bodies, the direct algorithm must summate over $(N-1)$ bodies. In order to evaluate force interactions of the system composed of the N bodies, $N \cdot (N-1)$ computations are needed; that is nearly $\sim N^2$ computations. For a galaxy with 10^{12} stars, 10^{24} computations are needed. To study evolutionary stages of such galaxy, we will need at least tens of thousands of such timesteps. Of course, it is possible to use third Newton's law of motion, which holds that "to every action there is always opposed an equal reaction" (Newton, 1687). This reduces the number of required computations to a half, i.e. $\frac{N}{2} \cdot (N-1)$. However, computational complexity is still $O(N^2)$. It means that the number of computations is approaching $c \cdot N^2$, where c is some constant. This direct (brute-force) computation provides a high accuracy for the price of huge time complexity. Therefore, number of bodies that can be simulated is dramatically limited.

General-purpose computers are becoming more advanced so that N-body simulations can account with more and more bodies. Computations on a graphics card in personal computer with a high processing speed called GPGPU (General-Purpose computation on Graphics Processing Units) are becoming available. Single-purpose computers like GRAPE (built around specialized pipeline processors, literally "GRAvity PipE") introduced in 1990 and computers with FPGA (Field Programmable Gate Array) are increasing a widespread usage and the accuracy of N-body simulations.

Advancement not only in the power of computer hardware, but also in the development of sophisticated algorithms made possible studies of large systems of bodies like galaxies or galaxy groups. Still, the gravitational influence is long reaching. Therefore, it is impossible to neglect in computations bodies behind some spatial distance. However, astronomical systems have properties that can be utilized in designing "faster" computational methods. These methods lead to the reduction of required differential equations that must be solved in order to obtain new positions and velocities of bodies in the modeled system. A small error is introduced in exchange for a large speedup.

A lot of methods for a faster simulation of gravitational force was invented. However, it is possible to divide them into two basic groups.

- Methods based on finite differences, finite elements, grid or mesh methods, Eulerian codes – e.g. particle in cell (PIC), particle-mesh (PM), particle-particle/particle mesh (P3M) – computes even in regions with a low number of bodies, grid density same everywhere even in areas without bodies, geometrical restriction, useful mainly for homogenous particle distributions. Used from 1960s: forces are evaluated in each of the cell of the grid; fast Fourier Transform (FFT) is then used to solve the gravitational potential from a density distribution interpolated onto a regular mesh.
- Methods based on a particle system, grid-less methods, Lagrangian codes – adaptive for non-homogenous particle distributions without the loss of speed, does not waste time in regions

with a low number of bodies. Invented in 1980s. Tree methods: Once a tree is built, it can be re-used as an efficient search method for other physics such as smoothed particle hydrodynamics (SPH).

State-of-the-art simulation codes like GADGET-2 (Springel, 2005) combine both techniques into a single hybrid code.

4.5 Hierarchical approach to *N*-body problem

How can we find out the gravitational influence of the galaxy M31 in the constellation of Andromeda[38] to the Earth? The number of stars in the Andromeda galaxy is of order 10^{12}. If we look at this galaxy on the night sky, we cannot distinguish its individual stars, because of the great distance of 730 kpc; we see it as a patch of light. The same physical intuition can be applied to the gravity. If we are computing the gravitational influence of the distant Andromeda galaxy, we can substitute the whole Andromeda with a single mass point, which position will be located at the center of mass of that galaxy (see Figure 4–3).

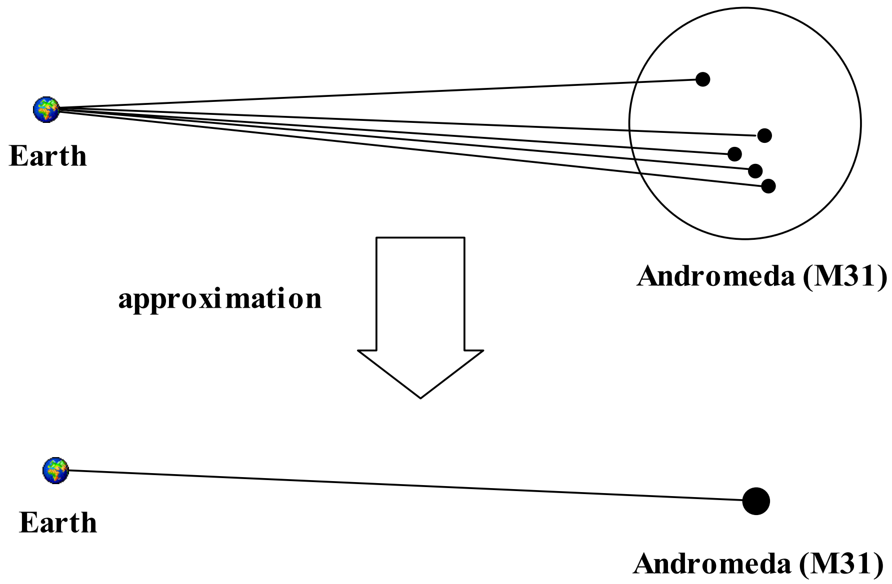


Figure 4–3: Gravitational potential of the distant system of bodies approximated as the potential of a single body.

Barnes and Hut (1986) introduced such a method for solving *N*-body problem that requires less computation than the direct *N*-body method. The gravitational influence of distant bodies can be combined into a single group so that the whole group acts as a single body. This method is called the Barnes-Hut algorithm, tree-code, or hierarchical tree-code.

[38] Means that M31 galaxy lies beyond the constellation of Andromeda.

4.6 Building tree hierarchy, space decomposition

A basic computer structure representing a physical spatial distribution of bodies in a space is a tree. The root of the tree (overall simulated volume) encompasses all bodies in the simulated system. The tree is constructed from the root by splitting the simulated volume into rectangular cubes (cells, domains). In three dimensions, the division of the parent cube into eight daughterly cubes (potential new nodes) constructs every node of the tree. The tree structure is therefore called octal-tree or simply oct-tree. The division is accomplished by splitting each of the Cartesian dimension to a half. This repeatedly continues until the cube contains more than one body. If there is only one body in the cube left, it is stored as a leaf in the tree structure. When the cube is empty, it is ignored. The tree is composed of a hierarchical arrangement of leaves, nodes and the root (top-most node) (Figure 4–4). The leave represents the mass point itself, in the hierarchy is on the lowest position. The tree is usually reconstructed during every timestep.

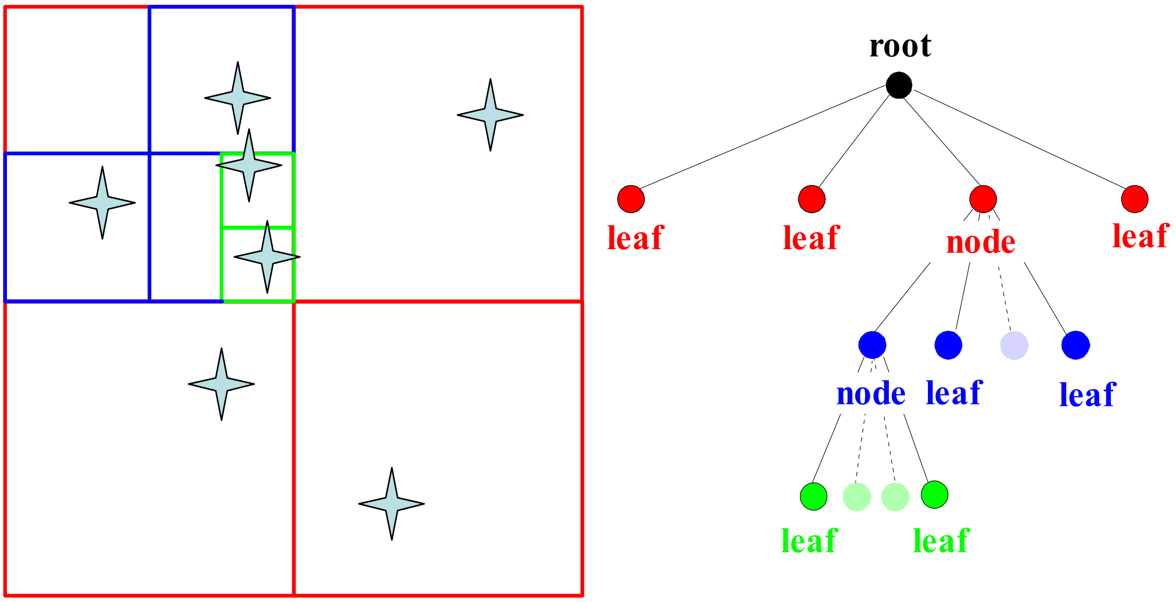


Figure 4–4: A two-dimensional simulation volume with stars is divided to four areas (left). A related quad-tree (right) is constructed in a clockwise pass starting at the bottom-right square of the simulation volume. Empty squares of the simulated volume are ignored in the tree and are illustrated as transparent circles connected with dotted branches to the parent node.

The tree method is suitable for astronomical systems. It is adaptive to a non-homogenous distribution of bodies, because it automatically adjusts to the particle distribution. In the case of uniform distribution, the tree has the same depth everywhere. On contrary, in a typical galaxy with more stars in the center, the tree is automatically deeper at the center and shallower on the edge (see Figure 4–5). This is the source of troubles for simulations with grid-based codes.

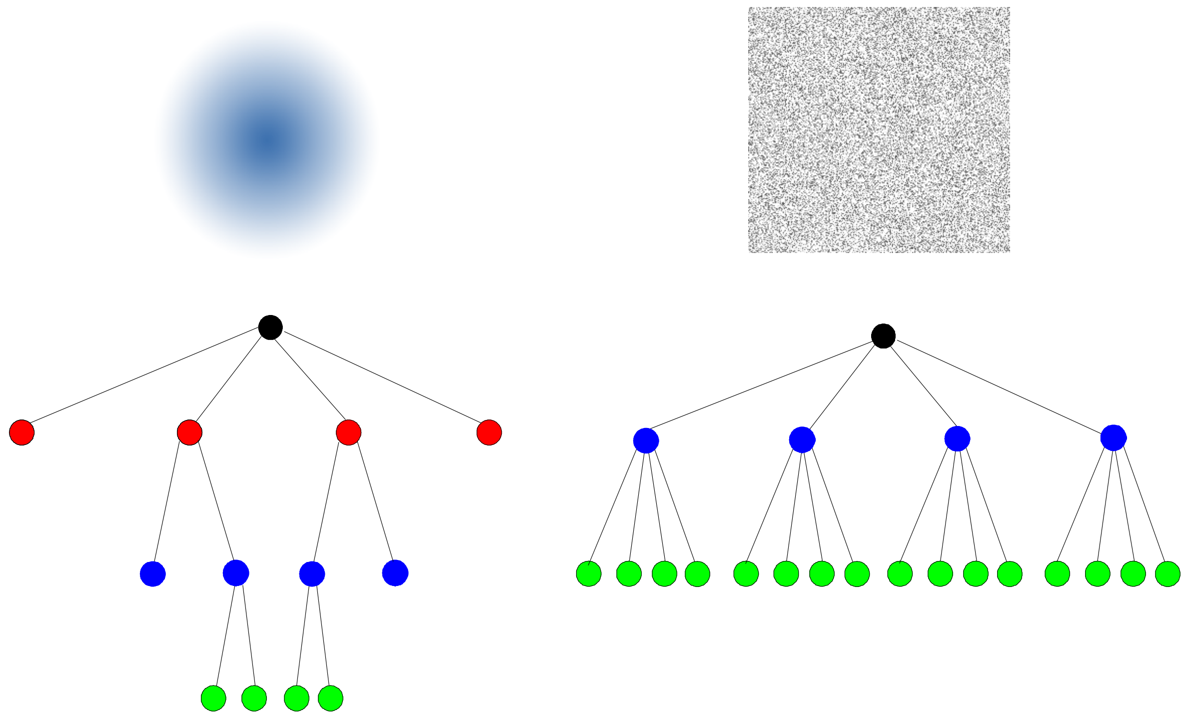


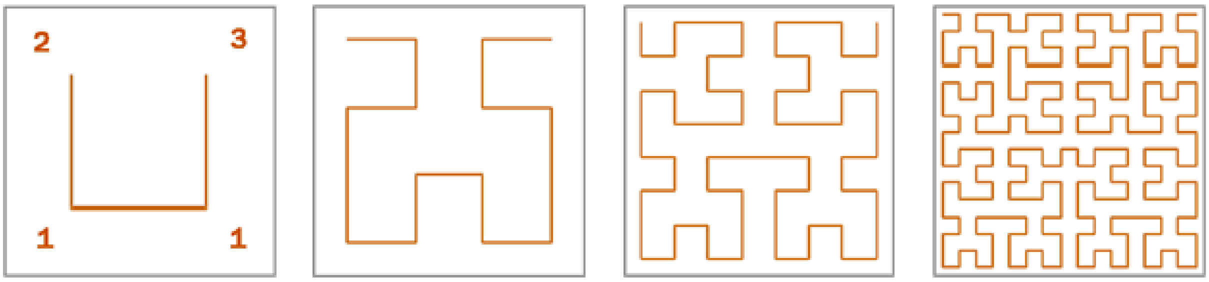
Figure 4–5: Galaxy-like (left) and uniform (right) distributions. The adaptability of the tree method is very useful in galaxy calculations with a highly varying particle distribution.

4.7 Division of space with Hilbert's space-filling curve

The simulated volume can be optionally divided by a space-filling curve[39]. The space-filling curve completely fills up the simulated volume by passing through the every point of simulated volume. It is physically meaningful to divide the simulated volume continuously.

Hilbert's curve in 2D can be constructed recursively starting with an initial curve C_0. We can identify orientations of the curve C_0 with numbers (1), (2), and (3) (see Figure 4–6). The subsequent curve scheme is determined by the previous through replacing both (1) with the smaller version of the same orientation of the scheme and by replacing ends (2, 3) with rotated and mirrored versions of the scheme. On the next level is applied the same algorithm on each part of the previous scheme.

[39] The usability of such division becomes apparent when the simulation has to be paralelized.



C_0, level 0 C_1, level 1 C_2, level 2 C_3, level 3

Figure 4–6: Pattern of Hilbert's curve in two dimensions.

In three dimensions, Hilbert's curve copying is somewhat more complicated. Again, the subsequent orientation of Hilbert's curve can be determined from its predecessor through a rotation. Let us define the initial orientation of Hilbert's curve (level 0) as on Figure 4–7. In the following step (level 1), the cube will be divided and traversed with Hilbert's curve in a way marked in Figure 4–8.

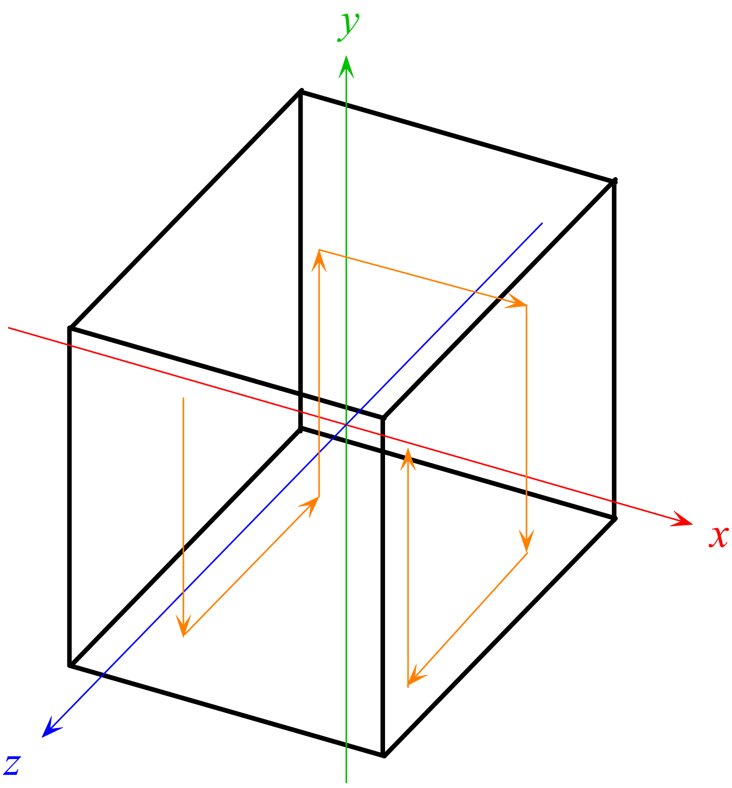


Figure 4–7: The initial orientation of Hilbert's curve (level 0, root cube) in the three-dimensional simulated volume. The division of this cube will construct level 1 of the oct-tree in a marked way.

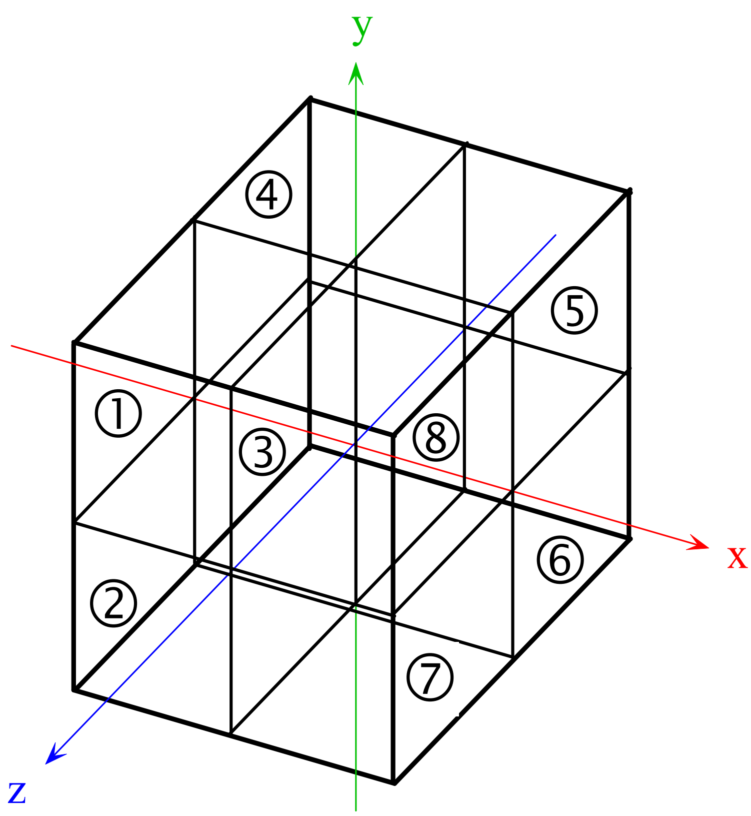


Figure 4–8: Level 1 division of the three-dimensional Hilbert's curve.

Rules for copying to the subsequent step are now following. Initial Hilbert's curve (level 0) will be rotated as follows:

1. Rotate around y-axis by $-90°$ and around x-axis by $+90°$.

2. Rotate around y-axis by $+90°$ and around z-axis by $+90°$.

3. Same as (2).

4. Rotate around x-axis by $+180°$.

5. Same as (4).

6. Rotate around y-axis by $-90°$ and around z-axis by $-90°$.

7. Same as (6).

8. Rotate around y-axis by $+90°$ and around x-axis by $+90°$.

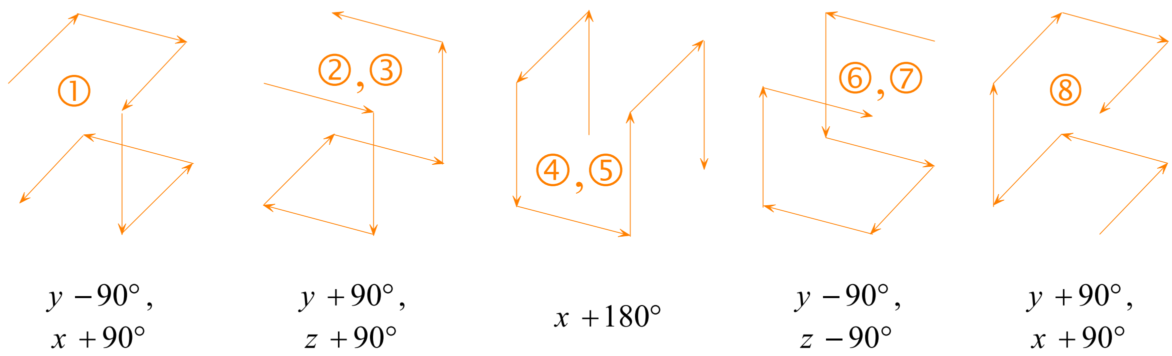


Figure 4–9: The traversal of the simulated volume by Hilbert's curve.

4.8 Computing centers of mass

Every node contains information describing a mass distribution of bodies that fall within this node. Once the tree construction phase is finished, information about a position, velocity and total mass of node's descendants is computed and stored into each node. This calculation is performed in one pass from leaves to the root. For higher accuracy, quadruple or higher order moments should be also computed. However, such calculations lead to higher requirements on a simulation time and computational slowdown.

4.9 Force evaluation

The pass through the tree from its root computes force acting on every single body in the system. It will take the body from the current node (for the first time, it is the root node) and it will receive its descendant (nodes or leaves from one of eight possible sub-nodes of the root). If the center of mass of the current node is sufficiently distant from the selected body, the force is computed as the force acting between the selected individual body and the node. This is the heart of the speed-up of force computations.

If the distance between the selected body and the node (its center of mass) is not sufficient then the node is "opened" and a distance check is performed between all of its leaves or sub-nodes and the selected body (see Figure 4–10). This is executed repeatedly. The worst-case scenario is that the algorithm will dig-down all the way from the root to individual leaves (individual bodies) and the computational complexity will be the same as in the case of the direct *N*-body simulation.

When is the node "sufficiently distant"? A criterion determining sufficient distance is a *multipole acceptability criterion or MAC.* Barnes and Hut (1986) introduced criterion

$$d > \frac{l}{\theta}, \tag{4.16}$$

where d is the distance between the selected body and the mass center of node, l is the length of cube's edge (see Figure 4–11) and θ is the opening angle. Parameter θ determines the speed-up of the algorithm and accuracy at the same time. It is suggested to choose this parameter in the range $0,7 \leq \theta \leq 1$ (Hernquist, 1987). Smaller values of θ are leading to the opening of a larger number of nodes, to a better accuracy of force computation and to computational slow-down.

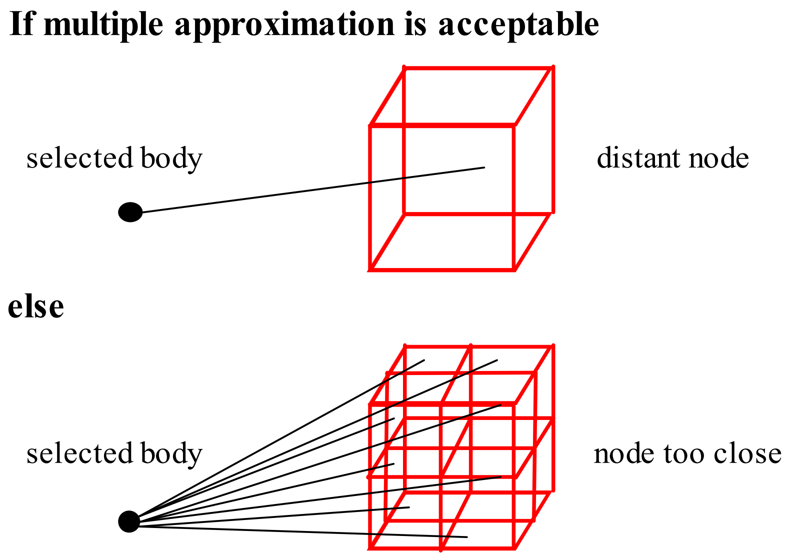


Figure 4–10: If the distance between the selected body and the node is sufficient, the computation of force is performed between them. Otherwise, the distance criterion is applied to all descendants of the node.

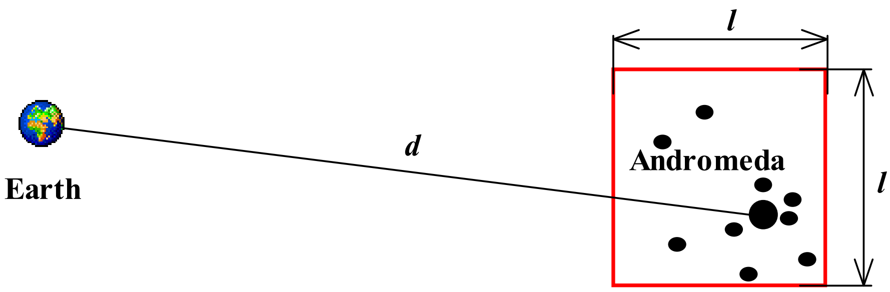


Figure 4–11: The meaning of parameters *d* and *l* in the Barnes-Hut criterion.

This approach leads to the reduction of required interactions and to the decrease of computational complexity to $O(N \cdot \log N)$. It is a substantial improvement over the direct summation method with $O(N^2)$ complexity (see Figure 4–12). To further improve a performance and reduce a total processor time required for a simulation run, I parallelized this part with OpenMP[40].

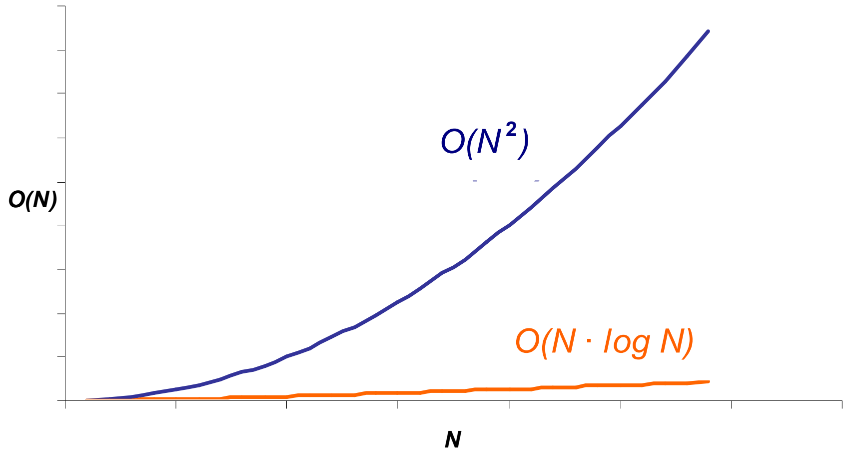


Figure 4–12: A superior scaling $O(N \cdot \log N)$ of the hierarchical method for solving the *N*-body problem as compared to the direct method with quadratic $O(N^2)$ complexity. For the same number N of bodies, fewer computations are required with the hierarchical method.

4.10 Grouping

Interaction evaluation and MAC testing are located in the computationally most-intensive part of the algorithm. Therefore, it is convenient to look for optimizations of this time-critical part of code. We can suppose that bodies that are spatially close to each other will interact with the nearly same nodes (Barnes, 1990). We should therefore create groups of close bodies according to their spatial location. We can utilize cubes (nodes) produced during the tree construction. When such encompassing cube contains some limited number of bodies (my simulations used always 32 bodies), we can call it the group and add this group (cube) into the list of groups. During the force evaluation, we do not choose the single body; instead we choose the whole group.

Nodes or leaves interacting with the selected group might be split into two lists (Becciani et al., 2000). The first list contains bodies (nodes or leaves) that are sufficiently distant and affect the group's center of mass, and therefore all bodies in the selected group. The second list contains bodies that are too close and interaction with them must be computed individually for each body in the group.

[40] This parallelism is classified as a shared memory parallelism. It delivers benefits on hyper-threading or multiple-core processors.

Again, we must introduce some criterion that will decide to which list should be the node or leave inserted. We can encompass all bodies in the group with a sphere of radius (Becciani et al., 2000)

$$r_{sphere} = c \cdot l_G, \tag{4.17}$$

where c is a parameter determining an accuracy and l_G is the length of the cube's edge. If the separation s (see Figure 4–13) between the group's center of mass and the distant node is greater than r_{sphere}, the distant node is inserted to the first list. Otherwise, the distant node is ascribed to the list of close bodies. Finally, all bodies from the group are added to the second list.

Parameter c determines the accuracy and is chosen to be $c = \frac{3 \cdot \sqrt{3}}{2}$.

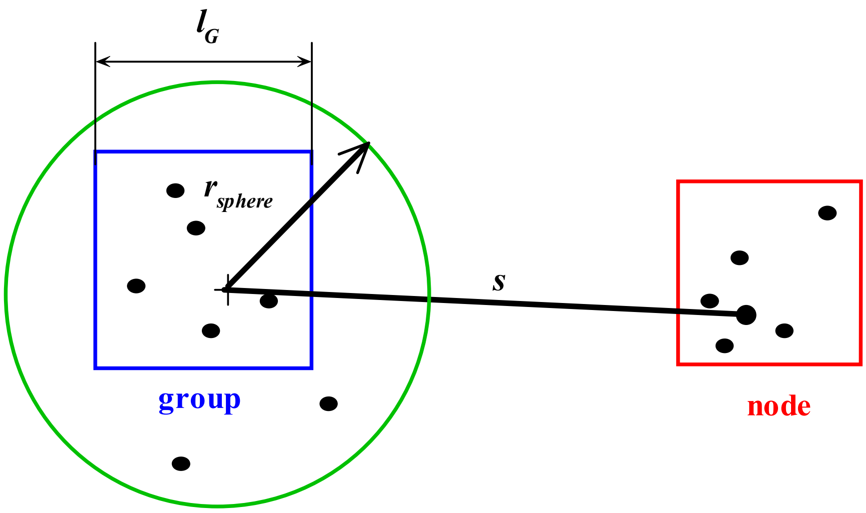


Figure 4–13: The meaning of parameters l_G, r_{sphere} and s in the grouping strategy.

4.11 Exploding galaxies

Galaxies are collisionless systems, which mean that its components (stars) do not affect each other by close encounters. Orbit of a single star is affected almost entirely by an overall potential of the whole galaxy, not by other star that is nearby. On contrary, star clusters are collisional systems, which mean that also the close encounters of bodies (stars) are important in computation and paths of individual bodies might be substantially altered by close passes of neighbors. As the notion suggests, collisional systems are so dense that stars may eventually get so close to each other that they may collide. Therefore, star clusters are sometimes called *dense stellar systems*. If you are interested in the modeling of dense stellar systems, read Hut and Makino (2006).

While using a constant timestep, two bodies can get very close to each other so that unbearable numerical error is introduced. An integration with an improperly big timestep (therefore called overstep problem) might lead to an anomalously high velocity of a body (star), which as a result leave a gravitationally bound system of bodies. When such encounter occurs in a numerical simulation, the star is unnaturally accelerated and flies out of the galaxy (therefore called disk-heating problem). This is a direct effect of discrete numerical calculations. In differential equation is movement described continuously and smoothly.

Close encounters of stars in a real galaxy are improbable with regard to a low spatial distribution of stars. In order to treat the galaxy as the collisionless system, bodies *i* and *j* in Equation (4.4) should never get close to each other (not even during the collision of galaxies). Therefore a *gravitational softening (smoothing)* must be employed to avoid nonrealistic acceleration caused by the close encounters (Aarseth, 1963). Small number called a *softening length* ε must be added to the denominator in Newton's universal law of gravitation (r_{ij} is replaced by $r_{ij}+\varepsilon$). The softening length ε must be small when compared to galaxy dimensions. The softening essentially smoothens out microphysics that cannot be resolved in the simulation. For large distances, the small number ε has little effect; model is more collisionless. Stars interact through the customary "softened" gravitational force where an acceleration $\vec{a}_i$ is expressed as

$$\vec{a}_i = G \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{m_j}{\left(r_{ij}+\varepsilon\right)^3} \cdot \left(\vec{r}_j - \vec{r}_i\right). \tag{4.18}$$

A mathematical formulation of the gravitational force depends on an inverse square of a distance between bodies *i* and *j*. The mass point representing the star is essentially an infinitely deep potential well that can be expressed as Dirac's δ-function[41] in the point's location. When these two bodies are very close to each other $\left(r_{ij} \to 0\right)$, the evaluation of acceleration together with the large timestep may bring a very large number that may introduce numerical errors or even lead to infinity. The gravitational softening will therefore eliminate numerical errors introduced when two bodies approach each other. Other solution to the close encounter problem is a variable timestep or cutoff radius, which reduces interaction at short ranges.

4.12 Code limitations

The difference between $N \cdot \log N$ and N^2 is immense and many more stars can be simulated. Nevertheless, the real galaxies have billions of stars and realistic simulations of galaxy dynamics will stay unreachable for some time. The value of *N* is too large for a simulation where each body represents the single star. This number is even beyond the power of modern computers, the speed of existing computers and sophisticated algorithms is still too limited for the number of bodies. Therefore, each individual particle in the simulation represents rather a large association of stars.

On contrary, solar system simulations are computed with an interest on specific bodies (planets, moons, asteroids, comets, spaceships) and their particular precise orbit. *N*-body simulations of

[41] The delta function is strange "function", which is zero everywhere except at the origin, where it is infinite.

galaxies are more of a statistical nature, they do not testify about orbits of individual stars. Individual bodies do not represent individual stars in our galaxy simulation.

The hierarchical and numerical approximation may lead to unacceptable errors in the simulation. Errors increases with each timestep in general. When simulating for several thousand time-steps, these errors will add up. Whatever code we use, we should test it in analytic limits where possible. We should compare our simulations with observations from telescopes and check whether our results are meaningful. We should use our intuition and ask what the output should be. The computer only does what we tell it to do, and will happily perform meaningless simulation. With the impressive power and scope of numerical techniques, we should be always aware of numerous potential pitfalls. Although remarkably robust, numerical simulations must be used with care if the results are to be meaningful. When we start computing, we should be careful and not stop thinking. Yet dynamics in galaxies is worth of study.

4.13 Concluding remarks

In Chapter 4, we have shown how to simulate the effect of the gravitational field and of Newton's laws of motion to move the stars around. I described my implementation of Barnes-Hut algorithm for many-body simulation and novel geometry-based construction of the 3-dimensional Hilbert's curve. Simulation code works in four steps. First, a tree is constructed by space decomposition from a list of bodies that form the simulated system. Space is divided utilizing Hilbert's self-similar space-filling curve. Groups of close bodies are created. Second, centers of mass of individual nodes are computed. Third, accelerations are computed with the Barnes-Hut algorithm. Fourth, new positions and velocities are computed. Thanks to this algorithm, all simulations will be fully self-consistent, i.e. no rigid potentials will be employed.

"Animals do not care about the evolution of the universe. Nor do many humans."
— Stephen Hawking —

5 GALAXY MODELS

In this chapter, we will show how to create a computer model of a galaxy in order to study galaxy dynamics. The creation of initial conditions for galaxies is some kind of black magic. A century ago, nobody even knew that galaxies exist. The Milky Way galaxy and the universe were synonymous. Now, astronomers believe that galaxies mainly contain the matter of form other than we know from our everyday life.

5.1 Galaxies

A galaxy is a massive ensemble of stars and other material orbiting about a common center and its constituents held together by the mutual gravitational interaction. Galaxies come in the variety of global shapes and internal morphologies. They can be, however, broadly classified into two major types: elliptical and disk galaxies.

To our understanding, before the formation of first galaxies were in the universe immense shapeless low-density diffuse clouds of light elements. Due to gravity, they started to contract and collapsed into smaller fragments of the size of galaxies. During runaway gravitational collapse, where stars formed quickly in large numbers and started to shine at once before the gravitational collapse finished, an elliptical protogalaxy was born. The energy of gravitational collapse was not lost (dissipationless collapse) and was converted to the chaotic motion of stars. If the primordial gas was shrunken by the gravity more slowly, the gas have had enough time to start rotation and settled into a regular disk galaxy where it formed stars. Galaxy's appearance therefore mirrors its formation. This is known as the gravitational collapse theory of a galaxy formation. In Chapter 6 will be shown that a subsequent evolution is important as well.

5.1.1 Elliptical galaxies

Elliptical galaxies are not flat ellipses, but they are three-dimensionally elliptical (water-melon like). Orbits of stars do not show a systematic rotation in such galaxies and are largely chaotic.

Elliptical galaxies are smooth, featureless, almost spherical. Elliptical galaxies are slowly rotating objects (Bertola and Capaccioli, 1975) supported by pressure, i.e. their dynamics is dominated by the irregular motion of stars (elliptical galaxies are kinematically hot).

Some of the elliptical galaxies are thought to be very old, because they contain red giant stars that are known to be old and very little or no gas and dust. The lack of blue young stars shows that the elliptical galaxies do not form stars currently. These elliptical galaxies reached their shape after the collapse of initial protogalaxy cloud. Some ellipticals might be newer and result from a more recent evolution (see Chapter 6).

Elliptical galaxies are both the most massive galaxies containing up to a few trillion of stars (central dominant or cD galaxies at the centers of galaxy clusters) and the least massive galaxies with a few million of stars.

Hubble space telescope (HST) imaging of massive elliptical galaxies revealed super-massive black holes in their centers. Giant elliptical galaxy M87, for example, contains central supermassive black hole of 2 to 3 billion solar masses ($M_{\odot}$).

Black holes

Black hole is an extremely dense object, where the gravitational field is so powerful that nothing can escape. We can categorize black holes through their mass as stellar black holes (BHs) with masses $M_{BH} < 10^2 M_{\odot}$, intermediate black holes (IMBHs) with masses $10^2 M_{\odot} < M_{IMBH} < 10^5\ M_{\odot}$, and supermassive black holes (SMBHs) with masses $M_{SMBH} > 10^5\ M_{\odot}$ (Miller, 2006).

5.1.2 Disk galaxies

Disk

A thin disk consists of relatively young stars, open clusters (loose clusters of stars), middle age stars like the Sun, gas and dust (interstellar matter or ISM). Observations of disk galaxies are showing that these galaxies have the very thin disk whose radius is of order 10 kiloparsecs and thickness is of order 100 parsecs.

The most of the stars in a disk galaxy travel on nearly circular orbits. Disk galaxies are rotationally supported, i.e. the regular circular motion of stars (disk galaxies are kinematically cool) dominates them. Stars in disk galaxies usually rotate with a constant velocity in the range of $100\ \mathrm{km \cdot s^{-1}}$ to $300\ \mathrm{km \cdot s^{-1}}$ with only a low dispersion of the velocity of order $\sim 10\ \mathrm{km \cdot s^{-1}}$.

Therefore, the angular momentum must govern the structure of disk galaxies. The rotation of the disk prevents its gravitational collapse in a radial direction. We all live in the Milky Way galaxy, where stars usually rotate with a velocity of $220\ \mathrm{km \cdot s^{-1}}$ and with a low velocity dispersion of about $40\ \mathrm{km \cdot s^{-1}}$.

It is estimated that there are approximately 10^{11} stars in the disk of the Milky Way galaxy in total, most of these stars have a slightly lesser mass than the Sun have. The Sun lies approximately 8.5 kiloparsecs from the center of the Milky Way. Disk galaxies contain mainly blue stars that are very massive and are known to live shortly. Because these young stars still exist in disk galaxies, these galaxies are referred to as young galaxies. These stars are born mainly in spiral arms. The spiral arms look like that they contain a more mass than their surroundings. However, there is at most 5 percent more of the mass in the spiral arms than outside of them. The formation of young massive bright blue and ultraviolet stars makes the spiral arms look so bright. The gas is compressed in the location of spiral arms and therefore the star formation takes a place mainly in arms. Mature disk galaxies now contain of order 10^{10} solar masses of gas (10^{10} $M_{\odot}$).

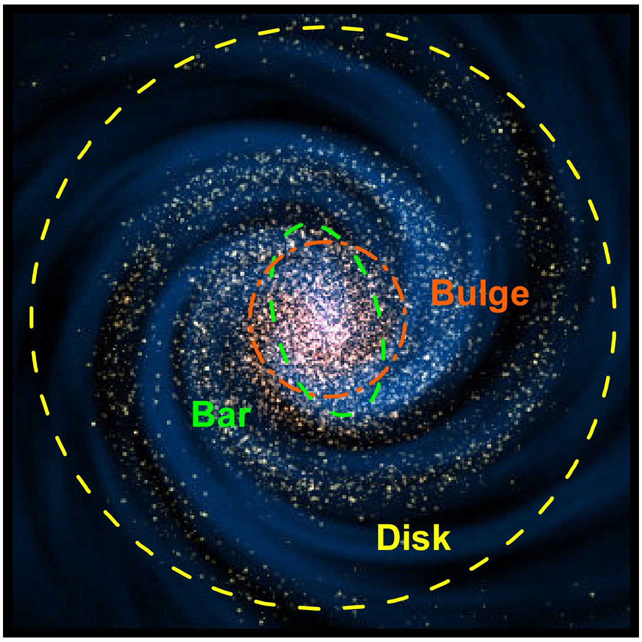


Figure 5–1: A face-on view of a disk galaxy with spiral arms, bar and bulge.

Bulge

A central bulge is a small spherical component dominated by old red stars. This galactic component of a disk galaxy resembles a miniature elliptical galaxy. The bulge surrounds the central super massive black hole. The Milky Way galaxy harbors the black hole with the mass of about 3.6 million solar masses (Eisenhauer et al., 2005). Supermassive black holes lurk at the centers of most, if not all, galaxies (Silk and Rees, 1998).

Bar

Bars are a common feature among disk galaxies. These bars have various morphologies in the sense of length, thickness, and shape. From surveys of spiral galaxies is estimated that about 75 percents of spiral galaxies have bars or ovals (Grosbøl et al., 2002). The Milky Way galaxy also has a bar (Binney, 1995).

Stellar halo

A roughly spherical halo of galaxy contains *globular clusters* (GCs), i.e. isolated dense stellar clusters of millions of stars, and other ISM. Like in elliptical galaxies, the motion of stars in globular clusters is chaotic. Globular clusters usually contain old red (giant) stars and are therefore very old. The stellar halo has a mass in the range of 15 to 30 percent of the mass of disk. The diameter of the halo is approximately the same as the diameter of the disk.

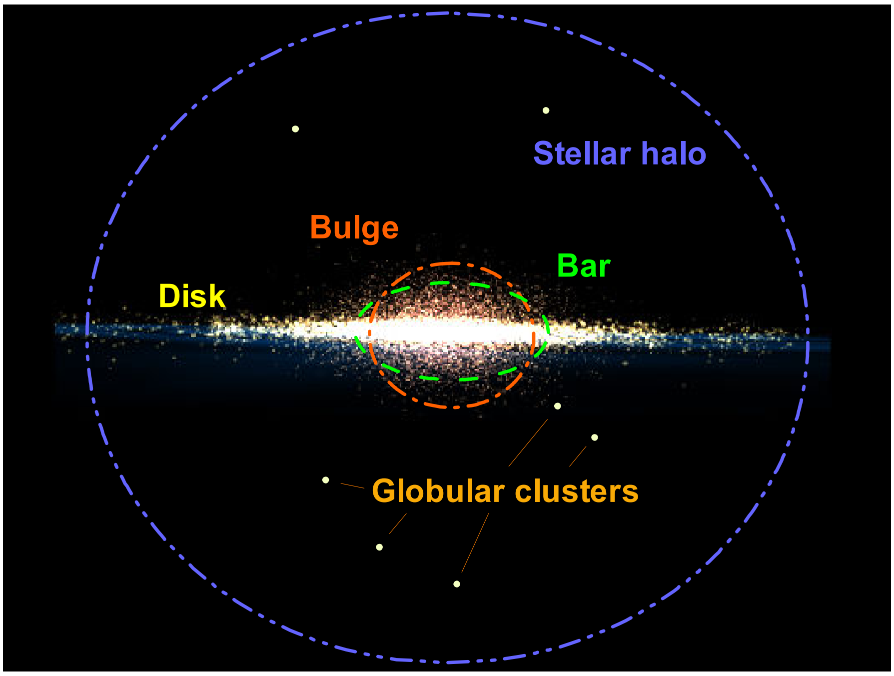


Figure 5–2: The disk galaxy as on Figure 5–1, but seen edge-on.

5.2 Modeling galaxies

Galaxies did not appeared suddenly out of nothing as we will show in Chapter 6. However, it is necessary for the sake of clarity to make some simplifying hypotheses. In this chapter, we will study the evolution of galaxies by performing simulations of isolated systems that are in the steady state (equilibrium[42]).

Elliptical galaxies contain mainly stars and very little gas. They can be readily approximated with mass bodies representing stars. Disk galaxies contain larger quantities of gas, but still there is a significantly larger proportion of stars than of the gas. Galaxy is in the first approximation a large collection of stars or with even greater idealization, the ensemble of mass bodies hold together by

[42] Stellar density in the disk is not changing over time (i.e. galaxies are not expanding or contracting along any direction).

the gravity. We can suppose that many of the properties we study in galaxy dynamics can be understood using a very simple approximation composed of following models.

Mathematical review

First, I will review some useful mathematical notions used throughout the text.

Vector quantities like $\vec{r}$ are usually expressed as $\vec{r} = r_x \cdot \vec{i} + r_y \cdot \vec{j} + r_z \cdot \vec{k}$, where $\left(r_x, r_y, r_z\right)$ are positions (components of a vector) along unit base vectors (directions) $\vec{i}, \vec{j}, \vec{k}$.

The summation convention is used during repetitive additions. For example, the equation

$$\phi = \sum_{i=1}^{N} \phi_i \tag{5.1}$$

can be rewritten as

$$\phi = \sum_{i=1}^{N} \phi_i = \phi_1 + \phi_2 + \ldots + \phi_N . \tag{5.2}$$

We will encounter Hamilton's operator ∇ (pronounced "nabla" or "del"). It is defined as

$$\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k} . \tag{5.3}$$

We will also encounter Laplace's operator Δ. It is defined as

$$\Delta f = \nabla\nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} . \tag{5.4}$$

5.3 Gravity

In secondary school physics we are taught that the gravity is a force acting remotely and instantaneously between all bodies as was stated by Sir Isaac Newton[43], based on his observations of nature and reasoning. According to Newton's theory "every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distances apart".

In the mathematical description of gravity, we will start with a scalar gravitational potential field rather than a vector force field. Suppose we have some arrangement of mass points fixed in space and we are interested in their cumulative effect on some given point in space. Total gravitational

[43] Isaac Newton (1642-1727)

potential of *N* bodies with masses $m_1, m_2, \ldots, m_N$ in a certain point in space is additive and is given by the total sum of all potentials from individual bodies $\phi_1, \phi_2, \ldots, \phi_N$ at distances $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N$ from the given point in space as

$$
\begin{aligned}
&\phi\left(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N; m_1, m_2, \ldots, m_n\right) = \\
&= \phi_1(\vec{r}_1, m_1) + \phi_2(\vec{r}_2, m_2) + \ldots + \phi_N(\vec{r}_N, m_N) = \\
&= -G \cdot \sum_{i=1}^{N} \frac{m_i}{\left|\vec{r}_i\right|}.
\end{aligned} \tag{5.5}
$$

The configuration of *N* point masses (the *N*-body system) can be generalized to a continuous mass density distribution function ρ, which is a function of position $\vec{r}$. If we write $\rho\left(\vec{r}\right)$ as a function of coordinates r_x, r_y, r_z then $\rho\left(r_x, r_y, r_z\right) \mathrm{d}r_x\ \mathrm{d}r_y\ \mathrm{d}r_z$ is the mass contained in a little box of volume $\mathrm{d}r_x\ \mathrm{d}r_y\ \mathrm{d}r_z$, located at the point r_x, r_y, r_z. A mass density $\rho\left(\vec{r}\right)$ in a specific place is given by the sum of mass sources (point masses) in an infinitely small volume element. The mass density distribution function ρ is the function describing the distribution of matter (stars) in the system (galaxy). A transition from a discrete to continuous form can be approximately expressed as $\int_V \rho\ \mathrm{d}V \cong \sum_{i=1}^{N} m_i$.

In a similar way, the gravitational potential generated by the continuous mass distribution instead of individual bodies as sources of the potential can be expressed continuously. Summation (5.5) is turning into integration in places (macroscopically) continuously filled with matter.

The gradient of the potential is then the gravitational force acting on a body with a unit mass, apart from a minus sign. The exploring body of the unit mass will experience at any point given by a position $\vec{r}$ force

$$
\vec{g}\left(\vec{r}\right) = -\mathrm{grad}\ \phi(\vec{r}) = -\nabla\ \phi(\vec{r}). \tag{5.6}
$$

This quantity $\vec{g}$ is called the intensity of gravitational field.

With continuous functions, we may use Poisson's equation (Greiner, 2004)

$$
\begin{aligned}
\mathrm{div\ grad}\ \phi(\vec{r}) &= 4 \cdot \pi \cdot \mathrm{G} \cdot \rho(\vec{r}) \\
\nabla^2 \phi\left(\vec{r}\right) &= 4 \cdot \pi \cdot G \cdot \rho\left(\vec{r}\right) \\
\Delta\ \phi(\vec{r}) &= 4 \cdot \pi \cdot \mathrm{G} \cdot \rho\left(\vec{r}\right).
\end{aligned} \tag{5.7}
$$

Here we get a connection between the gravitational potential ϕ and the mass density ρ. Poisson's equation serves as one of the principal governing equations in our examination of stability, structure and dynamics of self-gravitating systems. The mass density required for the generation of the potential can be found by solving Poisson's equation. On the other side, from the known mass density, we can determine the gravitational potential through Poisson's equation.

Different components of galaxies (disk, bulge… and halo) have their own mass density function within the global gravitational potential. Poisson's equation gives us a relation between them. For a self-gravity model, we must find the global gravitational potential, in which the mass density of all parts of a galaxy is combined as

$$\nabla^2\phi = 4\cdot\pi\cdot G\cdot\left(\rho_{disk}+\rho_{bulge}+\ldots+\rho_{halo}\right). \tag{5.8}$$

5.4 Initial conditions for spherical systems

As was described in Chapter 4.1, initial conditions for the *N*-body system must be set up. The most convenient is to start with elliptical galaxies. Systems with a spherical symmetry can be described by functions depending only on a one-dimensional distance r from the center of the system. In the centrally symmetric mass distribution with the spherical symmetry are the mass density ρ and gravitational potential ϕ same in each direction with the origin at the center of the system. The spherical symmetry therefore implies $\rho\left(\vec{r}\right)\rightarrow\rho(r)$ and $\phi\left(\vec{r}\right)\rightarrow\phi(r)$. Poisson's equation in one dimension is expressed as

$$\frac{\mathrm{d}^2\phi(r)}{\mathrm{d}r^2} = 4\cdot\pi\cdot G\cdot\rho(r). \tag{5.9}$$

5.4.1 Uniform sphere

A uniform sphere is one of the simplest approximations of the spherical system with the uniform density distribution function (volume density) given by

$$\rho = \begin{cases} \dfrac{3\cdot M}{4\cdot\pi\cdot b^3}, & \text{for } r\leq b \\ 0, & \text{for } r>b \end{cases}, \tag{5.10}$$

where M is the mass of the sphere and b is its radius. As we would expect from the mass density distribution function ρ, outside the sphere potential falls off as in Keplerian case. This is known as Newton's theorem. It states that "a body inside a spherical shell of matter experiences no net gravitational force from the shell" (the mass exterior to the shell has no effect) and that "the gravitational force on a body lying outside a closed spherical shell experiences a gravitational force which is the same as if all the matter in the shell was concentrated at a point at its center" (Greiner, 2004).

5.4.2 Isothermal sphere

Isothermal sphere has interesting property since its rotational velocity is unchanging with radius (it is constant). Therefore the isothermal sphere is useful for modeling flat rotational curves (more on Chapter 5.11). The mass density distribution function of isothermal sphere is given by

$$\rho(r) = \rho_0 \cdot \left(\frac{r}{a}\right)^{-2}, \tag{5.11}$$

where ρ_0 is a central density and a is a scale length. This distribution function (see Figure 5–3) gives the system with an infinite mass and must be therefore truncated at some distance from the center. Cut-off radius must be imposed for a finite total mass.

5.4.3 Plummer's and Hernquist's model

We should now turn to more realistic density distribution functions that mimic spherical components of galaxy. Plummer's (1911) model is such a simple approximation with the mass density distribution function given by

$$\rho(r) = \frac{3 \cdot M}{4 \cdot \pi} \cdot \frac{a^2}{\left(a^2 + r^2\right)^{\frac{5}{2}}}. \tag{5.12}$$

Another more realistic model is Hernquist's (1990) model given by

$$\rho(r) = \frac{M}{2 \cdot \pi} \cdot \frac{a}{r \cdot (a + r)^3}. \tag{5.13}$$

Plummer's and Hernquist's models are members of the same family of models (Evans and An, 2005). They can be both described by the mass density distribution function as

$$\rho = \frac{(p+1) \cdot M}{4 \cdot \pi} \cdot \frac{a^p}{r^{2-p} \cdot \left(a^p + r^p\right)^{2+\frac{1}{p}}}, \tag{5.14}$$

where parameter p is a positive number. The case $(p = 1)$ is Hernquist's model and the case $(p = 2)$ Plummer's model.

5.4.4 Jaffe's model

Another realistic model is Jaffe's (1983) model expressed as

$$\rho(r) = \frac{M}{4 \cdot \pi} \cdot \frac{a}{r^2 \cdot (a+r)^2}. \tag{5.15}$$

5.5 Units and scales

It is often useful to reduce units and equations describing a physical system to a dimensionless form, both for physical insight and numerical convenience. Let us imagine that we are using the gravitational constant in SI units that is of order 10^{-11}. However, typical distances in the world of galaxies are of order 10^{21} in SI units. A finite numerical representation of real numbers in a digital computer will most likely yield to large numerical round-off errors when we will compute with numbers of tens orders different. Therefore, we will use non-dimensional units throughout our simulations. We adopt $G = 1$ for Newton's gravity constant, a total mass of system $M = 1$ and a scale length a is also set equal to 1. As a general rule, all quantities in this text are given in these intrinsically scale free units unless otherwise noted.

It is useful to summarize some commonly used units:

$$\begin{aligned} 1 \text{ pc} &= 3.086 \cdot 10^{16} \text{m} \\ 1{,}000 \text{ km} \cdot \text{s}^{-1} &= 1.02 \text{ kpc} \cdot \text{Myr}^{-1} = 1.02 \text{ pc} \cdot \left(1{,}000 \text{ yr}\right)^{-1} \\ 1 \ M_\odot &= 1.989 \cdot 10^{30} \text{ kg.} \end{aligned}$$

Let us consider a practical choice of units on measures of galaxies. A typical disk galaxy has a size of orders of kiloparsecs, contains hundreds of billions of stars, and a rotational velocity in such galaxy is usually up to $300 \text{ km} \cdot \text{s}^{-1}$. The models may be therefore compared with real galaxies using, for example, the following scaling

$$\begin{aligned} [L] &= 1 \text{ kpc}, \\ [M] &= 10^{11} \ M_\odot, \\ [v] &= 300 \text{ km} \cdot \text{s}^{-1}, \end{aligned}$$

where we employed a convention to write $[x]$ for the unit of quantity x. $[L]$ is a unit length, $[M]$ is a unit mass, and $[v]$ is a unit velocity. With these, the unit time can be derived as

$$[T] = \frac{[L]}{[v]} = \frac{1 \text{ kpc}}{300 \text{ km} \cdot \text{s}^{-1}} = \frac{3.086 \cdot 10^{19} \text{m}}{300 \cdot 10^3 \text{ m} \cdot \text{s}^{-1}} = 102.9 \cdot 10^{12} \text{ s} = 3.263 \cdot 10^6 \text{ yr}. \tag{5.16}$$

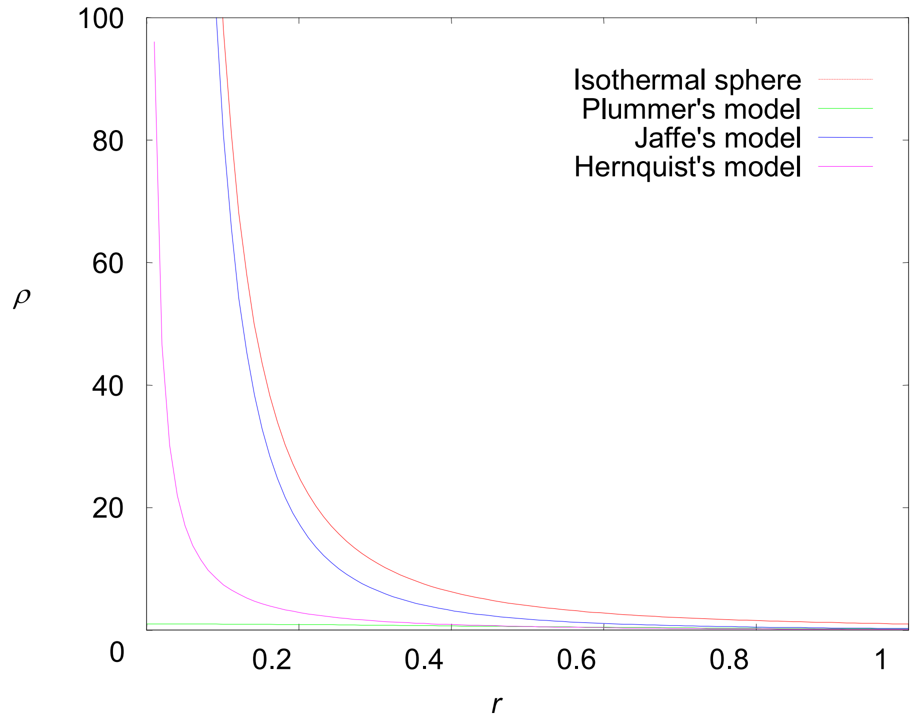


Figure 5–3: Density profiles of spherical components for modeling galaxies. All have $G = M = a = 1$.

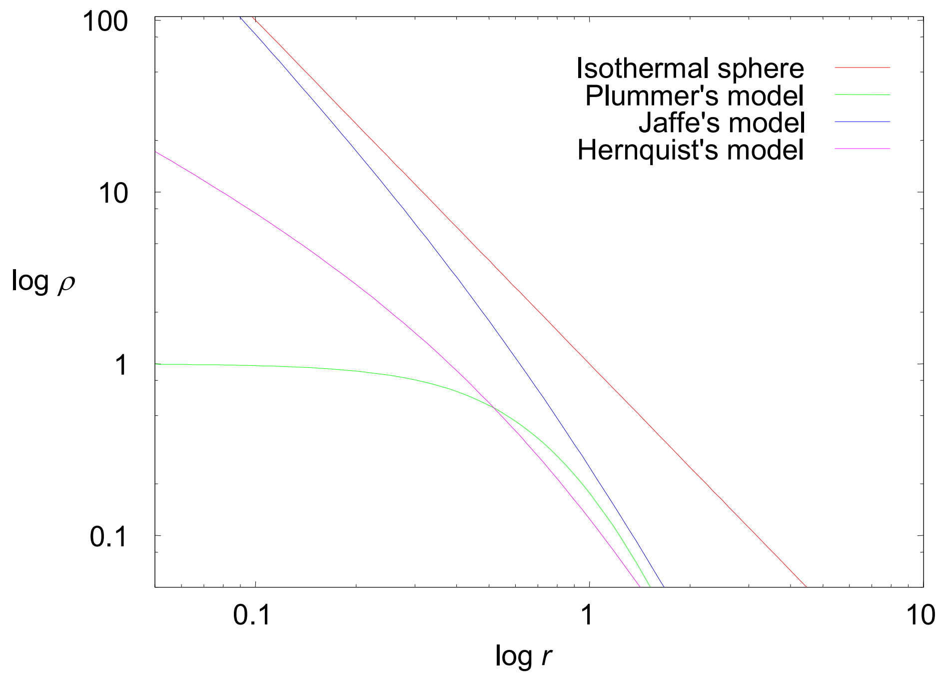


Figure 5–4: Density profiles of spherical components for modeling galaxies on logarithmical scale axes. All have $G = M = a = 1$.

5.6 Positions of bodies in galaxy

A probability $\mathrm{d}P$ that we will find a body (i.e. star) in the volume of spherical shell limited by radii $\left(r, r+\mathrm{d}r\right)$ relates to the radial mass density distribution function $\rho(r)$ as

$$\mathrm{d}P = 4 \cdot \pi \cdot \rho(r)\, \mathrm{d}r\,. \tag{5.17}$$

First, we will introduce the amount of mass in the spherical system bounded by the sphere with a radius r. This quantity is called a *cumulative mass distribution* $m(r)$ and is given by

$$m(r) = \int_0^r 4 \cdot \pi \cdot r'^2 \cdot \rho\left(r'\right) \mathrm{d}r'\,. \tag{5.18}$$

In numerical formulation is the density profile $\rho(r)$ numerically integrated with the radius r changing between a minimal radius equal to zero and a maximum radius $r_{\max}$. The cumulative mass distribution $m(r)$ of a model is tabulated on a grid with points spaced logarithmically in r. To check and compare numerical results with analytic solutions, we have plotted the analytic and numeric $m(r)$ of Plummer's model to see the difference (Figure 5–5).

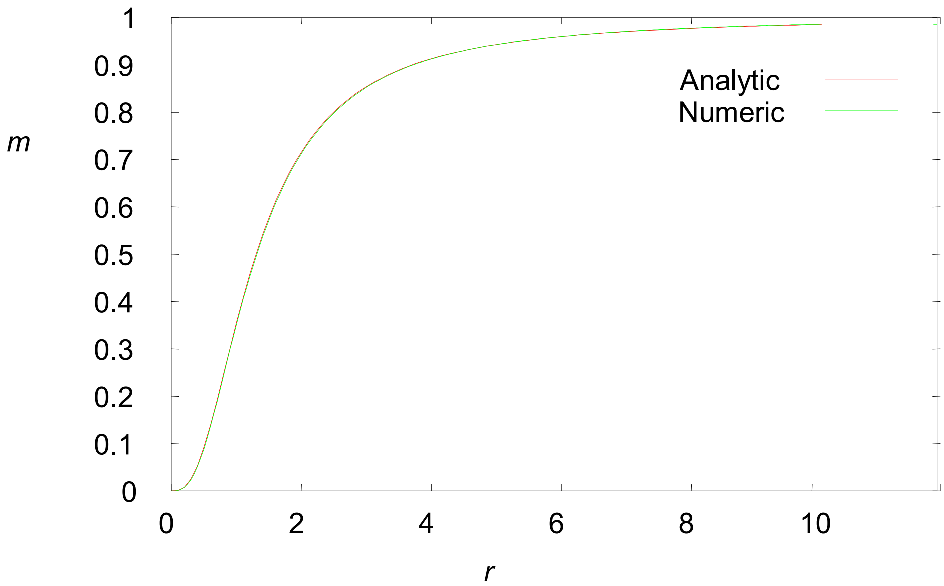


Figure 5–5: The cumulative mass of Plummer's model computed in an analytic and numeric form. The numeric representation is so precise that no deviation from the analytic expression can be seen. The numeric representation (green curve) completely covers analytic representation (red curve).

The cumulative mass distribution $m(r)$ is essentially a probability distribution function (PDF). Numerical libraries contain a variety of random number generators, but how can we realize an arbitrary PDF?

Given the known PDF, it can be realized by a random (Monte Carlo[44]) sampling from the PDF. We can find the maximum m_M of the function $m(r)$ (overall mass of the system of bodies); we also know the maximum radius $r_{\max}$. Then, we generate a couple of random numbers with a uniform (also called normal as well as Gaussian) distribution. We generate a random number *R* with the uniform distribution in an interval $<0;1)$ and expand it (multiply by $r_{\max}$) to $<0; r_{\max})$ range. Then we generate a second random number *P* with the uniform distribution in interval $<0;1)$ and expand it (multiply by m_M) to $<0; m_M)$ range. If the value of function $m(r=R)$ is equal or smaller than the random number *P* (so that the number *P* is under the graph of point $m(R)$) then we will accept the distance *R*. Otherwise, we reject this number, generate again two new random numbers *R* and *P* and perform the test again. The method is called *acceptance-rejection method*. It was used by John von Neumann[45] and is therefore sometimes called "*von Neumann's method*".

We may discover also other method for sampling from the arbitrary PDF. We may invert the function $m(r)$ to a function $m^{-1}(r)$ and thus obtain $r(m)$. We can find the maximum value m_M of the function $m(r)$ for $r = r_{\max}$. Then we generate the random number *M* from the uniform distribution and expand it to interval $<0; m_M)$. Finally, we will receive the position of the body by evaluating $r(M)$.

Up to now, all functions were one-dimensional. Now, we must scatter bodies to all three dimensions. The initial radius r of a body is randomly determined according to the radial surface density distribution function $r(m)$. The initial angles θ and ϕ are drawn uniformly between 0 and $2\cdot\pi$. Particles are then positioned in three dimensions using

$$\begin{aligned} p_x &= r\cdot\sin\theta\cdot\cos\phi \\ p_y &= r\cdot\sin\theta\cdot\sin\phi \\ p_z &= r\cdot\cos\theta. \end{aligned} \tag{5.19}$$

[44] Monte Carlo methods are widely used in physics, mathematics or biology.

[45] John von Neumann (1903-1957)

5.7 Velocities of bodies

To have a body on a stable orbit, its kinetic energy E_{kin} must be exactly the same as a potential energy E_{pot}, i.e., $E_{kin} = E_{pot}$ or $E_{kin} - E_{pot} = 0$[46]. The kinetic energy is defined as $E_{kin} = \frac{1}{2} \cdot m \cdot v^2$, where *m, v* are the mass and velocity of the body, respectively. The potential energy is defined as $E_{pot} = -m \cdot \phi(r)$, where r is the distance of the body from the center of the system. The potential energy characterizes a mutual force between bodies – the potential energy is *shared* at least between two bodies. The gravitational potential $\phi(r)$ is always negative. The potential energy E_{kin} is resulting from the interaction of the body and the companion system.

The velocity for the stable orbit can be derived as

$$\begin{aligned} &E_{kin} = E_{pot} \\ &\frac{1}{2} \cdot m \cdot v^2 = -m \cdot \phi(r) \\ &v(r) = \sqrt{-2 \cdot \phi(r)}. \end{aligned} \tag{5.20}$$

The body will stay in the system until its kinetic energy E_{kin} will not be greater than the potential energy E_{pot} of gravity that binds this body to the system. When $E_{kin} > E_{pot}$, the body will leave the system with so called escape velocity v_{esc}. This velocity can be easily derived for the spherically symmetric system, when we take into account Newton's theorems. Then the whole system is looking like the mass point of a mass *M* with the gravitational potential $\phi(r) = -G \cdot M \cdot \frac{1}{r}$ and the escape velocity required for the body to escape infinitely far from the gravitational field will be

$$\begin{aligned} &v_{esc}(r) = \sqrt{-2 \cdot \phi(r)} \\ &v_{esc}(r) = \sqrt{\frac{2 \cdot G \cdot M}{r}}. \end{aligned} \tag{5.21}$$

The body will stay in the system for $0 \leq v \leq v_{esc}$.

[46] The gravitational field is conservative, which means that energy used to move between two points within the field is independent of the path taken. The fixed constant has no effect on the dynamics of the system and therefore can be chosen to be zero.

Phase-space distribution function

An equilibrium spherical system depends on the phase-space coordinates $(\vec{r}, \vec{v})$ (Lynden-Bell, 1962). The structure of a galaxy is defined by a distribution function $f(\vec{r}, \vec{v})$ in the imaginary space called phase-space with 6 dimensions (three spatial coordinates for positions and three corresponding velocity components) at each moment (time) *t*. The phase-space is given by the product of configuration space (positional space, represents generalized coordinates) and velocity space (represents generalized momentum).

Such a system of bodies is subjected to the potential ϕ that is completely described by its phase-space distribution function $f(\vec{r}, \vec{v}) = f\left(r_x, r_y, r_z; v_x, v_y, v_z\right)$. It is a probability density of finding some mean number $\mathrm{d}N$ of stars $\mathrm{d}N = f\ \mathrm{d}^3\vec{r}\ \mathrm{d}^3\vec{v} = f\ \mathrm{d}r_x\ \mathrm{d}r_y\ \mathrm{d}r_z\ \mathrm{d}v_x\ \mathrm{d}v_y\ \mathrm{d}v_z$ in a small box $\mathrm{d}^3\vec{r}$ near a position $\vec{r}$, i.e. between r_x and $r_x + \mathrm{d}r_x$, r_y and $r_y + \mathrm{d}r_y$, r_z and $r_z + \mathrm{d}r_z$, and velocities in the range $\mathrm{d}^3\vec{v}$ about $\vec{v}$, i.e. between v_x and $v_x + \mathrm{d}v_x$, v_y and $v_y + \mathrm{d}v_y$, v_z and $v_z + \mathrm{d}v_z$. The probability density is never negative.

We have already found the position of the star in the system (a 3D position in the configuration sub-space of 6D phase-space) as was described in Chapter 5.6. Now, we must find the range of velocities $\left\langle 0; v_{esc} \right\rangle$ in the phase-space for the concrete position and also a correct statistical weight that tells us what probability distribution function is for $0 \leq v \leq v_{esc}$. As was shown in Equation (5.21), the escape velocity is given by the gravitational potential energy that comes from the gravitational potential of the system at the given point of space. The velocity distribution function at the position of each particle can be found from the distribution function $f(E)$. The distribution function f is assumed to be zero $f(E \geq 0) = 0$ for energies $E = \frac{1}{2} \cdot v^2 + \phi(r)$ greater than zero (while assuming a unit mass), because the stars with the positive energy will leave the system.

We must find the concrete phase-space distribution function $f(E)$ for the desired models. The probability isotropic distribution function *f* of any spherical model depends only on the overall specific energy *E* (energy per unit mass) of the system of stars and is available as (Eddington, 1916)[47]

$$f(E) = \frac{1}{\sqrt{8} \cdot \pi^2} \cdot \frac{\mathrm{d}}{\mathrm{d}E}\left(\int_E^0 \mathrm{d}\phi \frac{1}{\sqrt{\phi - E}} \frac{\mathrm{d}\rho}{\mathrm{d}\phi} \right). \tag{5.22}$$

[47] Complete derivation of Eddington's formula is given by Hut and Makino (2004).

Abel integral transform is required to relate density profile $\rho(r)$ of models and the distribution function $f(E)$ to the potential generated by $\phi(r)$. However, analytical distribution functions are known for only a handful of models, such as e.g. Plummer's model. In order to generate any model that is more realistic, one has to find the steady state distribution function $f(E)$ that reproduces the desired density $\rho(r)$ numerically.

Gravitational potential

In order to solve Equation (5.22), we must find the gravitational potential ϕ connected to the mass density ρ (that we have for various spherical models) through Poisson's equation (5.7). Let us remind Poisson's equation in one dimension given by

$$\frac{\mathrm{d}^2\phi(r)}{\mathrm{d}r^2} = 4\cdot\pi\cdot G\cdot\rho(r)\,. \tag{5.23}$$

The cumulative mass distribution $m(r)$ relates to the gravitational potential ϕ through Poisson's equation as follows

$$\begin{aligned}
m(r) &= \int_0^r 4\cdot\pi\cdot r'^2\cdot\rho\left(r'\right)\mathrm{d}r' = \\
&= \frac{1}{G}\cdot\int_0^r \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\phi(r)\right) = \\
&= \frac{1}{G}\cdot\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\phi(r)\right)\Bigg|_0^r = \\
&= \frac{1}{G}\cdot r^2\frac{\mathrm{d}}{\mathrm{d}r}\phi(r) \\
&\Rightarrow \frac{\mathrm{d}\phi(r)}{\mathrm{d}r} = G\cdot\frac{m(r)}{r^2} \\
&\Rightarrow \phi(r) = \int_0^r G\cdot\frac{m(r')}{r'^2}\mathrm{d}r'.
\end{aligned} \tag{5.24}$$

In the numerical formulation is the gravitational potential $\phi(r)$ determined by numerical integration of the cumulative mass distribution $m(r)$ and is tabulated on a grid between the minimal radius equal to zero and radius equal to ten times the maximum radius $\left(10\cdot r_{\max}\right)$ with grid points spaced logarithmically in r. The cumulative mass distribution $m(r)$ is obtained by the interpolation of the grid points.

Determination of distribution function

The integrand in Equation (5.22) depends on the derivative of the density $\frac{\mathrm{d}\rho}{\mathrm{d}\psi}$. We must therefore express mass density ρ as the function of the gravitational potential ϕ. Both the mass density ρ and the gravitational potential ϕ are obtained from a numerical interpolation of $\rho(r)$ and $\phi(r)$ at the logarithmically spaced points r and acquired values are placed on a grid that represents $\rho(\phi)$. The numerical derivation is then performed on the function $\rho(\phi)$. Numerical integration is applied to the integral from Equation (5.22) and the result is placed on the grid of ϕ evaluated at logarithmically spaced points r. Finally, Equation (5.22) must be differentiated; and we get the distribution function $f(E)$.

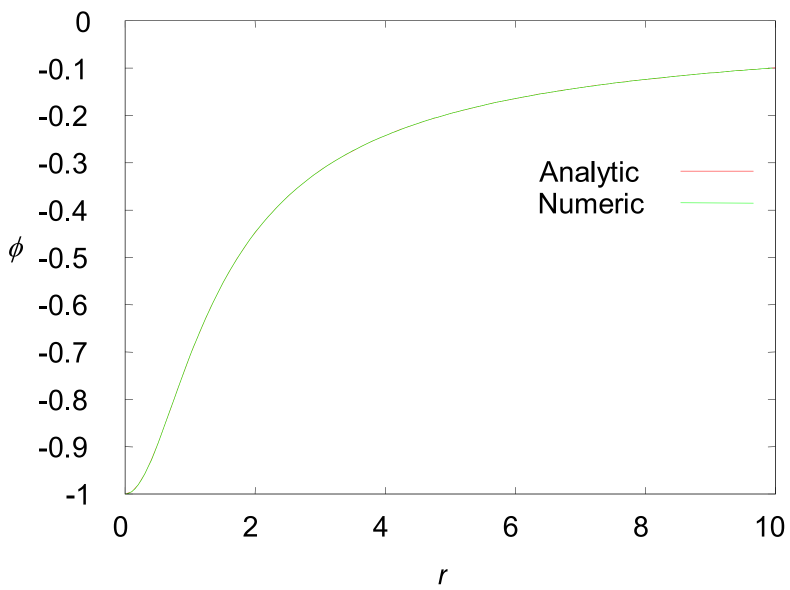


Figure 5–6: The gravitational potential of Plummer's model computed in an analytic and numeric form. The numeric representation is so precise that no deviation from the analytic form can be seen.

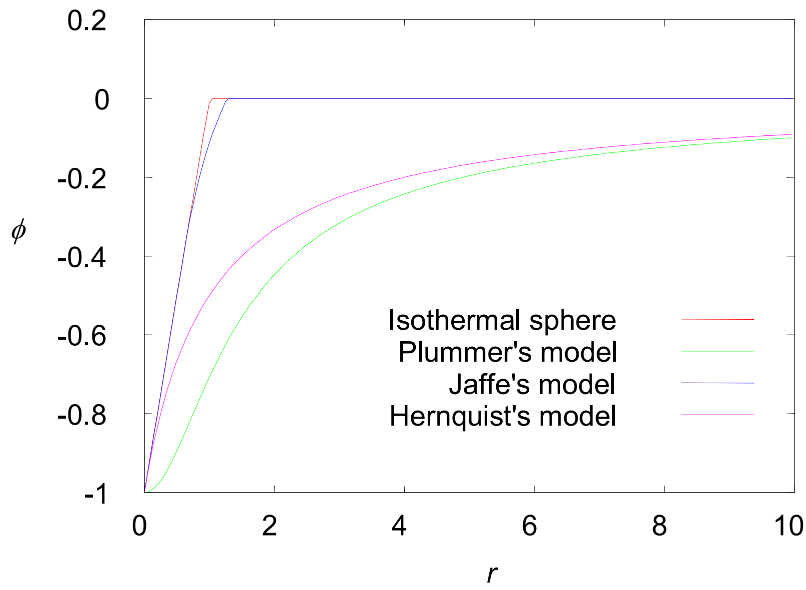


Figure 5–7: Numerically computed gravitational potentials of various spherical models.

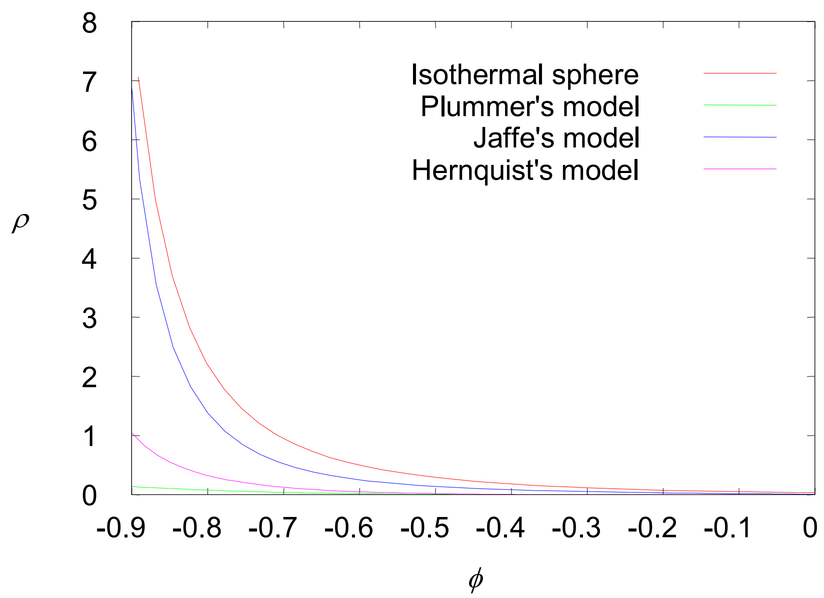


Figure 5–8: The mass density distribution ρ as the function of the gravitational potential ϕ.

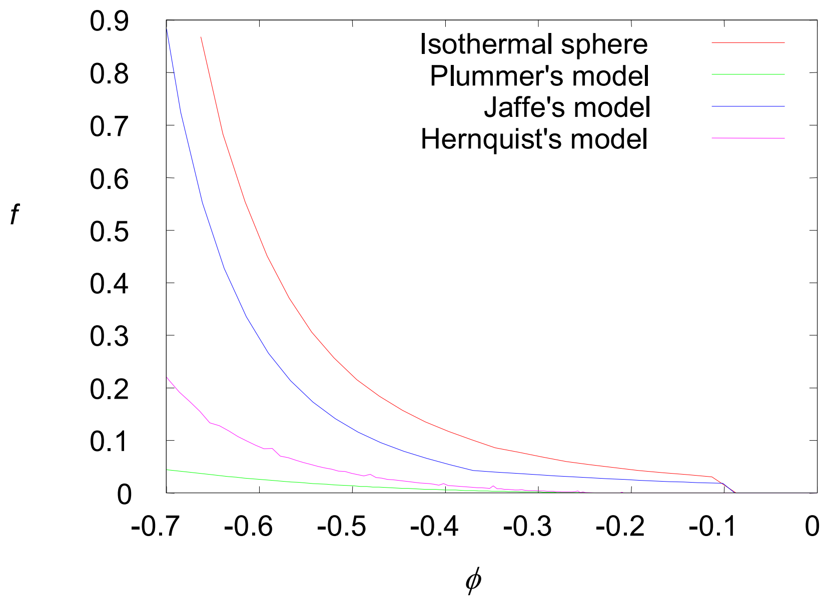


Figure 5–9: Numerically computed distribution function of various spherical models.

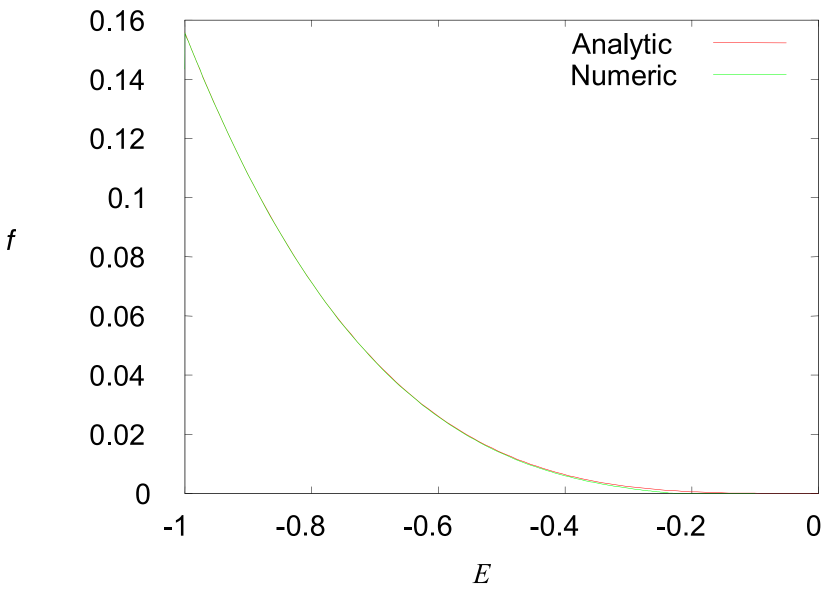


Figure 5–10: The distribution function $f(E)$ of the Plummer's model computed in the analytic and numeric form. A minute deviation can be seen as the function is approaching zero.

Once the *N*-body system is generated, its realization must be verified. Every closed system held together by the gravity must globally obey the virial theorem[48]. The virial theorem states that the mean values of the kinetic energy $\overline{E_{kin}}$ and potential energy $\overline{E_{pot}}$ are related as $\overline{E_{kin}} = -\frac{1}{2} \cdot \overline{E_{pot}}$ or $2 \cdot \overline{E_{kin}} + \overline{E_{pot}} = 0$ in the closed system. The total energy $\overline{E} = \overline{E_{kin}} + \overline{E_{pot}}$ is conserved.

Given a total mass *M* and the mean kinetic energy $\overline{E_{kin}}$ of the system of stars, we may get from the definition of the kinetic energy a *characteristic velocity*

$$v_{char}^2 = \frac{2 \cdot \overline{E_{kin}}}{M}. \tag{5.25}$$

We can also define a *characteristic length* (Meylan and Heggie, 1996)

$$R_{char} = -\frac{G \cdot M^2}{2 \cdot \overline{E_{pot}}}. \tag{5.26}$$

These characteristic quantities are sometimes known as the virial velocity and radius. Their ratio is an estimate of time that a typical star takes to cross the system. The crossing time is a measure of the time that take for a star to traverse the diameter of the system. The crossing time is defined (Meylan and Heggie, 1996) as

$$t_{cross} = \frac{2 \cdot R}{v}, \tag{5.27}$$

where R is a measure of the size of the system and v a measure of the mean stellar velocity. We choose R and v to be equal to R_{char} and v_{char}, respectively.

5.8 Spherical galaxies

Our first objective is a simulation of elliptical galaxies with an eccentricity of zero (spherical galaxies). We will perform the simulation of two spherical systems – Plummer's and Hernquist's models.

We will use a program for initial conditions generation to create a spherical galaxy consisting of stars with a position, velocity and mass each. We will evolve initial conditions with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 0.75$.

[48] Softened force imply that the system of particles in equilibrium no longer satisfies the conventional virial theorem (Selwood, 1987). Deviation will be small, however, and can be neglected in current investigations.

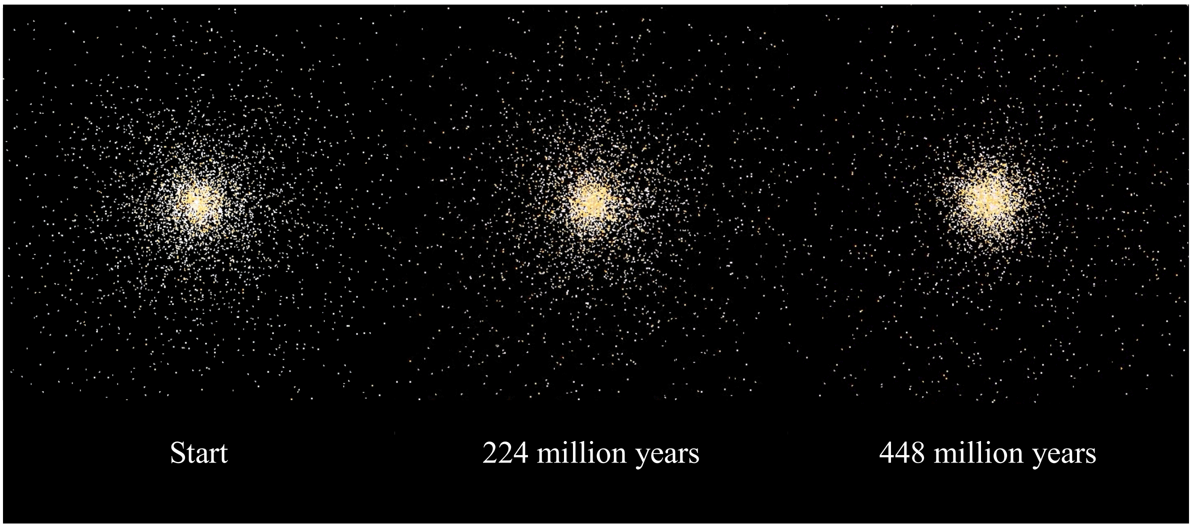
Model	Resolution (star particles)	Timestep	Steps
Plummer	5,000	0.0005	30,000
Hernquist	5,000	0.0005	30,000

Table 5–1: Parameters of simulation for Plummer's and Hernquist's model.

Model realization is in the dimensionless system of units. Gravitational forces were calculated after smoothing the mass distribution using the softening length $\varepsilon = 0.0001$. Model is comparable to a real-world dwarf spherical galaxy with following radius, mass and Newton's gravitation constant

$$\begin{aligned} r &= 5\ \mathrm{kpc}, \\ m &= 5 \cdot 10^{9}\ M_{\odot}, \\ G &= 6.67 \cdot 10^{-11}\ \mathrm{N} \cdot \mathrm{m}^{2} \cdot \mathrm{kg}^{-2}. \end{aligned}$$

The time unit is comparable to $74.6 \cdot 10^{6}\ \mathrm{yr}$, the timestep to $37.3 \cdot 10^{3}\ \mathrm{yr}$ and the overall simulation with $30,000$ steps covers $1.12 \cdot 10^{9}\ \mathrm{yr}$.



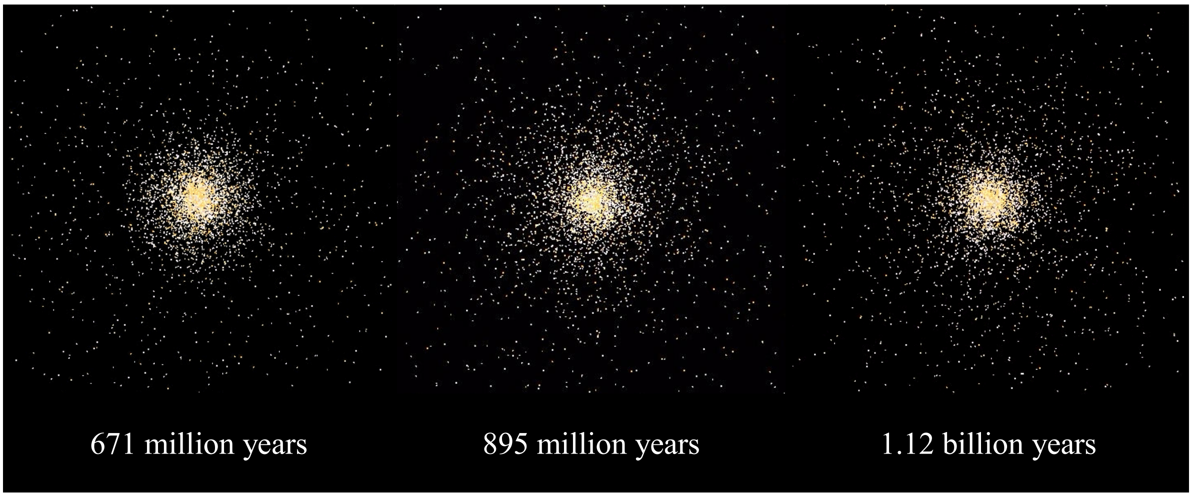


Figure 5–11: A time sequence of Hernquist's model in the *xy* plane. Practically no evolution can be seen.

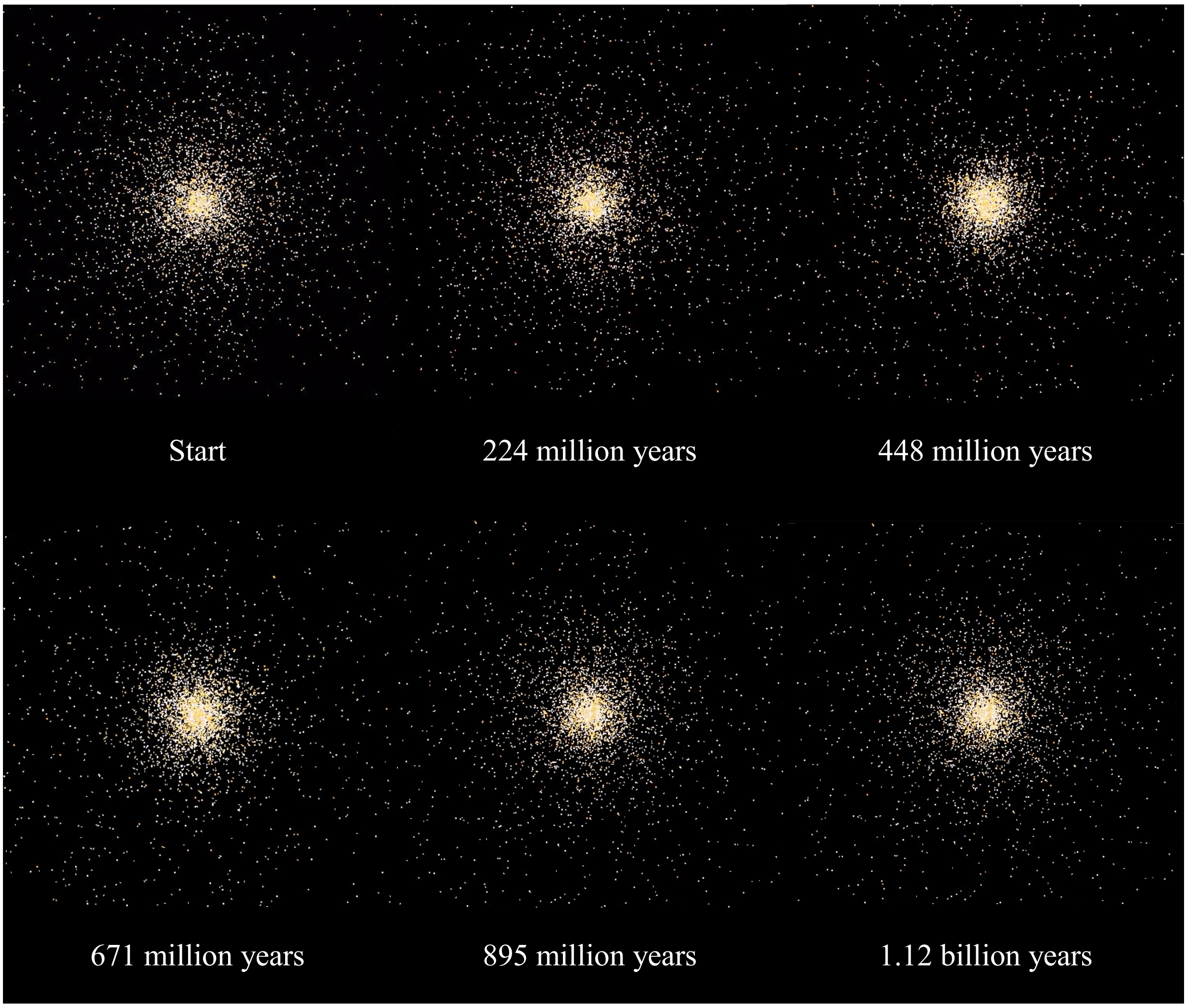


Figure 5–12: A time sequence of Hernquist's model in the *yz* plane.

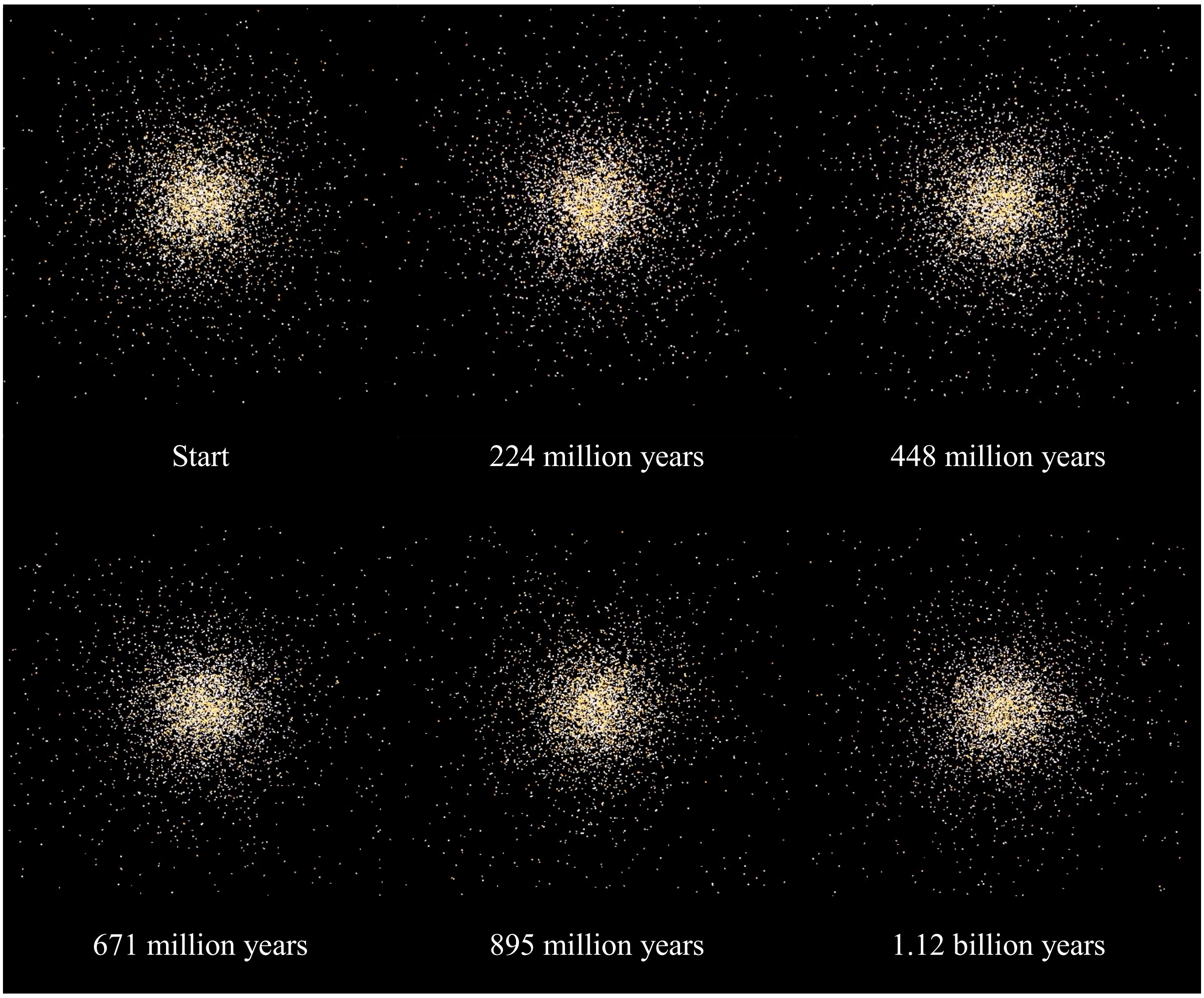
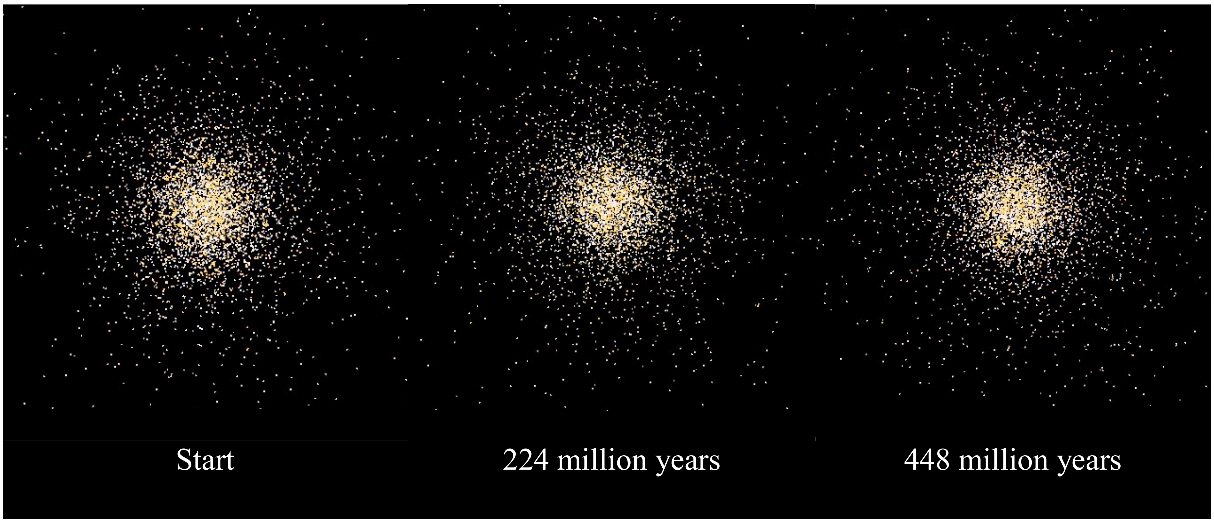


Figure 5–13: A time sequence of Plummer's model in the *xy* plane. Practically no evolution can be seen.



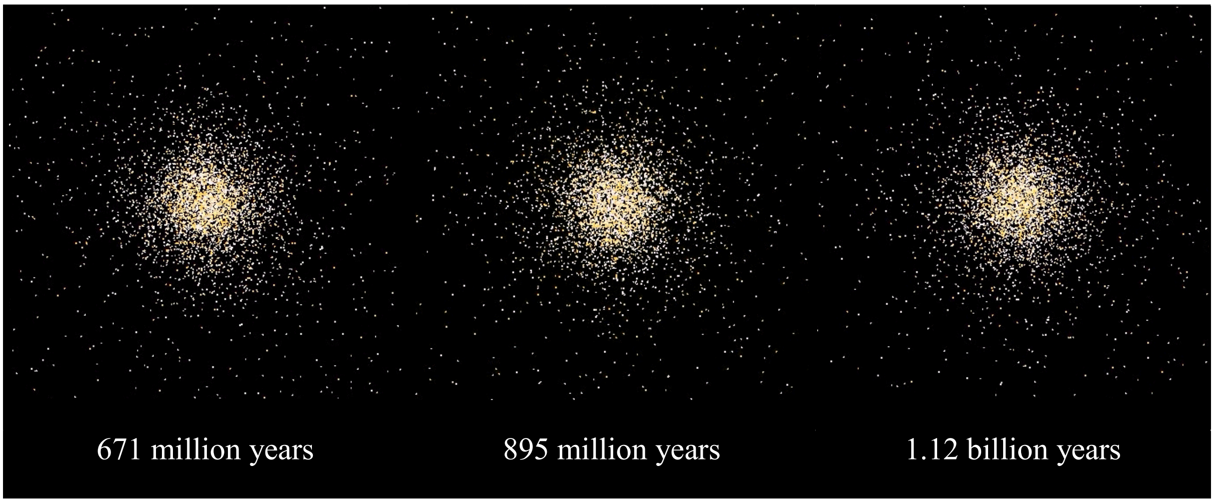


Figure 5–14: A time sequence of Plummer's model in the *yz* plane.

Stability test of the initial models shows a little evolution in these models. Both Hernquist's and Plummer's models are in the dynamic equilibrium from the beginning.

5.9 Initial conditions for flattened systems

A spiral galaxy is flat and can be represented by a flattened potential. The disk of the spiral galaxy is generally very flat and we will describe them as infinitely thin. We will suppose that disks are axisymmetrical and therefore, initial conditions will be set up in cylindrical coordinates.

An initial angle θ is drawn uniformly between 0 and $2 \cdot \pi$. In the *xy* plane are disk particles positioned using

$$\begin{aligned} x &= r \cdot \cos\theta \\ y &= r \cdot \sin\theta. \end{aligned} \tag{5.28}$$

Stars in the disk have only circular velocities (so called cold disk, where no random motions are induced). Disk rotation is generated to be counter-clockwise in the *xy* plane (right handed Cartesian coordinate system is completed with *z*-axis and disk rotation is chosen to by given by the right hand rule with a thumb along *z*-axis).

5.9.1 Kuzmin's disk

A reasonable approximation for the disk galaxy is Kuzmin's disk. Kuzmin found in 1956 the potential of an infinitesimally razor-thin disk with Plummer's potential in the plane of the disk. The initial mass density distribution (surface density) Σ of Kuzmin's disk in the disk plane is

$$\Sigma(r) = \frac{M}{2 \cdot \pi \cdot a^2} \cdot \left(1 + \frac{r^2}{a^2}\right)^{-\frac{3}{2}}, \tag{5.29}$$

where r is a distance from the center of the disk and a is a radial scale length. This mass density belongs to a more general family of disks introduced by Toomre (1963).

5.9.2 Exponential disk

Bodies in an exponential disk are distributed in the plane with the surface mass density distribution given by

$$\Sigma(r) = \frac{M}{2 \cdot \pi \cdot a^2} \cdot \exp(-\frac{r}{a}). \tag{5.30}$$

5.9.3 Kepler's disk

Kepler's disk has the potential of a point mass M given by $\phi(r) = -G \cdot \frac{M}{r}$. Its rotation is that obeyed by the planets in the solar system and Kepler's three laws. A circular velocity computed after introducing Kepler's potential is

$$v(r) = \sqrt{G \cdot \frac{M}{r}}. \tag{5.31}$$

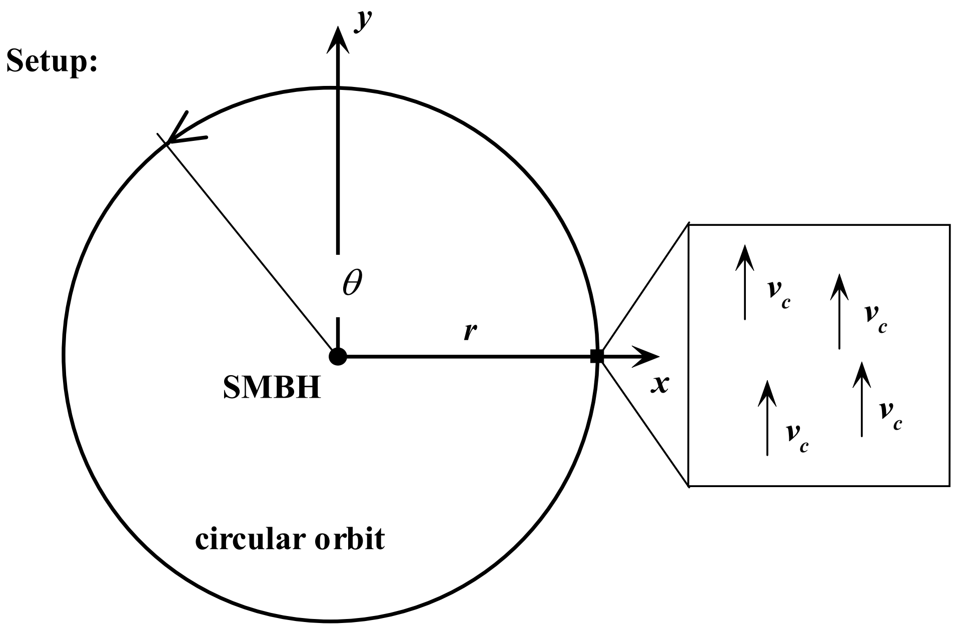


Figure 5–15: An initial setup of Kepler's disk with circular velocities.

5.10 Disk galaxies

Kepler's model

As the first approximation, we can model a disk galaxy with Kepler's model. As the planets in the solar system revolve around the Sun, also the stars (with planetary systems) in the disk galaxy are revolving on (nearly) circular orbits around the supermassive black hole at the center. The two massive bodies (SMBH and single star) are essentially unaffected by the third body (other stars).

Let us assume that the galaxy is made of two components: the stellar disk with uniformly distributed stars and the super-massive black hole. A model realization is in the dimensionless system of units. We will evolve initial conditions with the Barnes-Hut *N*-body simulation code.

Model	Resolution (star particles)	Timestep	Steps	Opening angle θ	Softening length ε
Kepler	5,000	0.0005	30,000	0.75	0.0001

Table 5–2: Parameters of the Kepler's disk simulation.

The conversion to physical units is performed by scaling the Kepler's digital galaxy to the system with known physical properties. If we scale it to the disk of the Milky Way galaxy, the unit length, unit mass, unit velocity, gravitational constant and unit time are

$$
\begin{aligned}
[L] &= 10 \text{ kpc}, \\
[M] &= 10^{11}\ M_{\odot}, \\
[v] &= 300 \text{ km} \cdot \text{s}^{-1}, \\
[G] &= 6.67 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}, \\
[t] &= 68.2 \cdot 10^{6} \text{ yr}.
\end{aligned}
$$

The timestep is therefore equal to $34.1 \cdot 10^{3}$ yr and the overall simulation covers $1.02 \cdot 10^{9}$ yr .

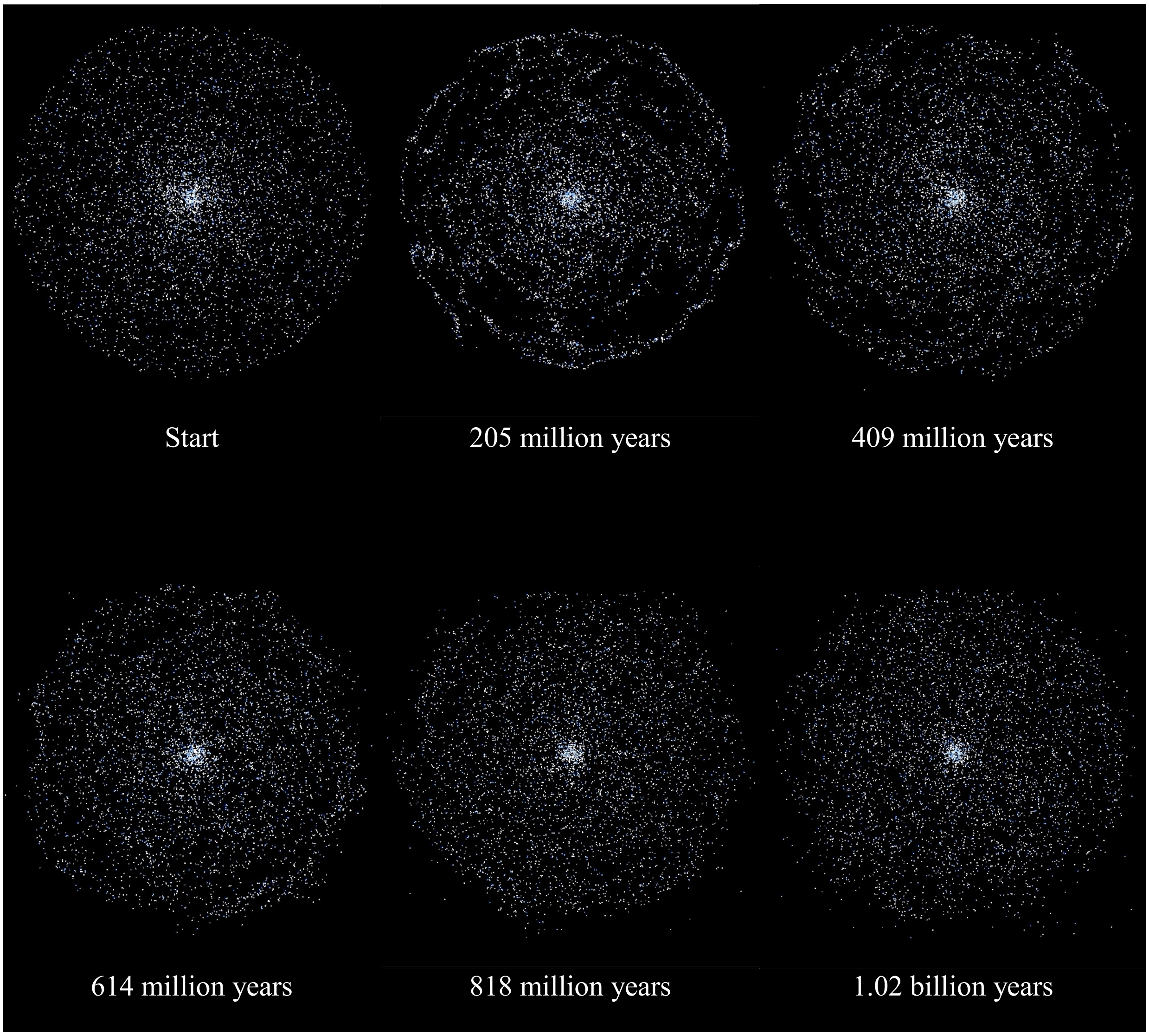


Figure 5–16: A time sequence for Kepler's disk model in the *xy* plane. The disk is ideally flat in the *yz* plane.

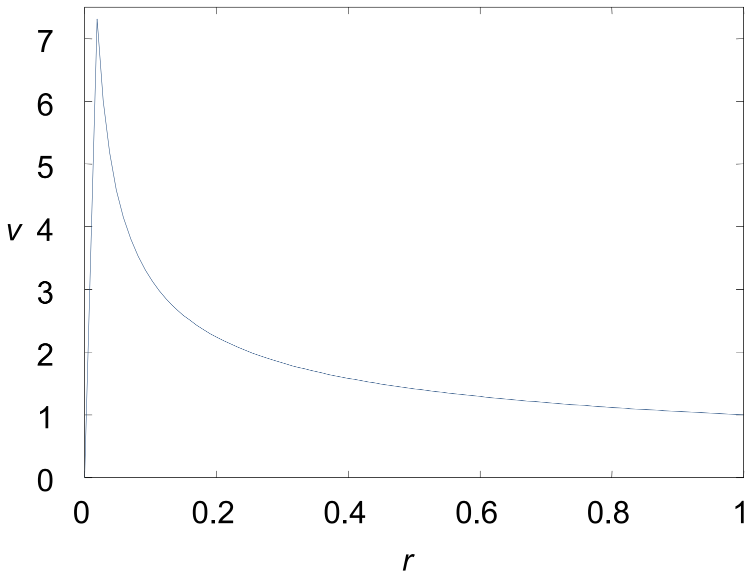


Figure 5–17: A rotation curve[49] of Kepler's model.

5.11 Rotation curves

We will now compare a rotation curve from the simulations of Kepler's disk with a rotation curve of a real galaxy as measured by a real experiment. Such work can be carried out with a radio telescope that traces neutral hydrogen (HI) atoms and carbon monoxide (CO) molecules. HI atoms are hard to observe, but CO molecules are copying the presence of HI atoms and their motion. Spectroscopy and the Doppler Effect permits us to measure the speed of gas orbiting around the center of the galaxy.

These observations of galactic rotation curves in disk galaxies have revealed that rather than falling-off as we can see in the plot of our computer simulation of Kepler's disk (see Figure 5–17) and that is in an agreement with Newtonian physics and observed mass, the rotational velocities remain constant at large radii. This is valid for rotation curves of the Milky Way galaxy and other real galaxies (Fuchs et al., 1998).

[49] Rotation curve describes the orbital velocity of stars around the common center.

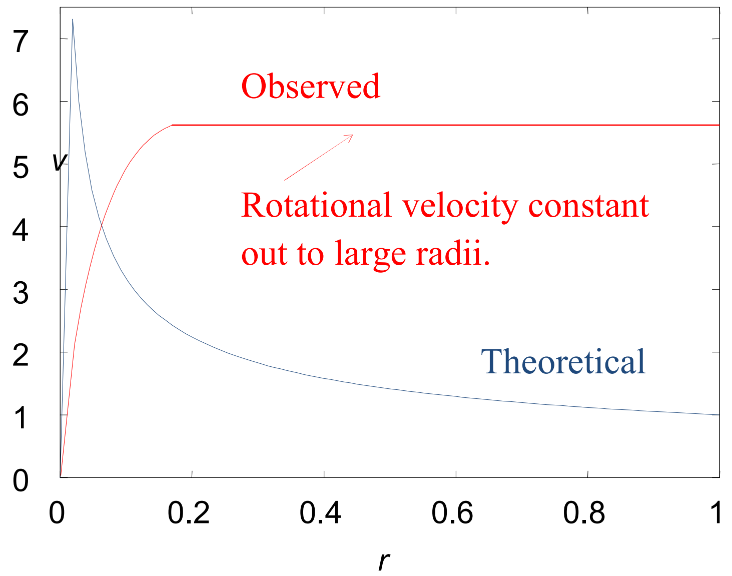


Figure 5–18: Rotation curves: Theoretical (based on Newton's law of gravity and visible matter) and experimental (based on observations).

Based on an astronomical observation of a mass distribution in disk galaxies can be inferred that the number of stars is dropping towards the edge of the stellar disk. From Newton's law of gravity, we obtain

$$\begin{aligned} F &= G \cdot \frac{M \cdot m(r)}{r^2} \\ M \cdot a &= G \cdot \frac{M \cdot m(r)}{r^2} \\ M \cdot \frac{v^2}{r} &= G \cdot \frac{M \cdot m(r)}{r^2} \\ v(r) &= \sqrt{\frac{G \cdot m(r)}{r}}, \end{aligned} \tag{5.32}$$

where $v(r)$ is the orbital velocity of a star at the distance r from the center of the galaxy. The cumulative mass inside radius r is

$$m(r) = \frac{v^2 \cdot r}{G}. \tag{5.33}$$

If the orbital velocity v is constant with increasing r (as is known from observations) then for $G = const.$ the only term in Equation (5.33) that can change with increasing r is $m(r)$. The cumulative mass $m(r)$ must grow even in vast distances from galaxy center. However, observed light or luminosity $L(r)$ that is believed to trace the matter distribution tends to a finite limit as we reach the edge of the luminous material in the galaxy. Therefore, the flatness of the rotation curve in large distances from the center of the disk, where number of stars is beginning to drop, cannot be explained by the visible mass alone.

Dark matter

It is supposed that invisible matter supplies this missing gravity. Jan H. Oort (1932) studying the motion of stars in the Milky Way galaxy was the first one, who noticed that some matter in our galaxy is "missing". The flat rotation curves of spiral galaxies at large galactocentric distance suggest that the halo of the invisible matter might exist around spiral galaxies; this dark halo might continue to contribute to $m(r)$ out to very large radii. Fritz Zwicky[50] studied the motion of galaxies within Coma cluster of galaxies in 1933. He discovered that galaxy velocities are so large that they should fly out of the galaxy cluster long ago. Since the cluster shows no signs of flying apart, some force of invisible source must hold it together. It is generally supposed that some additional matter invisible to us must gravitationally bound these galaxies together (Zwicky, 1937).

However, idea of dark matter (DM) was not accepted till extensive observational results of Rubin and Ford (1970). Visible parts of the spiral galaxy and the gas layers cannot account for the observed high rotation velocities in the outer parts of spirals, if the stellar mass-to-light ratio is constant with the radius. This is an indirect evidence that luminous portions of galaxies represent only a small fraction of the total galactic mass. Pioneering simulations of galaxies performed by Ostriker and Peebles (1973) also revealed that the inclusion of the massive halo will stabilize a galaxy disk against strong axisymmetric instability. Otherwise, the galaxy simply flies apart. The Milky Way and other disk galaxies may be therefore embedded in halos of unseen matter.

5.12 Activation of the disk

Orbits of stars in real disk galaxies are not perfectly circular. We will create more realistic models of the disk galaxy with a hot disk, where a velocity dispersion (fluctuation, deviation) is introduced. A way to quantify velocity dispersion in the disk is the Toomre's (1964) parameter Q expressed as

$$Q(r) = \frac{\sigma_{rad}(r) \cdot \kappa}{3.36 \cdot G \cdot \Sigma(r)}, \tag{5.34}$$

where $\sigma_{rad}(r)$ is the radial velocity dispersion, κ is the frequency of small oscillation about the circular radius (local Lindblad epicyclic frequency) and $\Sigma(r)$ is the stellar surface density. The stability criterion against axisymmetric perturbations requires for the stellar disk to set $Q > 1$.

[50] Fritz Zwicky (1898–1974)

For $Q < 1$, the disk is locally unstable and $Q = 1$ corresponds to the cold disk. The epicyclic frequency is opt as $\kappa(r) = \sqrt{2} \cdot w(r)$, where $\omega(r)$ is an angular velocity $\omega(r) = \dfrac{v(r)}{r}$.

Tangential (azimuthal) velocity dispersion $\sigma_{\tan}(r)$ is expressed as

$$\sigma_{\tan}(r) = \frac{3.36 \cdot G \cdot \Sigma(r)}{2 \cdot \omega(r)}. \tag{5.35}$$

Vertical velocity dispersion (dispersion in a direction, which is perpendicular to the disk plane) is chosen as the part of tangential velocity dispersion as

$$\sigma_Z(r) = 0.5 \cdot \sigma_{\tan}(r). \tag{5.36}$$

From Equation (5.36) can be seen that even if the initial mass density of the disk is planar, vertical dispersion will lead to the thickening of the ideally flat disk.

Hot disk realizations

We will create a disk with kinematically hot (supported by random motions, rather than the rotational motion) stars. The thickness of the disk is set such that Toomre's stability criterion is satisfied. The radial velocity dispersion $\sigma_r(r)$ corresponds to the value of the Solar neighborhood, i.e. $Q = 1.5$. The Gaussian-distributed random velocity dispersions are equal to $\sigma_{rad}(r)$, $\sigma_{\tan}(r)$ and $\sigma_Z(r)$.

Kuzmin's and exponential model

We will use a program for initial conditions generation to create a disk galaxy. A model realization is in the system of units in which the gravitational constant G, the total mass M, and the scale radius a, are all equal to unity. We will evolve initial conditions with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 0.75$ and the softening length $\varepsilon = 0.0001$.

Model	Resolution (star particles)	Timestep	Steps
Kuzmin	5,000	0.0005	30,000
exponential	5,000	0.0005	30,000

Table 5–3: Parameters of the Kuzmin's and exponential model simulations.

The timestep was set equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $1.02 \cdot 10^9$ yr.

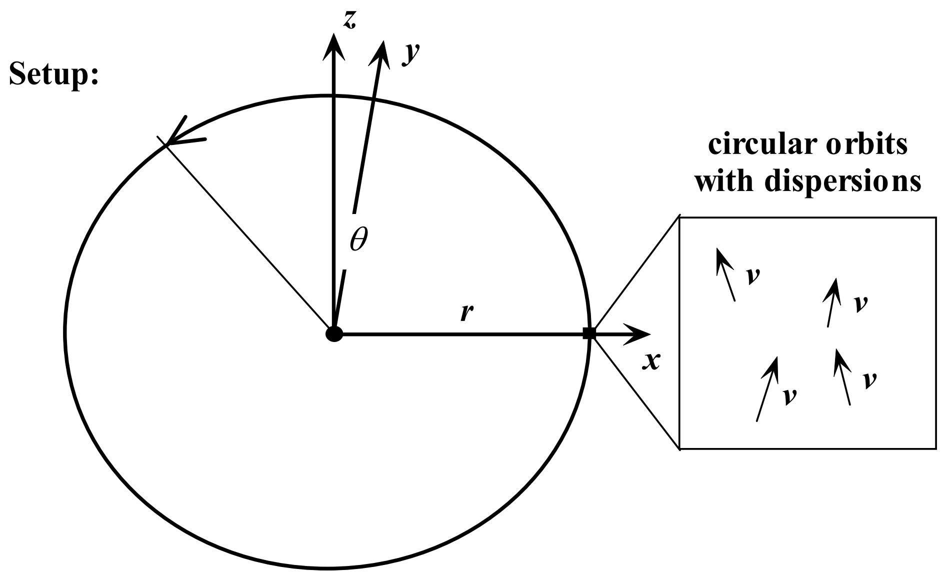
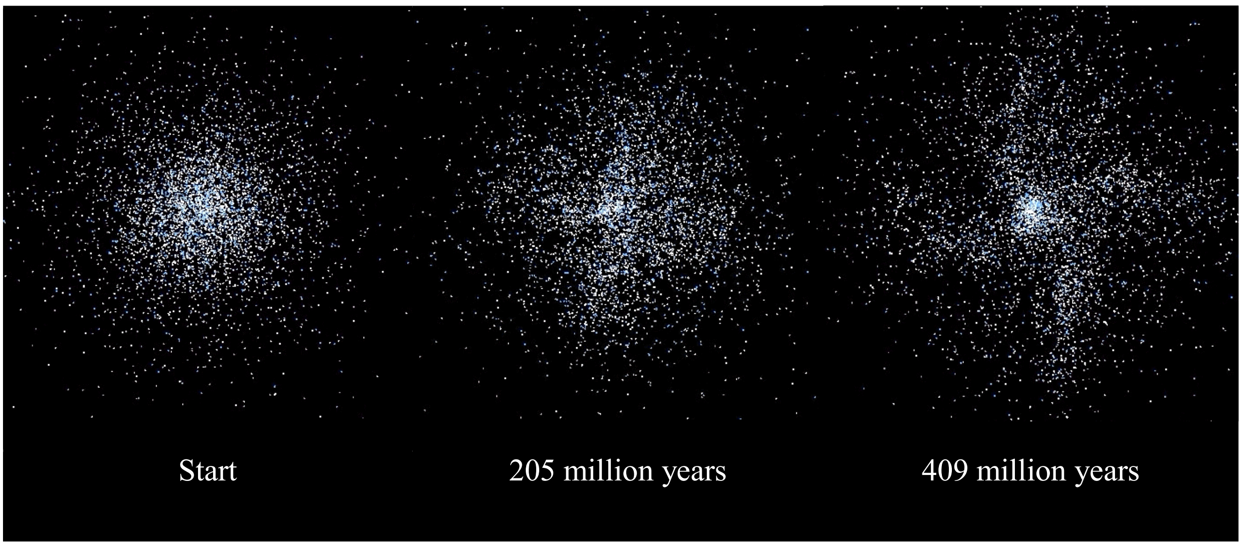


Figure 5–19: An initial setup for the hot disk with the velocity dispersion added to circular velocities.



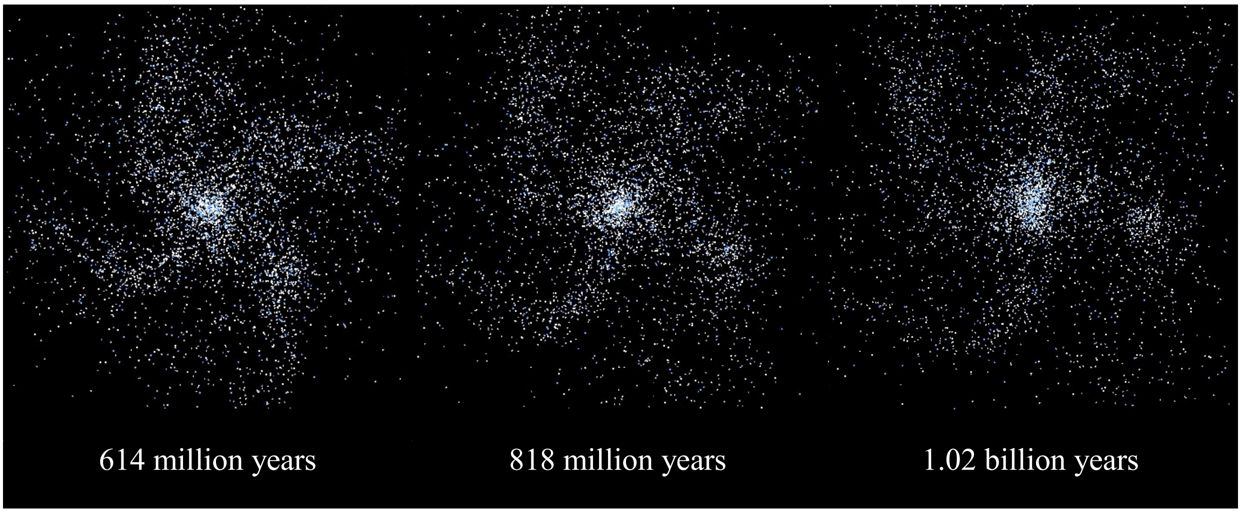
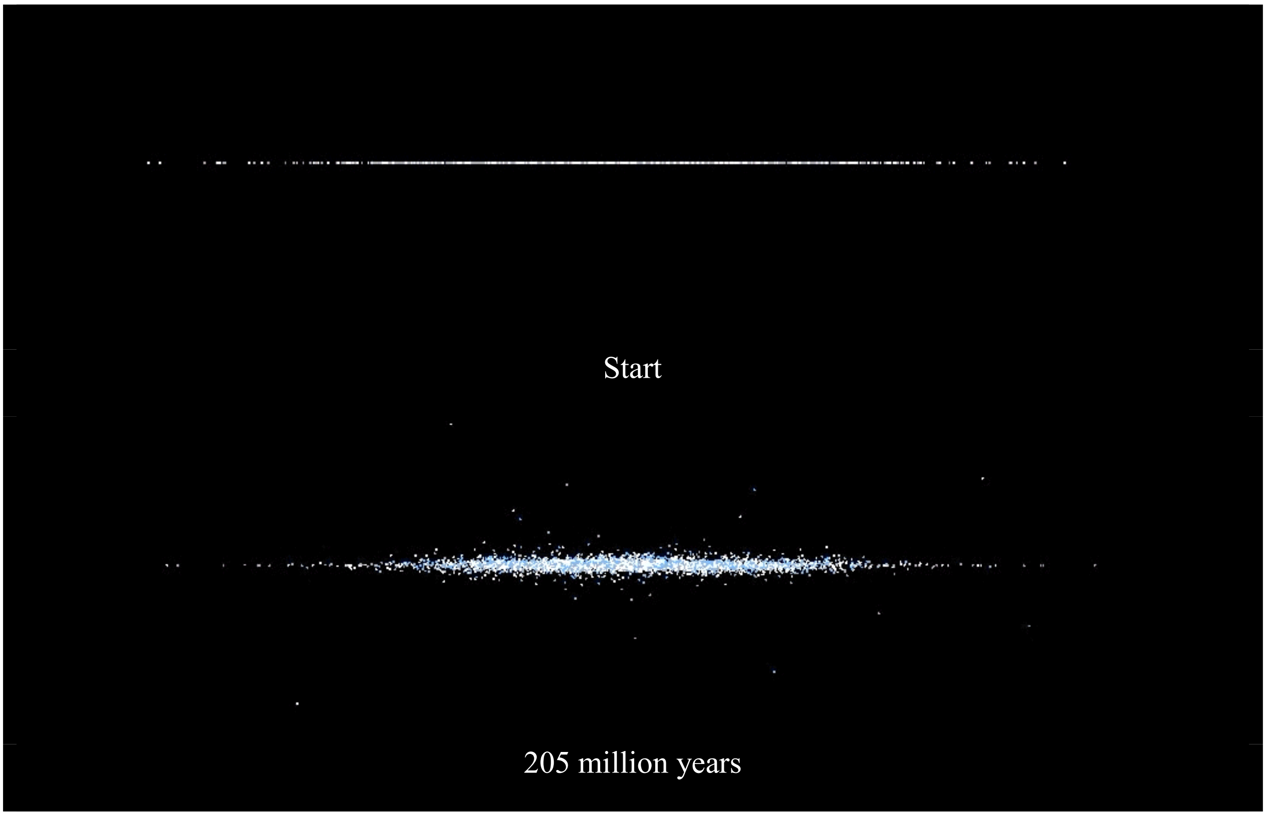


Figure 5–20: A time sequence for the hot exponential disk with $Q = 1.5$ in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.



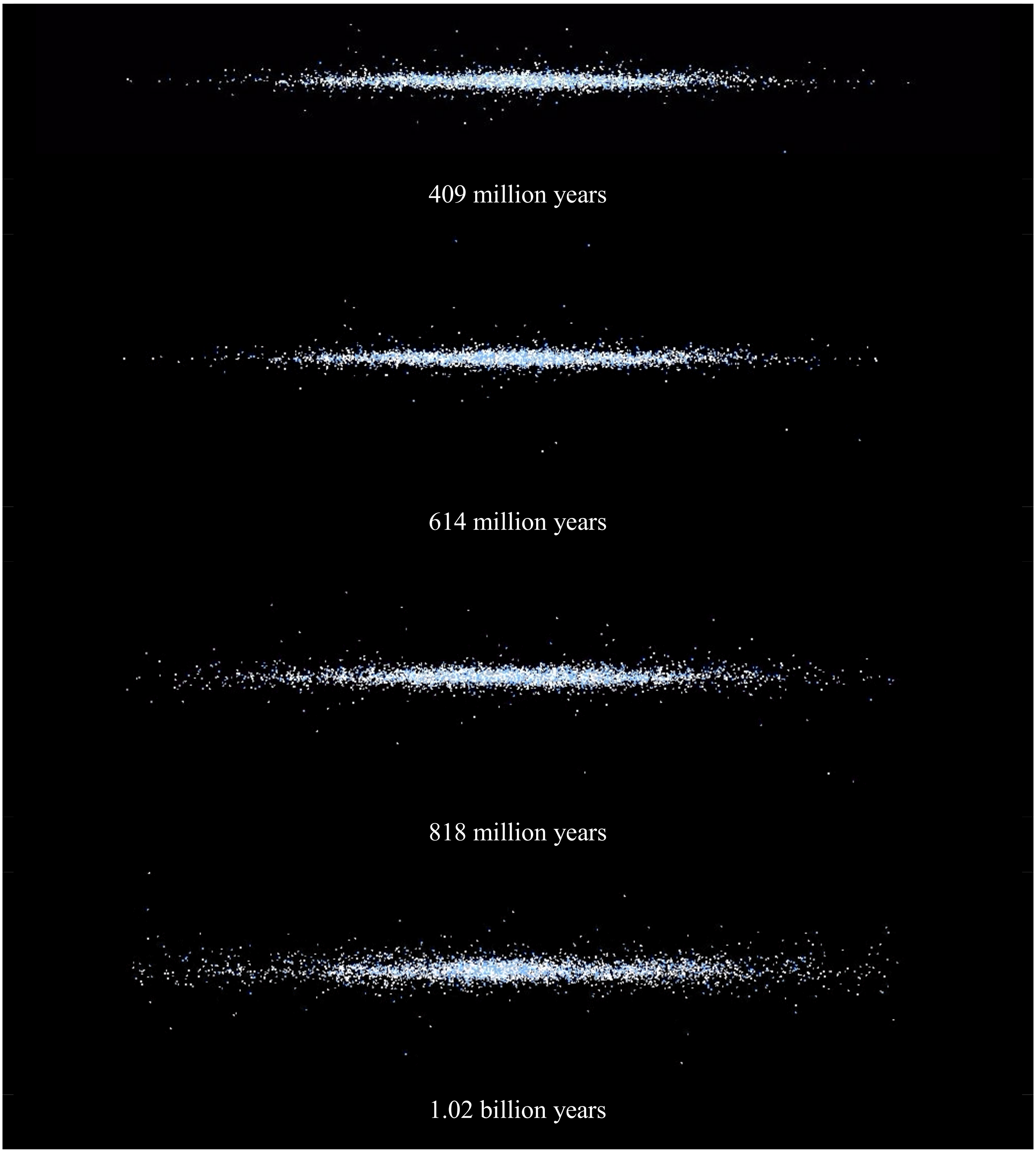


Figure 5–21: A time sequence for the hot exponential disk with $Q = 1.5$ in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm. The thickening caused by the random motion of stars can be seen.

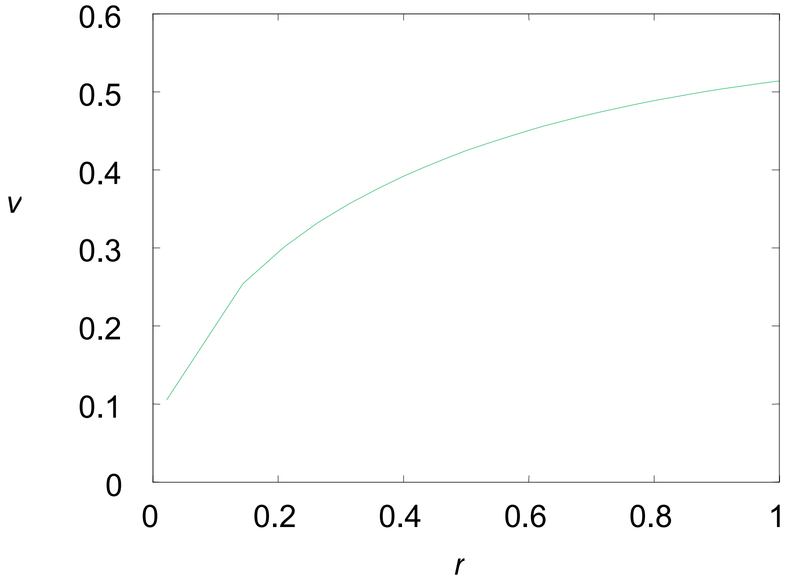
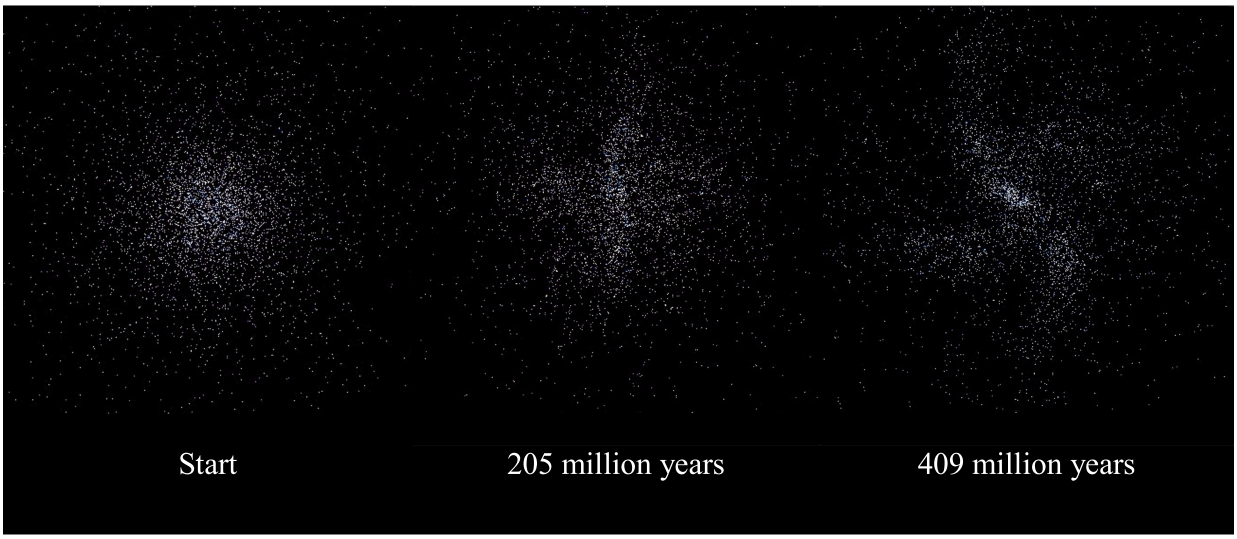


Figure 5–22: The rotation curve of the exponential disk.



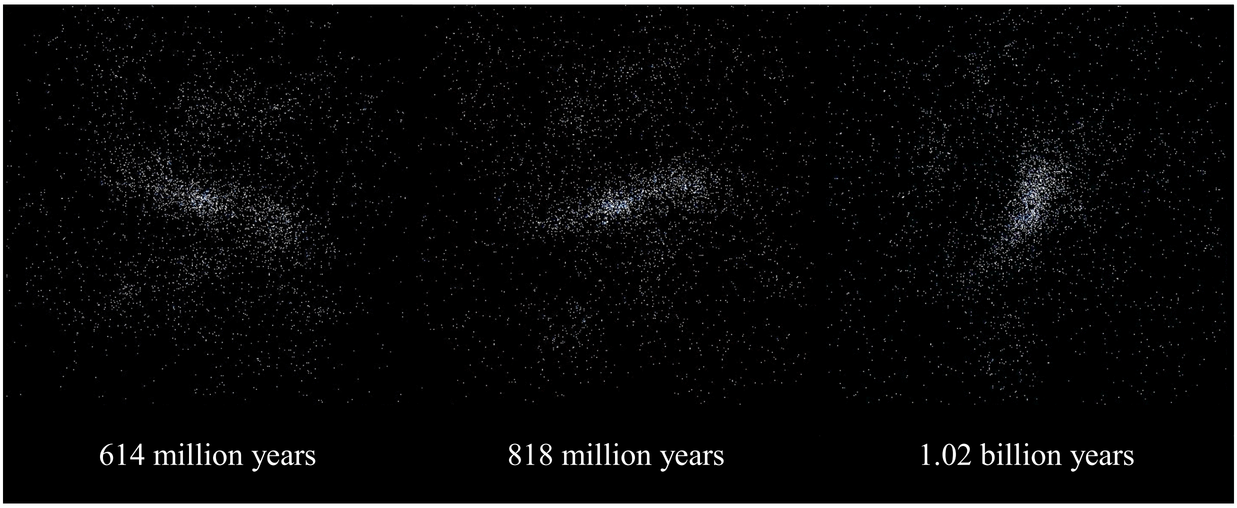
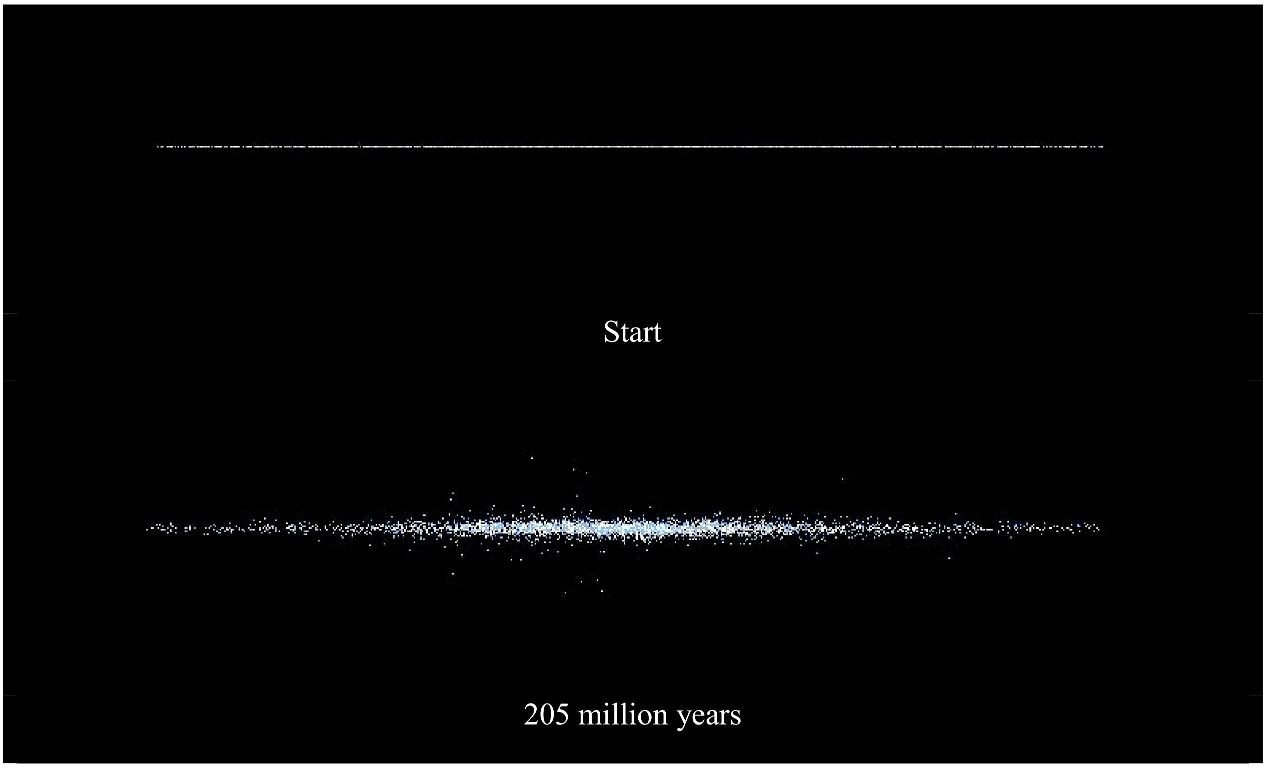


Figure 5–23: A time sequence for the hot Kuzmin's disk with $Q = 1.5$ in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.



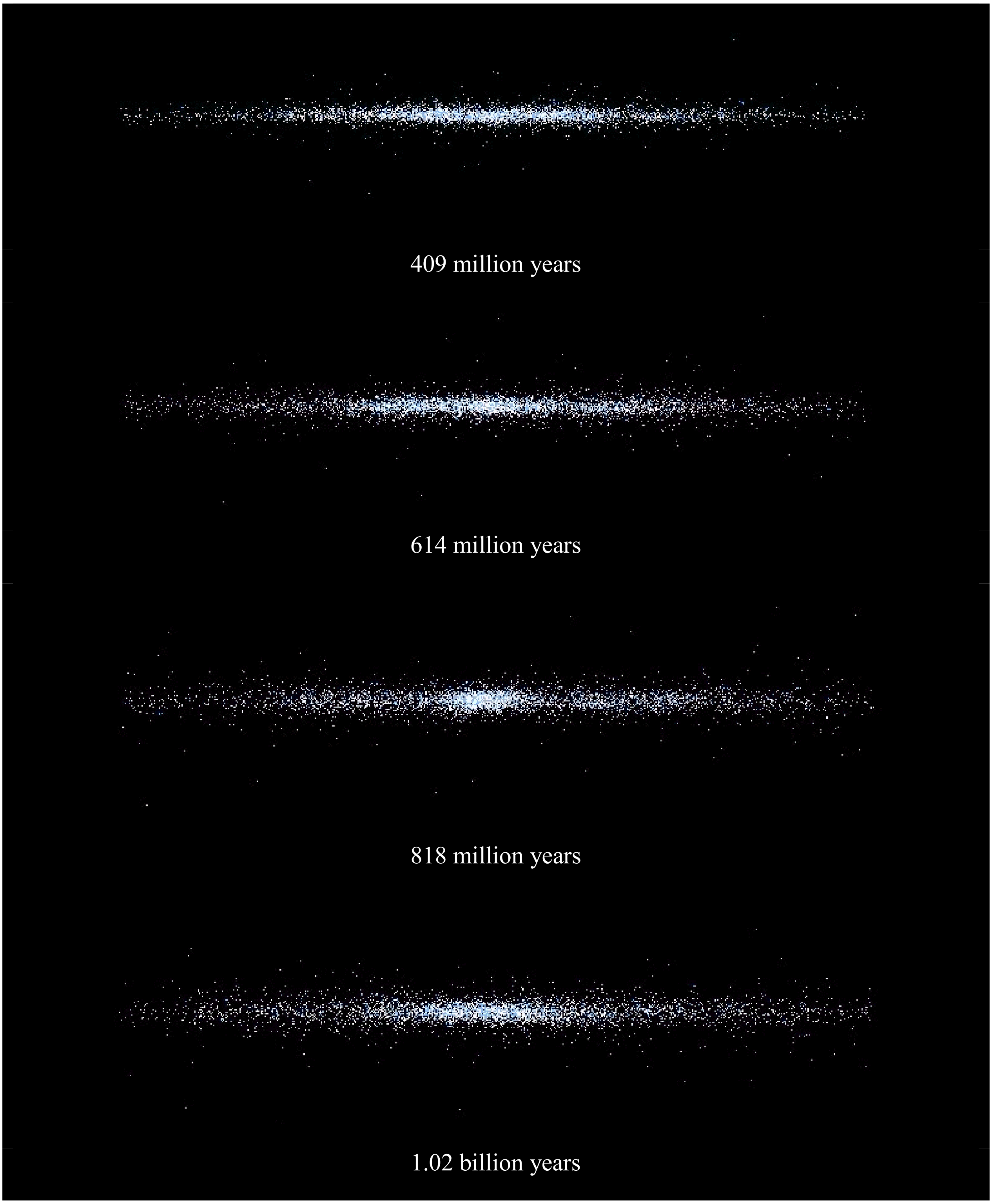


Figure 5–24: A time sequence for the hot Kuzmin's disk with $Q = 1.5$ in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm. The thickening caused by the random motion of stars can be seen.

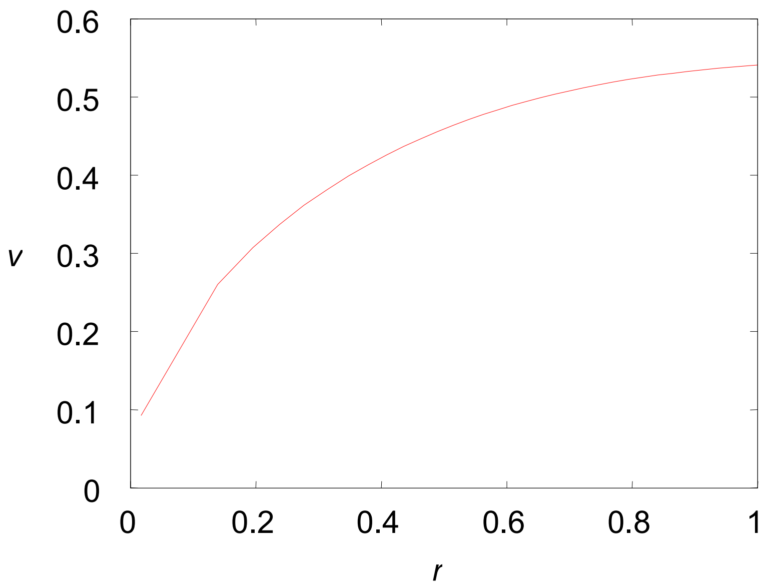


Figure 5–25: The rotation curve of Kuzmin's disk.

First, we will check the stability of the model. The whole simulation is showing a continuous evolution and changes of both disks. After the formation of a strong axisymmetric instability, the system relaxes to a steady state. After relaxation, the system forgets its initial conditions.

Large-scale deviations from the axial symmetry can be recognized in the simulations – the rise of a strong central bar and spiral arms. Initial conditions of the disk galaxy are developing into the bar and two or four spiral arms. Early *N*-body researchers (e.g. Hohl, 1971) already recognized the violation of axial symmetry in disks.

With a continuous evolution of the *N*-body system, the bar becomes weaker, but seems to last forever (exceeding the age of the universe) (Sellwood, 1981). The variation of mass breaks the axisymmetry and gravitation amplifies it by creating the spiral arms and bars. It is inevitable that a spiral galaxy will develop the axisymmetric instability at some point in its evolution.

5.13 Composite models

Galaxies can be better approximated with more than one component. Galaxies are composed of various parts that can be viewed in isolation as spherical or disk distributions. When we add these components together, we obtain a composite model of galaxy.

It is interesting that $\rho(r)$ does not have to be related to $\phi(r)$ through Poisson's equation. Multi-component model might be constructed, with components co-existing in the mutual gravitational potential $\phi_{total} = \phi_d + \phi_b + \phi_h$ (disk, bulge, halo). Our numerical spiral galaxy will be such a composite model of a disk, spherical bulge and halo. For each component of the combined model, the distribution function can be found separately.

Initial conditions are generated from the isotropic distribution functions $f_b(E)$, $f_h(E)$, and $f_d(E)$, which represents the bulge, disk, and halo, respectively. We assume that the total potential is spherical even when the potential is aspherical because of the disk. Influence to current investigations can be neglected.

More techniques for generating initial conditions were introduced. Barnes (1988) constructed isolated components and then slowly induced them into the presence of each other. Hernquist (1993) used local Maxwellian approximation. Kuijken and Dubinski (1995) employed a technique involving spherical harmonic expansion of the potential. Boily et al. (2001) used a method without the need of computing anisotropic velocity dispersions.

Galaxy experiments

Our objective will be a simulation of the composite model of the Milky Way galaxy. The most intuitive is to start with a total disk mass that is equal to the mass of 400 billion stars of solar mass. A bulge mass will be 10 % of the disk mass and stellar halo will contain 30 % of the disk mass (see Table 5–4). We will evolve initial conditions with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$ and the softening length $\varepsilon = 0.0001$. Model galaxy will be generated with the following units

$$
\begin{aligned}
[L] &= 10 \text{ kpc}, \\
[M] &= 10^{11} \ M_{\odot}, \\
[v] &= 300 \text{ km} \cdot \text{s}^{-1}, \\
[G] &= 6.67 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}, \\
[t] &= 68.2 \cdot 10^6 \text{ yr}.
\end{aligned}
$$

Disk		Bulge		Stellar halo	
M_D	4.0	M_B	0.4	M_H	1.2
R_D	3.0	R_B	1.5	R_H	3.0
Q	1.5				
N_D	4,000	N_B	1,000	N_H	2,000
Kuzmin		Plummer		Plummer	

Table 5-4: The Milky Way galaxy model G1.

In this system of units is the timestep equal to $34.1 \cdot 10^{3}$ yr and the overall simulation with 150,000 timesteps covers more than $5.1 \cdot 10^{9}$ yr .

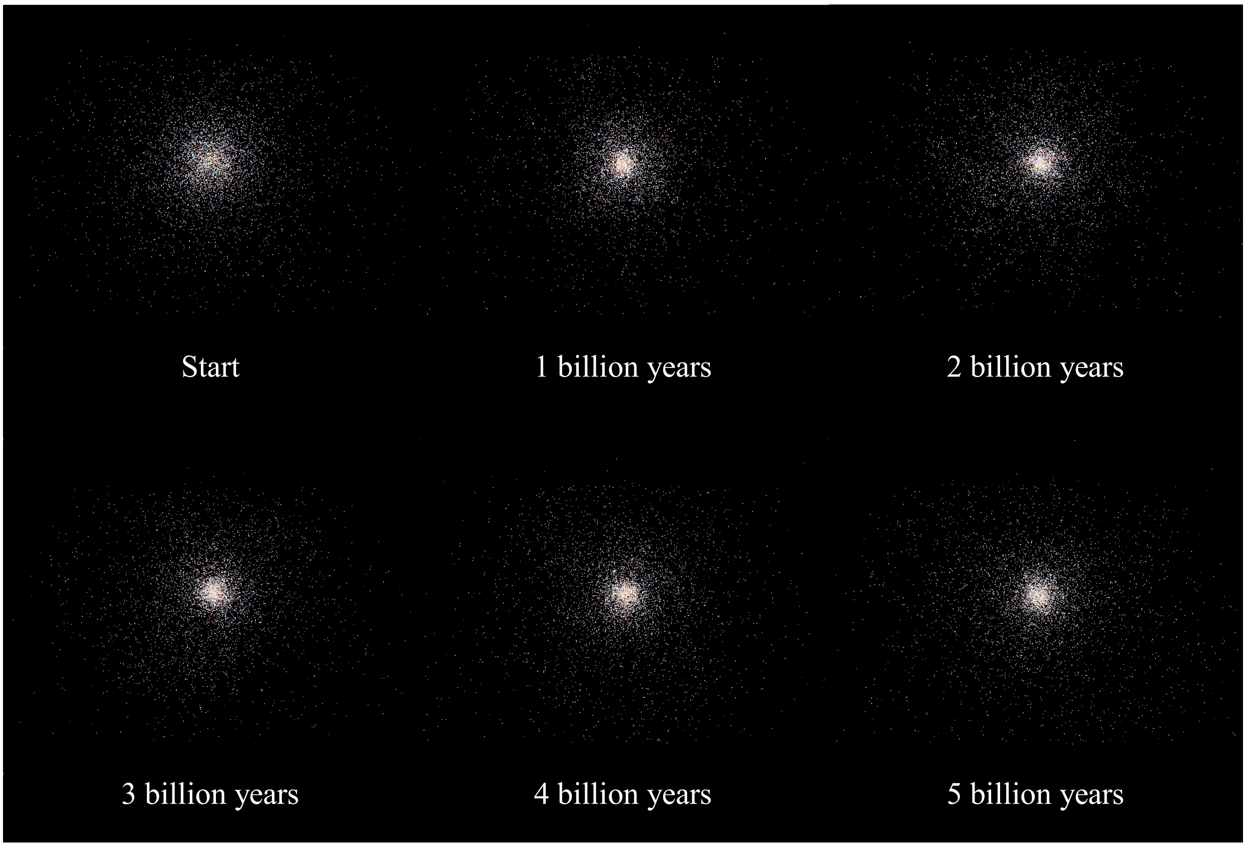
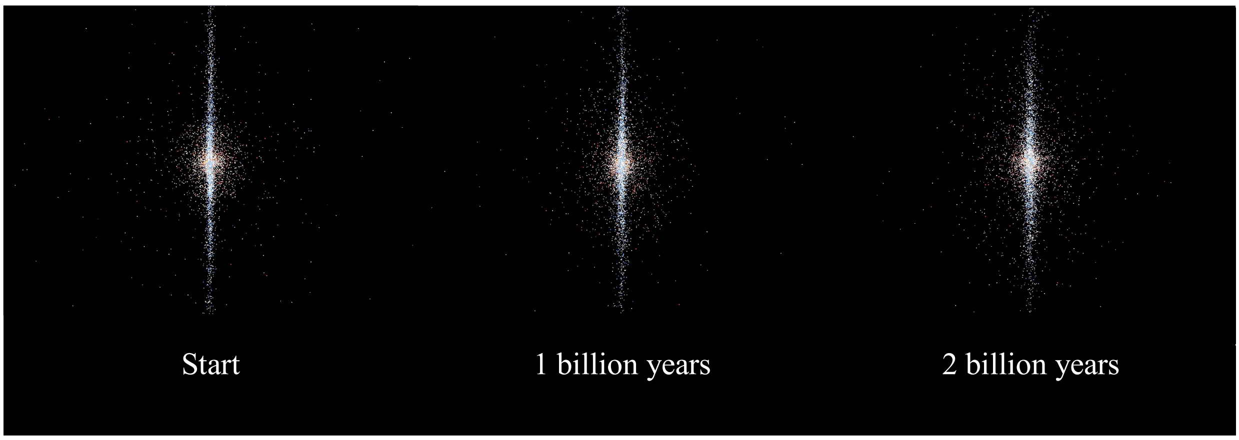


Figure 5–26: The evolution of the Milky Way galaxy G1 composite model in the *xy* plane.



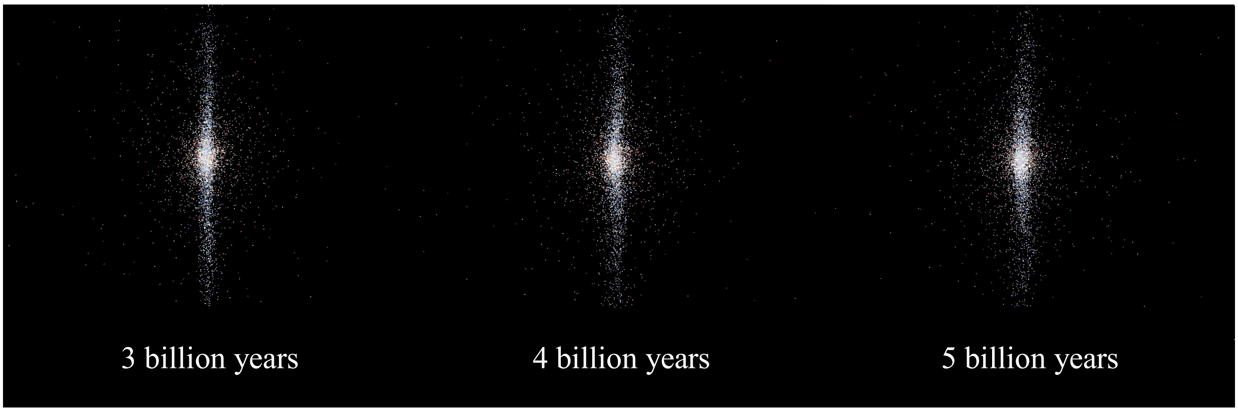


Figure 5–27: The evolution of the Milky Way galaxy G1 composite model in *yz* plane.

Milky Way with dark matter halo

In the Milky Way galaxy model G2, we will add the isothermal dark-matter halo component. Our galaxy is probably contained in the halo of dark matter particles with radius nearly 250 kpc that is about 20 times more massive than visible parts of Galaxy, leading to luminous-to-dark ratios of 1:20.

Disk		Bulge		Dark halo	
M_D	0.9	M_B	0.1	M_{DH}	20.0
R_D	12.0	R_B	1.5	R_{DH}	250.0
Q	1.5				
N_D	3,000	N_B	1,000	N_{DH}	3,000
Kuzmin		Plummer		Isothermal	

Table 5-5: Milky Way galaxy model G2.

Simulation parameters and units are the same as in the simulation of G1 model. The timestep is equal to $34.1 \cdot 10^3$ yr and the overall simulation with 150,000 timesteps covers more than $5.1 \cdot 10^9$ yr.

5.14 Concluding remarks

In Chapter 5, we have shown how to create a computer model of a galaxy in order to study galaxy dynamics. We found that the construction of galaxy in a controllable way is difficult. Due to immense complexity, all models are very artificial in comparison to real galaxies. In spite of that, the initial density distribution function of models are in good agreement with observations of real galaxies. The galaxy models created were single or multicomponent systems in stable dynamical equilibrium.

Initial conditions were generated as follows. Specifying the mass density distribution function, we first calculate the model's cumulative mass distribution function and corresponding gravitational potential. Then the mass density distribution function is expressed as a function of gravitational potential. The phase-space distribution function is calculated on the fly using a numerical formulation of Eddington's formula. Once the phase-space distribution function has been calculated, one can start to randomly sample particles from the distribution function. If the use of another mass density profile is requested, all that is necessary, is to override a virtual function "rho" according to the chosen mass density profile. Through this approach, many kinds of models may be constructed. Models created in this way are quickly getting into equilibrium.

We created realizations of an elliptical galaxy from various spherical models. The spherical models were in equilibrium from the beginning. We have created disk models that showed continuous evolution. We saw that dynamically cold disk without a dark halo spontaneously formed features resembling galactic bar and spiral arms. It has been shown that a self-gravitating disk system is unstable unless a certain velocity dispersion and dark halo were included. All models were evolved for more than 1 billion years and movies from all computations were produced.

"The past, present and future are only illusions, even if stubborn ones."
— Albert Einstein —

6

MODELING GALAXY INTERACTIONS

Why are not all galaxies the same? Why do galaxies show such a variety of properties? Why do galaxies come in such a wide variety of shapes and sizes? How do they evolve? Were galaxies predestinated at the moment of their birth? In this chapter, we will show how to study galaxy collisions and mergers with computer simulations.

6.1 Introduction

"Spiral nebulae" were once thought to be inside the main and single galactic continent, our Milky Way galaxy. When these "nebulae" were recognized as galaxies in their own right by Hubble[51] (1925) with vast distances between them, galaxies were then thought of as "island universes" slowly evolving in isolation. These early studies of galaxy dynamics were based on an extraordinary success of isolated system. The problem is that no real system is truly isolated. Galaxies have peculiar velocities in space relative to the general expansion of the universe (Hubble flow) and they can get close to each other.

In 1956, Fritz Zwicky as the first one spotted an enormous variety of extended features seen in interacting galaxies[52]. As galaxy catalogs began to grow (Arp, 1966), the number of anomalously looking peculiar galaxies was mounting. Distorted galaxies with arcs, tails, ripples, plumes,

[51] Edwin P. Hubble (1889–1953)

[52] Cited by Barnes and Hernquist (1992).

wings, shells, bridges, tails, and grand arms emanating from their bodies were discovered. These galaxies have blue colors suggesting that new stars are born within these galaxies (high star formation rate, SFR).

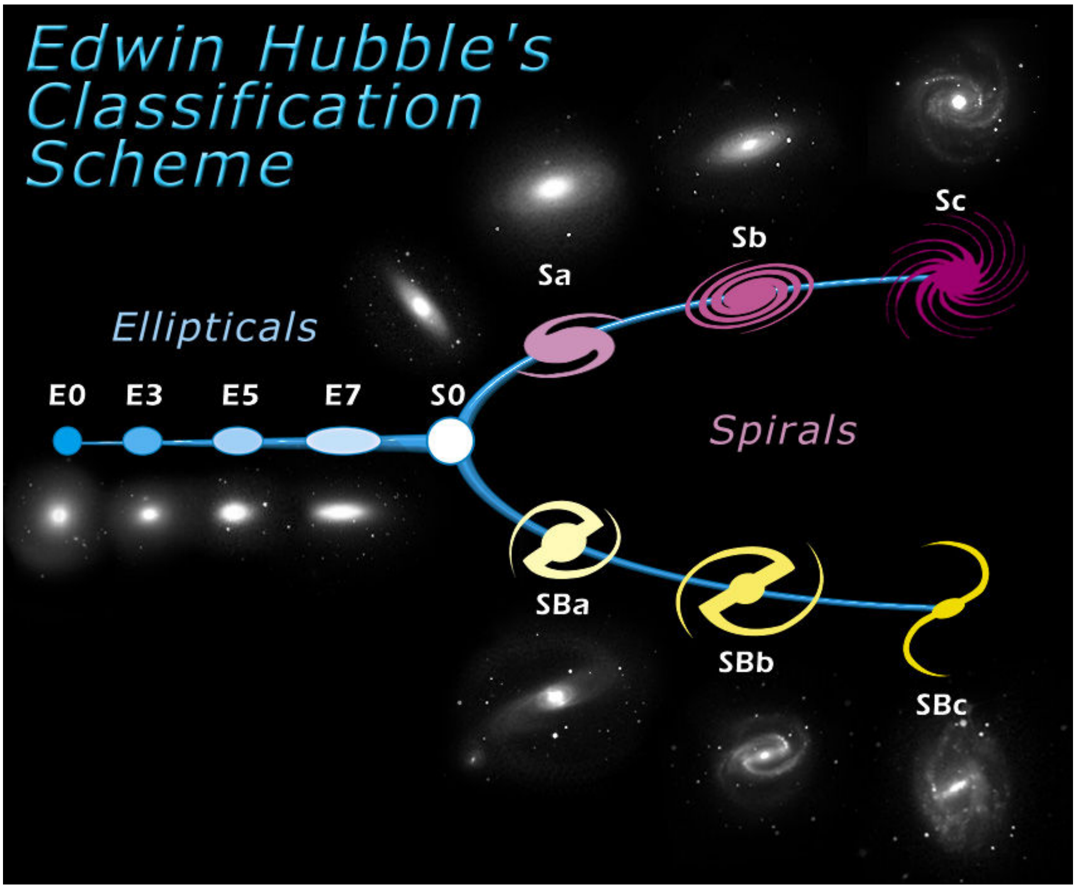


Figure 6–1: Galactic species in the Hubble's galactic family. The "tuning fork" classification scheme (Hubble, 1926) of elliptical (Ex), lenticular (S0), spiral (Sx), and barred spiral (SBx) galaxies. Courtesy Space Telescope Science Institute (STScI).

6.1.1 The structure of the universe

The currently most successful model of the universe is the Big Bang theory with inflationary ΛCDM[53] model that explains observed properties of our universe on super-galaxy scales. According to this standard cosmological model, our universe started with a very hot and very

[53] Lambda and Cold-Dark-Matter (LCDM or ΛCDM). Lambda describes the universe's increasing rate (acceleration) of expansion, cold dark matter stands for non-relativistic or cold (velocities below the speed of light), invisible (not interacting electromagnetically), gravitationally interacting non-baryonic particles.

uniform matter distribution. However, primordial small-scale quantum fluctuations[54] in the matter distribution (small inhomogenities) (Peebles, 1974) were inflated to huge dimensions after the Big Bang. Successive accumulation of matter was started in places with the higher concentration of matter, given by primordial fluctuations (inhomogenities) through the gravitational attraction. Weak density inhomogenities of baryonic matter[55] were amplified by the gravitation of dark matter and caused the growth of inflated fluctuations. A cooling gas was attracted by the clumps of dark matter forming visible parts of galaxies. Small systems composed of the dark matter and gas have continued to merge and formed larger objects. This process led to the formation of all structures – galaxies, groups, clusters, super-clusters of galaxies and to the overall large-scale structure of the universe (LSS) (White and Rees, 1978) – and continues up to this today. This structure formation scenario is known as the *hierarchical merging scenario*.

Gravitation reorganized initial fluctuations in the matter distribution into the structure, where stars are clustered in galaxies, galaxies are clustered into galaxy groups, galaxy groups form galaxy clusters, and galaxy clusters form super-clusters of galaxies. Some clusters are not comprised of small groups, but are simply large, galaxy filled clusters. They are called "rich clusters" (e.g. Coma cluster).

The fluctuations in the matter distribution (seeds of galaxies) grew during initial inflation phase in the early universe linearly and are therefore suitable for analytical studies. However, when the fluctuations began to grow in amplitude through gravitation and started to form galaxies and clusters of galaxies in non-linear process, the only way to study them are numerical simulations on computer.

6.1.2 Demography of galaxy clusters

Galaxy clusters are ensembles of a few tens to a few thousands of galaxies bound together by the gravity. Rich clusters with a high galaxy density contain a large number of elliptical[56] galaxies. A supergiant elliptical galaxy can be found at the very center of the rich cluster. Spiral galaxies are rare in rich clusters, but dominate in the space between clusters or in smaller groups with a low galaxy density[57]. This correlation between the galaxy type and the local galaxy density is called a Morphology-Density (MD) relation.

---

[54] Against the physics of large scales (macroworld), which we face in every day life and from which we gather our experience, while studying small scales (microworld) we encounter everlasting changes and unavoidable deviations of quantities called fluctuations.

[55] The amount of baryonic matter was not able to produce sufficient gravitation in order to create the observed large-scale structure of the universe under the assumption that Newtonian gravity and dynamics are the correct physical description shaping the large-scale structure of the universe.

[56] Rich clusters also contain lenticular galaxies (S0 in the Hubble's classification, with disk but no spiral arms).

[57] The Milky Way galaxy resides in such a small galaxy group called the Local Group.

Deep field images

The farther away a galaxy is, the longer takes to its light to get to us. Deep observations not only look to great distances, but also look back in time. The universe is expanding according to the Big Bang theory, and the more distant an object is, the more rapidly it is receding from us. A velocity at which the object is moving away from us can be determined from the incoming "light" by the shift in its spectral lines. By studying galaxies at ever higher redshifts z [58] (larger distances), we can observe the universe at an ever younger age. Over the cosmic time, we can see that galaxies are changing.

From the Hubble Deep Field (HDF) observations[59], we know that galaxy sizes are growing with the time and simulataneously the number of galaxies is dropping. Larger galaxies are built of the smaller ones through the gravitational assembly and thus supporting the hierarchical merging scenario. Up to 25 % of interacting and disturbed spiral galaxies can be seen on redshifts $z > 2$ (now, on a redshift $z = 0$, it is just 1 %). Spiral galaxies on large distances are smaller, many are disturbed, assymetrical and with high SFR. A lot of irregular and small fragments that appear to be undergoing gravitational interactions and mergers are observed (Dressler et al., 1994). MD relation therefore varies with the redshift. Giant ellipticals exist at redshifts $z = 3$ with about 50 % of their current stellar mass, while cD galaxies underwent significant merging events at redshifts $z < 1$.

An observational evidence for the hierarchical merging scenario can be seen in rich clusters of galaxies, like the MS1054-03, which is located at a redshift $z = 0.83$ (8.8 billion light years away with currently favored cosmological parameters). From the sample of 81 studied galaxies, 13 of them are in collision or are their recent remnants (van Dokkum et al., 1999). Spiral galaxies were once located in regions with small distances between galaxies. Nowadays, giant elliptical galaxies are on their place.

Galaxies are recognized as the product of evolution lasting nearly 14 billion years. It is clear that many galaxies interact with neighboring ones and galaxy interactions are believed to be the key evolutionary mechanism. Galaxy collisions are usual and gravitational interactions are responsible for many features seen in galaxies. Galaxies are metamorphosed by their mutual interactions. Galaxies are not longer seen as isolated and unchanging objects. Only a few galaxies did not experience interactions and mergers. Galaxies evolve, interact, merge and collide. The variation of structural properties of galaxies through the Hubble's sequence is caused mainly by the gravitational interaction of galaxies. These interactions explain the Hubble's galaxy classification and explain peculiar galaxies. Even our own galaxy is now undergoing morphological events through the strong interaction with surrounding dwarf galaxies.

---

[58] Redshift z is defined in wavelength as $z = \dfrac{\lambda_o - \lambda_e}{\lambda_e}$, where the subscripts *o* and *e* refer to observed and emitted.

[59] Based on the Hubble Ultra-Deep Field (HUDF) observations is estimated that there are about 125 billion galaxies in the observable universe.

6.2 Computer simulations

Erik Holmberg (1941) performed the first simulation of the *N*-body system representing interaction of two stellar systems. It was an analog method for *N*-body simulation. Holmberg replaced the gravity by the light (same $\frac{1}{r^2}$ law) and stars represented by *N* light bulbs. The light bulbs were replaced one at a time by a device connected to *N* photodetectors that measured luminous flux at a distance r from remaining light bulbs. The experimental system was a collection of 74 bulbs ($N = 2 \cdot 37$) set up on a board. Holmberg's algorithm had an excellent $O(N)$ linear complexity.

In 1960s, theoretical astronomers began to use digital computer simulations to study the effects of close encounters of galaxies and found that the peculiar properties could be explained through the gravitational interaction of galaxies. Computational astronomy began to emerge in order to understand to such peculiar galaxy behavior. Pioneering numerical simulations of colliding galaxies were performed by Alar and Juri Toomre (1972). They argued that galactic bridges and tails seen in some multiple galaxies were gravitationally induced during a close encounter of these galaxies. Increasing computational power led to more detailed simulations with more bodies representing each galaxy.

6.3 Configuration of galaxy interactions

Our universe is ancient and vast. It is not a static place, rather it evolves. Every possible configuration of initial conditions of galaxy interaction allowed by nature might occur or will occur. As the result, we can observe many structures and irregular galaxies.

In a two galaxy encounter, one galaxy will be called a *central galaxy* and second, usually less massive, a *companion galaxy* or a *satellite galaxy*. Disk orientations for galaxies are described by the orientations of their angular momentum vectors. As explained in Chapter 5.9, galaxy disks created by our initial conditions generator lie in the *xy* plane. Galaxies can be optionally reoriented before they are set on a collision course. Galaxies can be rotated according to rotation angles ϕ, λ and θ as shown on Figure 6–2 and then set on the close encounter trajectory.

To classify galaxy interactions, we must parameterize the orbits of involved galaxies. Let us use two different models for interactions: a *direct collision* and *Keplerian encounter*. An unlimited number of initial configurations can be obtained by changing parameters of these interactions. To reliably follow a galaxy interaction of originally isolated galaxies, an initial separation should be substantially larger than galaxy sizes. Galaxies should be separated by ~ 10–100 times their sizes. However, many real satellite galaxies are very close to their centrals, so we must sometimes break this rule.

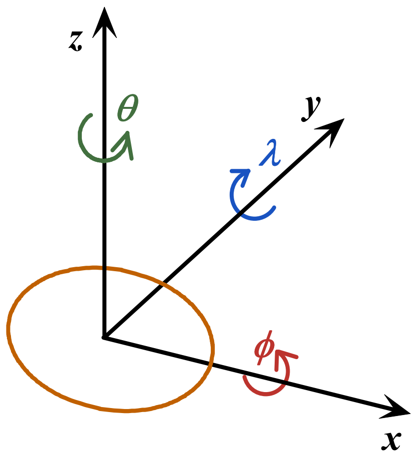


Figure 6–2: A disk situated in the xy plane can be rotated around all three axes. All rotations are given by the right hand rule: ϕ (measured in the yz plane around x), λ (measured in the xz plane around y), and θ (measured in the xy plane around z).

6.3.1 Direct collision

The central galaxy's center-of-mass (CM) is located at $(0, 0, 0)$ position. Initial conditions of the companion galaxy's CM are defined by the following set of parameters (Figure 6–3):

- An initial separation d_z between galaxies describes a shift between the central and companion galaxy in the z-axis direction.
- An initial deviation d_x of the companion galaxy from the central galaxy defines the distance of the companion galaxy's CM from the central galaxy's CM in the x-axis direction.
- An initial velocity of the companion galaxy v_{comp} describes the velocity of the companion galaxy's CM in respect to the central galaxy's CM against the x-axis direction.

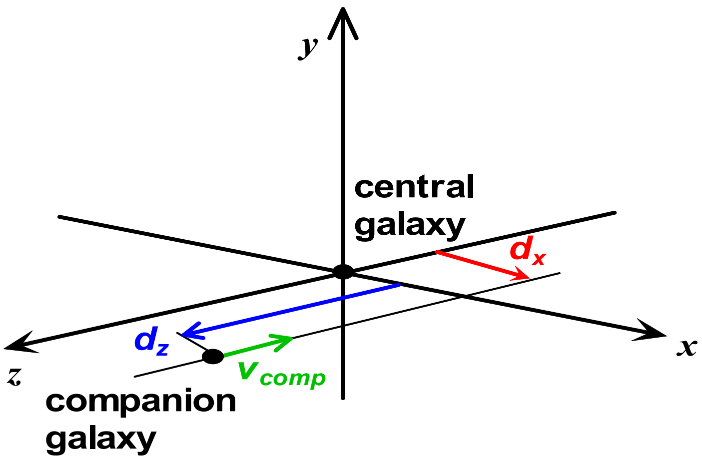


Figure 6–3: Parameters of the direct collision.

6.3.2 Keplerian encounter

Galaxies (centers-of-mass) are positioned on a two-body orbit that point particles of the same mass would follow. Keplerian encounter is described by the following orbital elements:

- A numerical eccentricity ε is an eccentricity of a conic section that is followed by the satellite galaxy. A circle has an eccentricity of zero; for an ellipse it is less than one; for a parabola it is equal to one; and for a hyperbola it is greater than one.
- A semi-major axis *a* (see Figure 6–4).
- A true anomaly *v* is an angle between the position of the satellite galaxy and its pericenter as seen from the central galaxy's CM situated in focus *F*.

A quantity relating the numerical eccentricity ε and the semi-major axis *a* is a linear eccentricity $e = \varepsilon \cdot a$.

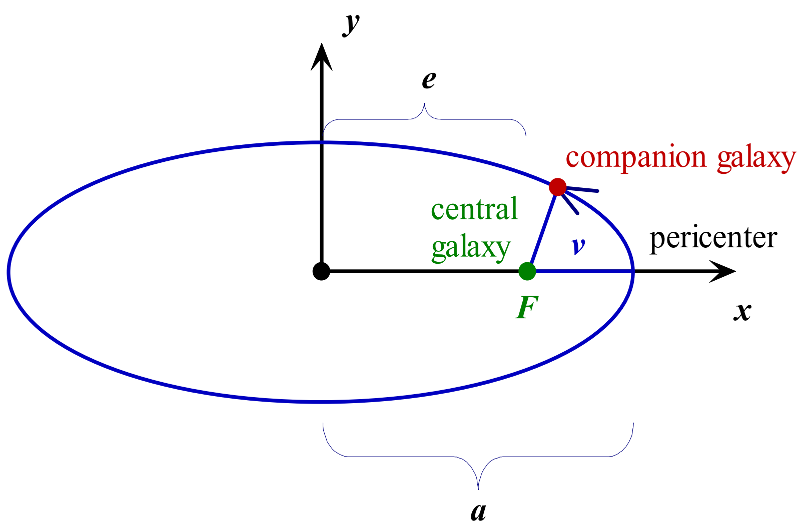


Figure 6–4: A schematic representation of initial conditions of two colliding galaxies in Keplerian elliptical encounter. The binary orbit lies in the *xy* plane. The central galaxy is located in focus *F* and the companion galaxy is located at the position given by the true anomaly *v*.

6.4 Mergers

If the relative speed of galaxies in the interacting system is high enough, they will only briefly meet and then will escape. An internal structure and orbits of galaxies will be only slightly affected. Otherwise, the system will merge.

6.4.1 Mergers of spherical galaxies

The simplest simulations of galaxy interaction involve spherical galaxies (González-García and Albada, 2005). As a trial simulation of galaxy interaction, we can set up two identical spherical galaxies on the direct collision course. Galaxies forming the interacting pair are created separately as described in Chapter 5. Spheroid is run in isolation for 1 Gyr (see Table 6-1) before being set into orbit.

Model	Resolution (star particles)	Timestep	Steps
Plummer	5,000	0.0005	30,000

Table 6-1: Parameters of spheroidal galaxy and its isolated evolution.

Initial conditions of interaction are evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 0.75$. Gravitational forces were calculated after smoothing the mass distribution using the softening length $\varepsilon = 0.0001$. The model is comparable to the interaction of a real-world dwarf spherical galaxy with the following radius, mass and Newton's gravitation constant

$$r = 5 \text{ kpc},$$
$$m = 5 \cdot 10^9 \ M_\odot,$$
$$G = 6.67 \cdot 10^{-11} \ \text{N} \cdot \text{m}^2 \cdot \text{kg}^{-2}.$$

The time unit is comparable to $74.6 \cdot 10^6 \text{ yr}$, the timestep to $37.3 \cdot 10^3 \text{ yr}$ and the overall simulation of interaction covers $3.9 \cdot 10^9 \text{ yr}$.

Run	d_z	d_x	v_{comp}	Total particles	Timestep	Steps
A	10.0	0.0	0.5	10,000	0.0005	105,000
B	10.0	0.3	0.5	10,000	0.0005	105,000
C	10.0	2.0	0.5	10,000	0.0005	105,000
D	10.0	5.0	0.5	10,000	0.0005	105,000

Table 6-2: Initial conditions for the direct interaction of spherical galaxies.

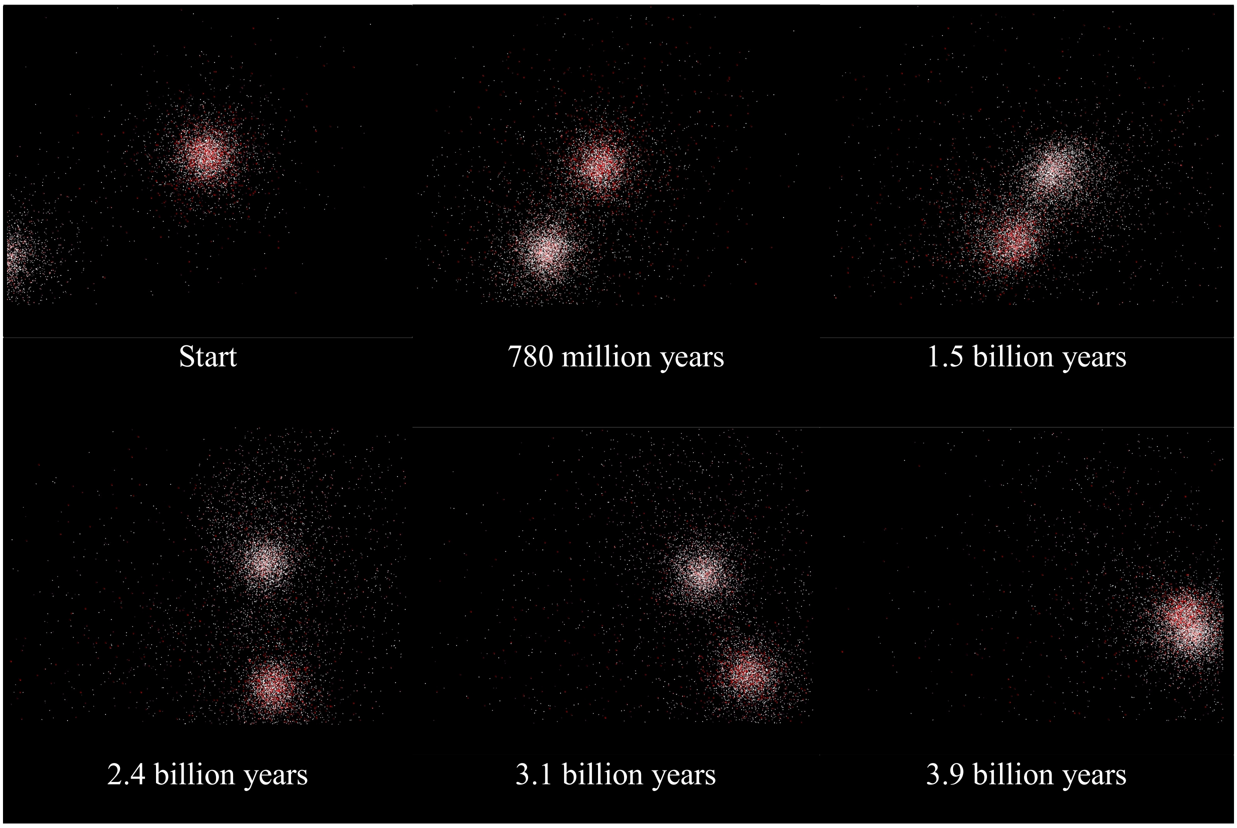
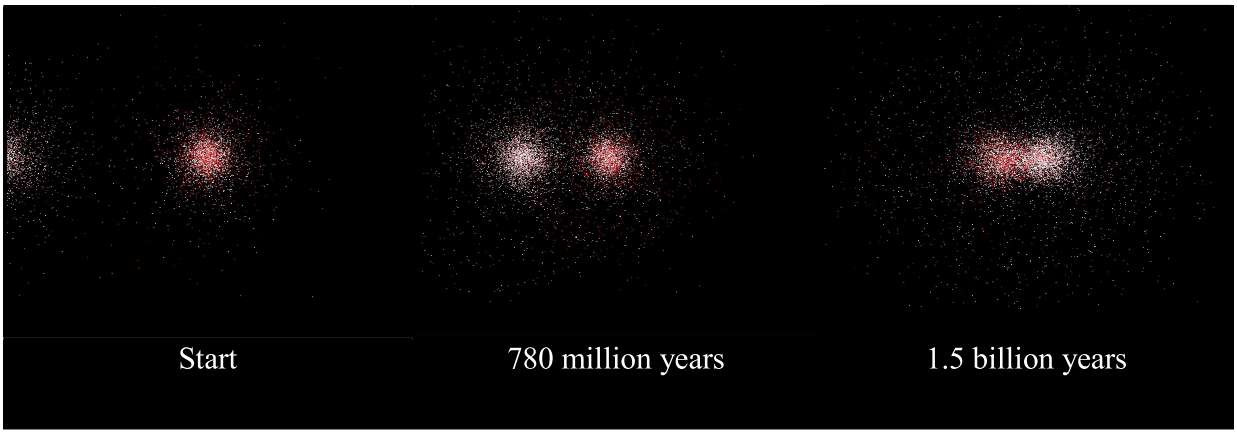


Figure 6–5: A time sequence of the “D run” of the direct collision of Plummer’s models in the *xz* plane evolved with the Barnes-Hut *N*-body algorithm.



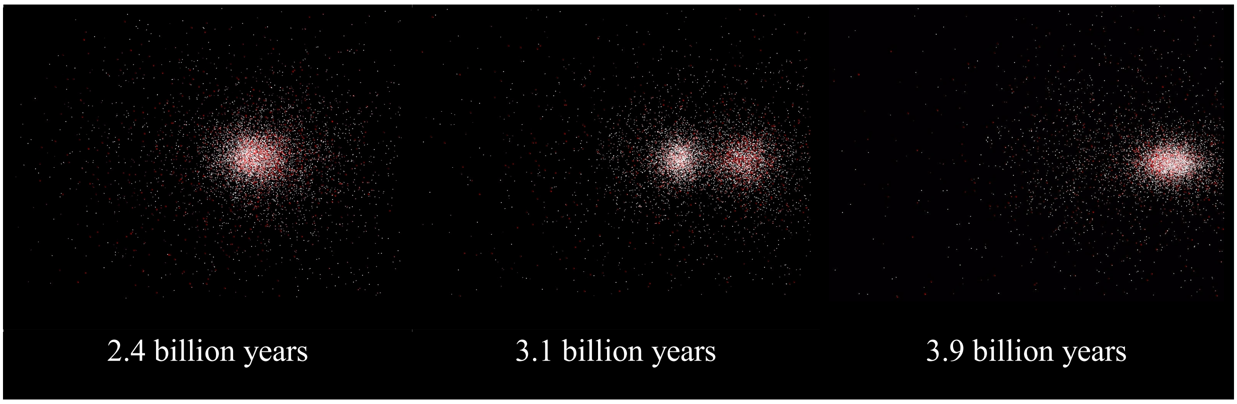


Figure 6–6: A time sequence of the "D run" of the direct collision of Plummer's models in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm.

Elliptical galaxies do not have an ordered motion of stars as spirals have. In spherical galaxies, which have a high anisotropy distribution in star velocities, it is harder to find spectacular tidal features. Merger remnants resulting from encounters of spheroids are again spherical galaxies.

6.4.2 Mergers of disk galaxies with satellites

Stars in spiral galaxies have ordered motion and so it is easier to find disturbances in such ordered motion caused by a tidal influence of the companion galaxy passing nearby.

An interaction of a large central galaxy interacting with a companion small satellite galaxy is called a *minor merger* (mass ratio of involved galaxies is smaller than 1/5). Here, the gravitational field of the more massive galaxy captures the smaller galaxy and gradually strips a material from this orbiting companion. As the satellite is stripped on its orbit, the stripped material is incorporated into the larger galaxy and loses its original identity as the satellite. The small galaxy is destroyed, while the massive galaxy retains its properties. This interaction is known under much more famous name *cannibalism*. Galactic cannibalism may strongly upset a disk structure of the central galaxy leading to the heatening of its stellar disk.

Dynamical friction

If a more massive body M is getting through a uniformly distributed sea of less massive bodies m, then bodies m are dislocated into a trailing wake behind the body M. In the wake is therefore an increased density of bodies m with respect to places outside the wake. Bodies m are accelerated and are gaining kinetic energy and momentum. Because of energy and momentum conservation laws in a system consisting of body M and bodies m, the body M is losing in energy and momentum, and suffers a steady deceleration. The body M is decelerated by this mechanism in the direction of its motion by the dynamical friction (Chandrasekhar, 1943).

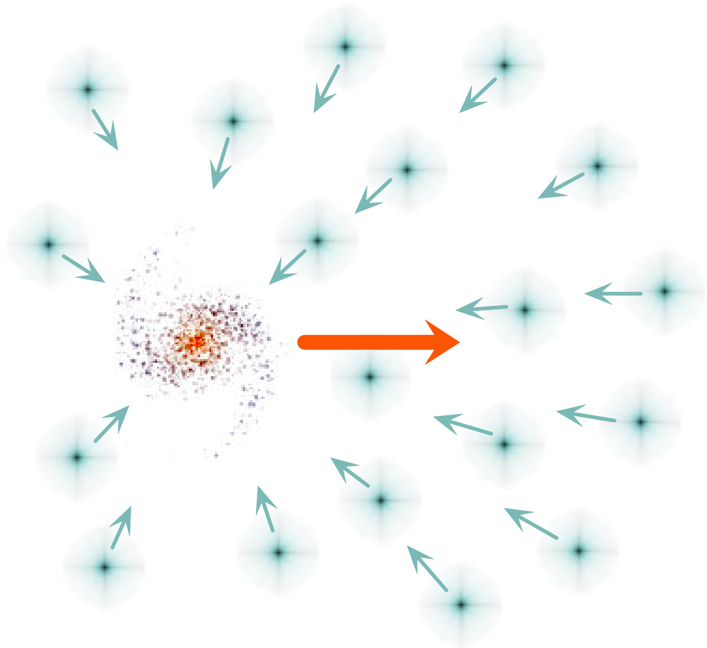


Figure 6–7: Smoothly distributed light particles (dark matter particles) attracted by the massive body (galaxy).

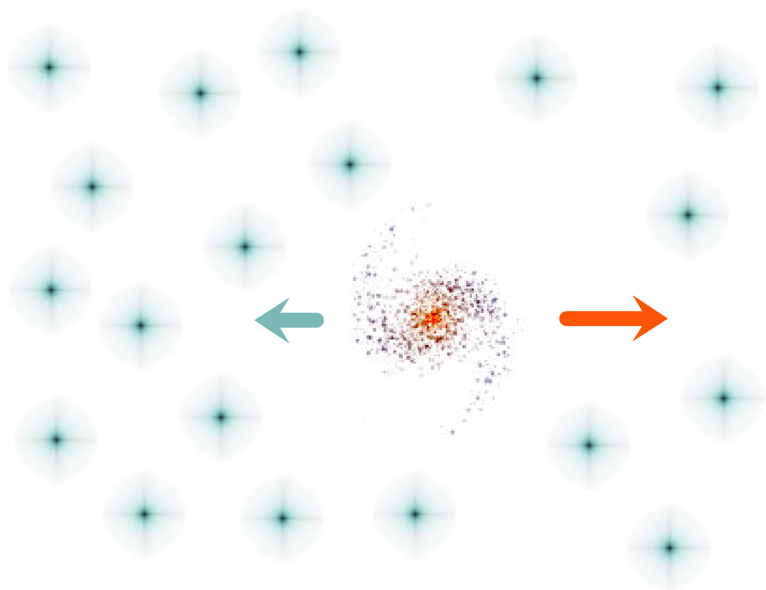


Figure 6–8: Light particles are dragged behind a massive body to form a „wake“. The massive body is decelerated.

A typical example of this mechanism is a dwarf galaxy flying through a dark matter halo (DMH) of a larger galaxy. The strong dynamical friction force slows down the moving dwarf galaxy that eventually merges with the large galaxy. The orbital velocity of the dwarf galaxy is reduced, causing it to spiral into the host galaxy (orbital decay). The extensive DMH and the dynamical friction force result in a more frequent merging between galaxies. DMH may extend many hundreds of kiloparsecs and can gravitationally interact and enhance the galaxy merger before and after the strong interaction of luminous parts of galaxies occur. Even when visible components of galaxies completely miss each other, their dark haloes may interact and lead the system to the merger.

Generic Models

A *generic model A* consists of the central Kuzmin's disk and the satellite Plummer's sphere that is set into orbit immediately without isolated evolution. Initial conditions of interaction are evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The collision is strongly off-center ($d_x = 0.8$).

Central disk		Satellite sphere		Direct collision		Evolution	
$M_{central}$	1.0	M_{comp}	0.005	d_z	1.0	Timestep	0.0005
$R_{central}$	1.0	R_{comp}	0.1	d_x	0.8	Steps	220,000
Q	1.5			v_{comp}	0.5		
$N_{central}$	4,000	N_{comp}	1,000				
Kuzmin		Plummer					

Table 6-3: Parameters of the generic model A.

With the scaling

$$
\begin{aligned}
[L] &= 10 \text{ kpc}, \\
[M] &= 10^{11}\ M_{\odot}, \\
[v] &= 300 \text{ km} \cdot \text{s}^{-1}, \\
[G] &= 6.67 \cdot 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}, \\
[t] &= 68.2 \cdot 10^6 \text{ yr}
\end{aligned}
$$

is the timestep equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $7.5 \cdot 10^9$ yr.

The simulation of generic model A did not lead to the merger of the disk and the sphere in the simulated time.

A *generic model B* consists again of the central Kuzmin's disk and the satellite Plummer's sphere that is set into orbit immediately. Initial conditions of interaction are again evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The collision is nearly perfectly polar ($d_x = 0.08$).

Central disk		Satellite sphere		Direct collision		Evolution	
$M_{central}$	1.0	M_{comp}	0.001	d_z	1.0	Timestep	0.0005
$R_{central}$	1.0	R_{comp}	0.1	d_x	0.08	Steps	220,000
Q	1.5			v_{comp}	0.5		
$N_{central}$	4,000	N_{comp}	1,000				
Kuzmin		Plummer					

Table 6-4: Parameters of the generic model B.

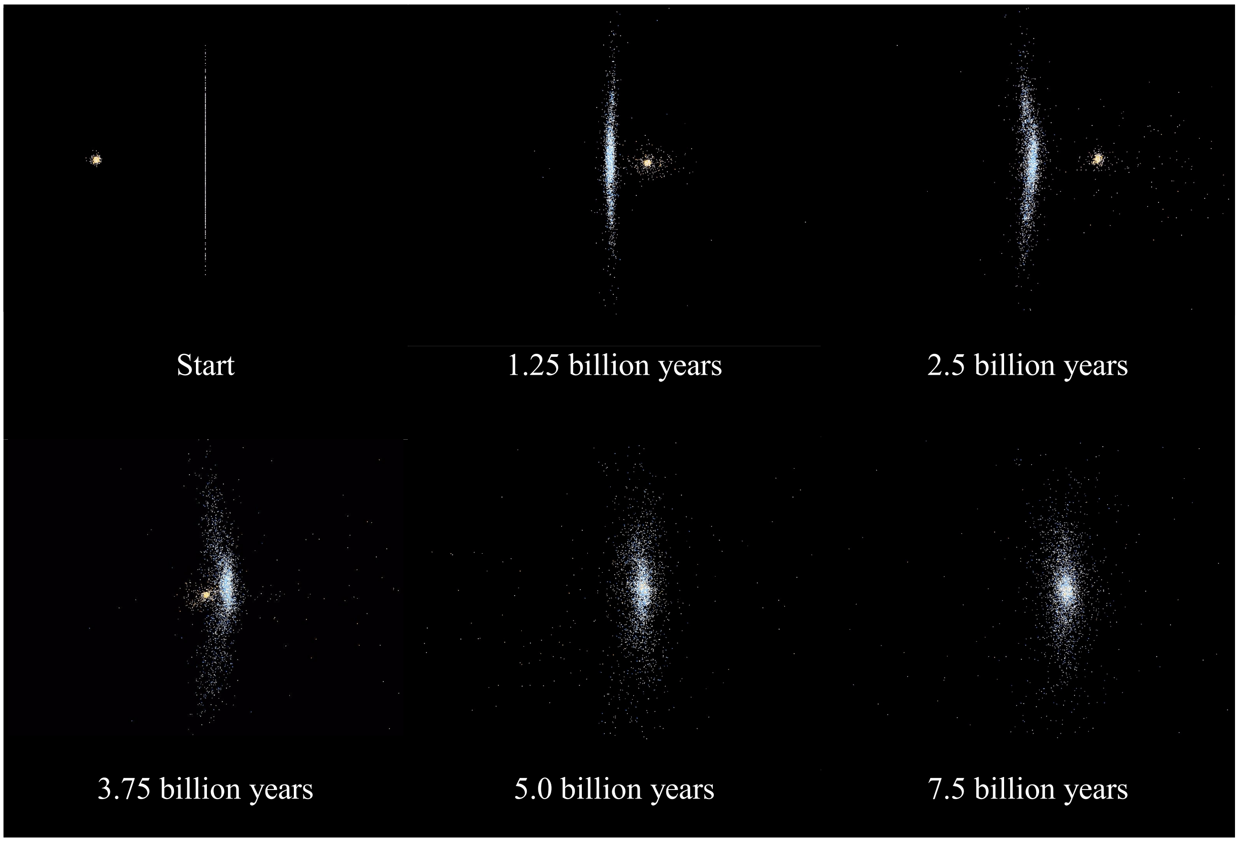


Figure 6–9: A time sequence for the "Generic model B" minor merger in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm.

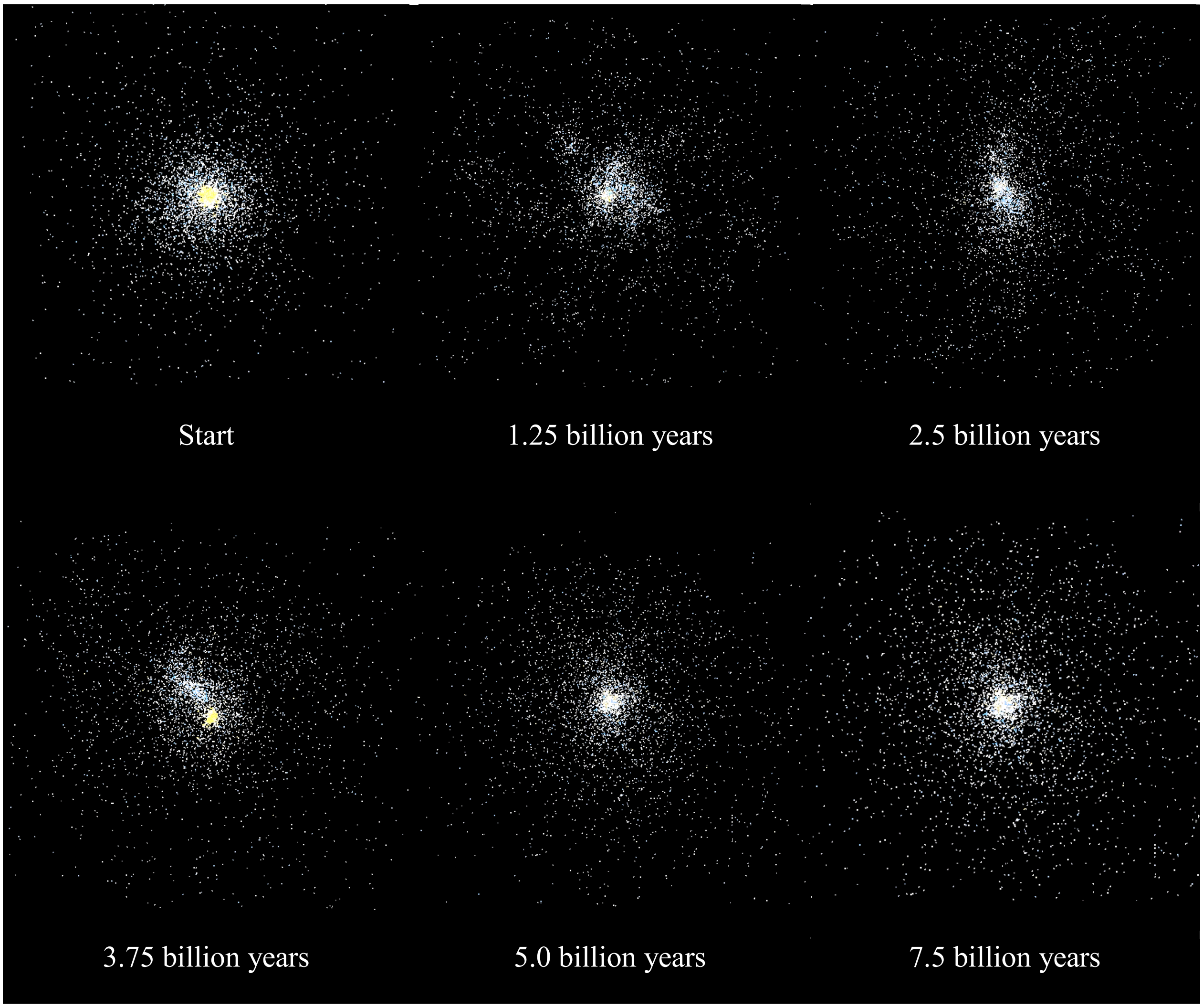


Figure 6–10: A time sequence for the "Generic model B" minor merger in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.

The simulation of the generic model B led to the merger of the disk and the sphere.

Milky Way galaxy system

Our own Milky Way galaxy is still in the process of galaxy evolution, growing through eating smaller companion galaxies. The Milky Way is currently accreting its small companions, the Magellanic Clouds and numerous nearby dwarf galaxies. The Milky Way's disk is thickening as a consequence of accretion of smaller companion galaxies (Buser, 2000).

Large and Small Magellanic Clouds

The largest companions of the Milky Way galaxy (MW) are the Large and Small Magellanic Clouds (LMC, SMC). These galaxies orbit the MW every few billion years. Tidal forces of the MW extracted from the Clouds a circumpolar stream of gas known as the Magellanic Stream that trails the LMC and SMC in their orbit around the MW and stretches over 100 degrees in the Southern Sky.

	LMC	SMC
mass within radius	$8.7 \cdot 10^9\ M_\odot$ †	$2.7 \cdot 10^9\ M_\odot$ ‡
radius	8.9 kpc†	3 kpc‡
position [kpc]	(–1.0; –40.7; –26.3)*	(14.8; –36.1; –41.9)*
velocity [$\mathrm{km \cdot s^{-1}}$]	(41.0; –200.0; –169.0)*	(60.0; –174.0; 173.0)*

Table 6-5: Mass, position and velocity of the Large and Small Magellanic Clouds. Parameters (†) from van der Marel et al. (2002), (‡) from Harris and Zaritsky (2006) and (*) from Kroupa and Bastian (1997).

Positions and velocities adopted for the LMC and SMC are given in Table 6-5 in Galacto-centric coordinates[60]. We will model the Clouds as Plummer's models with masses and radii given in Table 6-6. Initial conditions of interaction are evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The timestep is equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $3.58 \cdot 10^9$ yr.

Milky Way		SMC		LMC		Evolution	
$M_{central}$	10.0	M_{SMC}	0.007	M_{LMC}	0.01	Timestep	0.0005
$R_{central}$	3.0	R_{SMC}	0.3	R_{LMC}	0.9	Steps	105,000
Q	1.5						
$N_{central}$	5,000	N_{SMC}	2,500	N_{LMC}	2,500		
Kuzmin		Plummer		Plummer			

Table 6-6: Model parameters of the MW, SMC and LMC interaction.

[60] The coordinate system is centered on the Milky Way. The *x*-axis points from the Sun to the center of the Milky Way, the *z*-axis points to the North Galactic Pole (indicating height above the Galactic plane), and the third axis completes a right handed coordinate system.

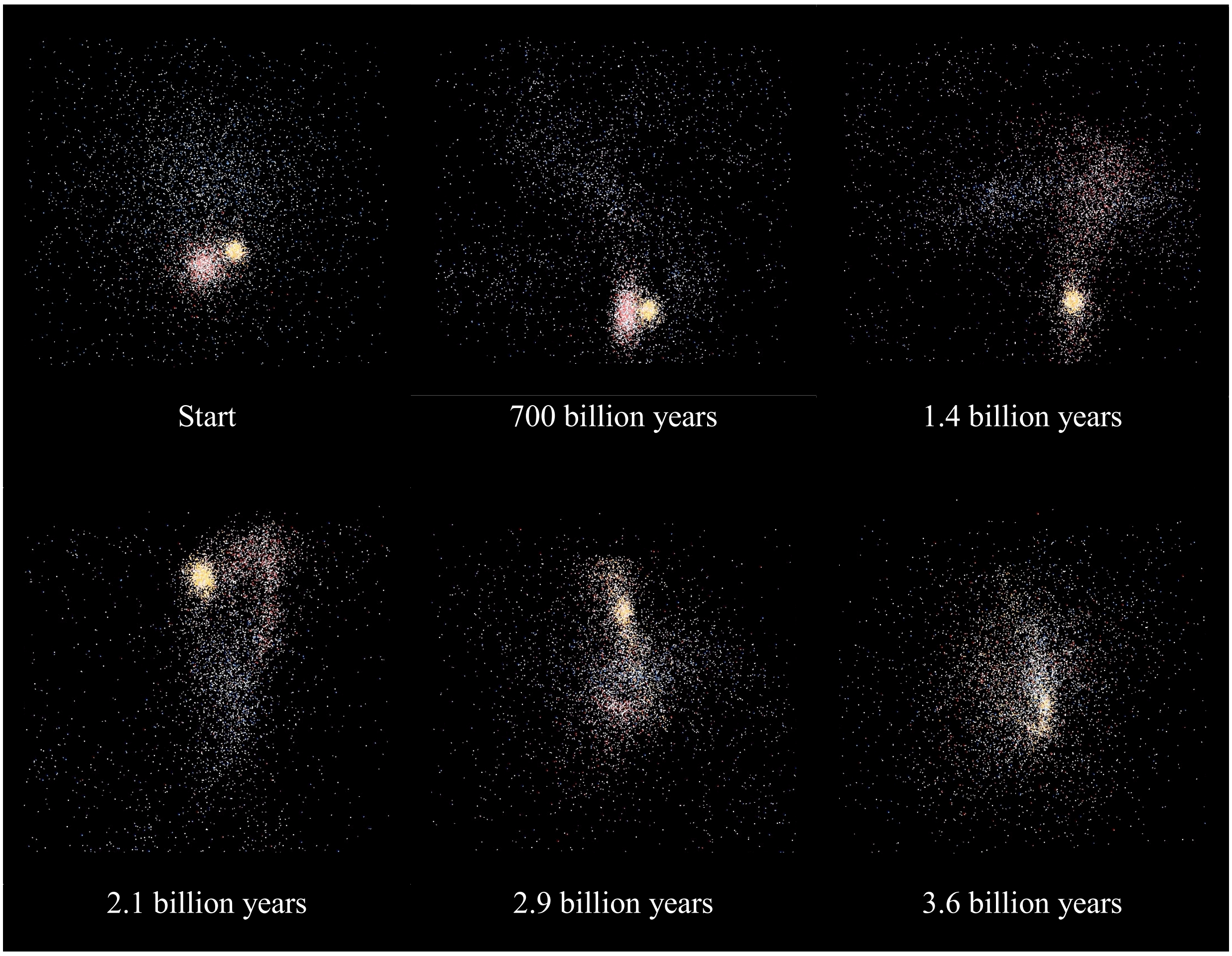
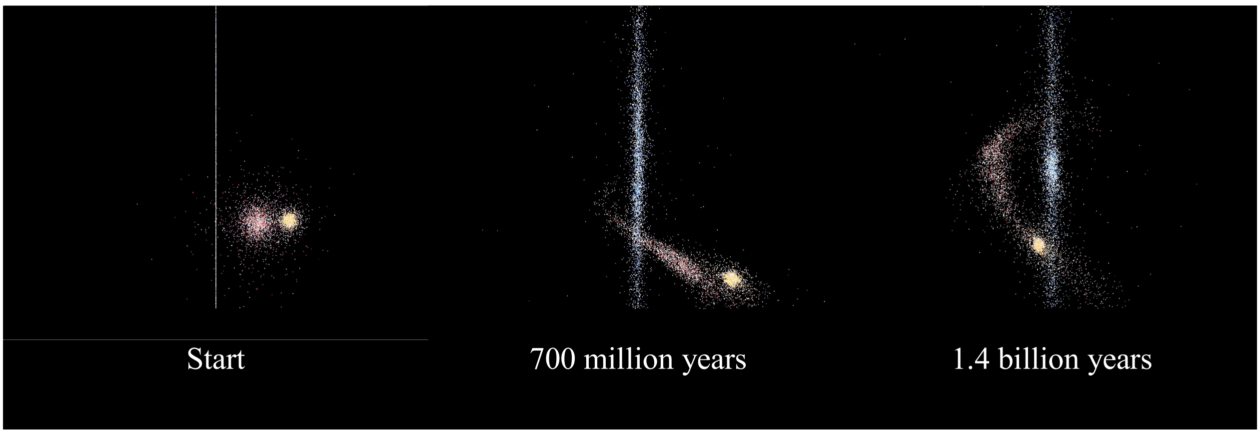


Figure 6–11: A time sequence for the Milky Way and the Small and Large Magellanic clouds merger in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.



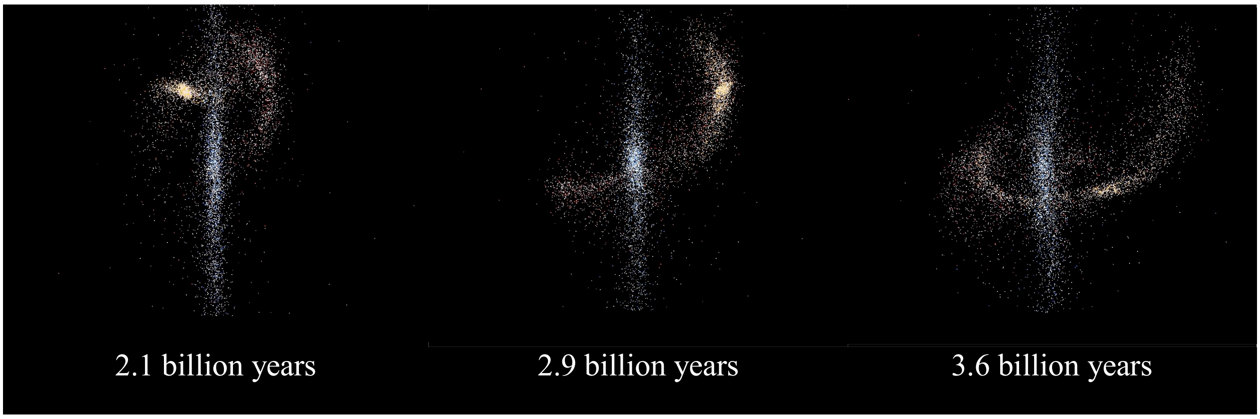


Figure 6–12: A time sequence for the Milky Way and the Small and Large Magellanic clouds merger in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm. The formation of a Magellanic Stream-like feature can be seen.

We can see that a lot of stars are being stripped from the Clouds (Magellanic Stream) and is incorporated into the MW's disk.

Sagittarius Dwarf Elliptical Galaxy

Sagittarius Dwarf Elliptical Galaxy (or SagDEG)[61] was discovered by Ibata et al. (1994). It is a small galaxy that is penetrating the disk of the MW. I have chosen to model SagDEG interaction with the MW according to parameters given in tables 6–7 and 6–8. Initial conditions of interaction were evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The timestep is equal to $34.1 \cdot 10^{3}\ \mathrm{yr}$ and the overall simulation covers $4.7 \cdot 10^{9}\ \mathrm{yr}$.

SagDEG	
position [kpc]	(16.2; 2.3; –5.9)
velocity [km·s^{-1}]	(238.0; –42.0; –222.0)

Table 6-7: Position and velocity of Sagittarius Dwarf Elliptical Galaxy in Galacto-centric coordinates (Read and Moore, 2005).

[61] Sagittarius Dwarf Elliptical Galaxy should not be confused with Sagittarius Dwarf Irregular Galaxy (SagDIG) discovered in 1977 and located at a distance 4.2 million light years from the Milky Way. SagDIG is not a satellite of the Milky Way galaxy, rather it is the satellite of the whole Local Group.

Milky Way		SagDEG		Evolution	
$M_{central}$	10.0	M_{sag}	0.001	Timestep	0.0005
$R_{central}$	3.0	R_{sag}	0.1	Steps	140,000
Q	1.5				
$N_{central}$	5,000	N_{sag}	2,500		
Kuzmin		Plummer			

Table 6-8: Model parameters of the MW and SagDEG interaction.

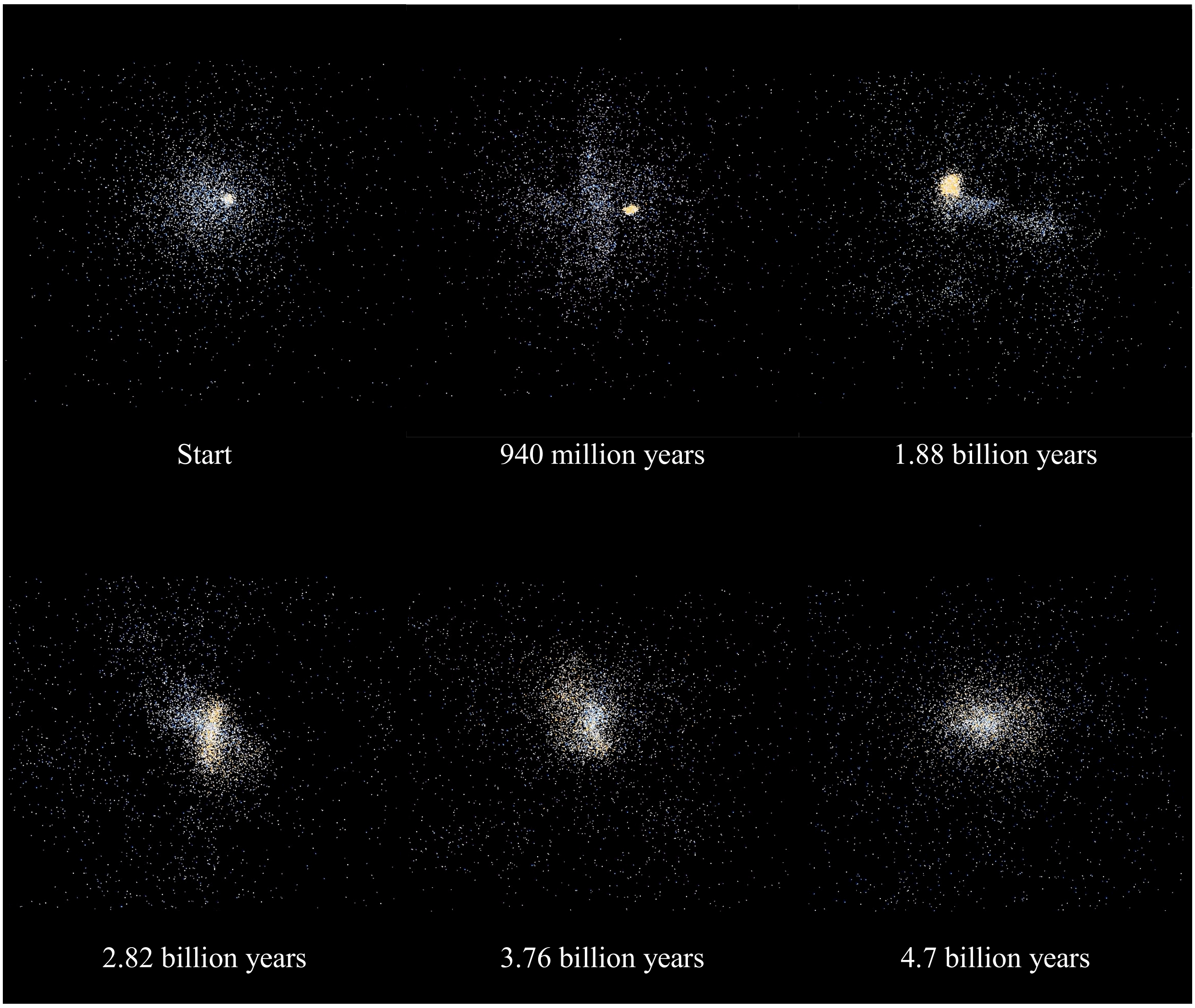


Figure 6–13: A time sequence of the SagDEG evolution in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.

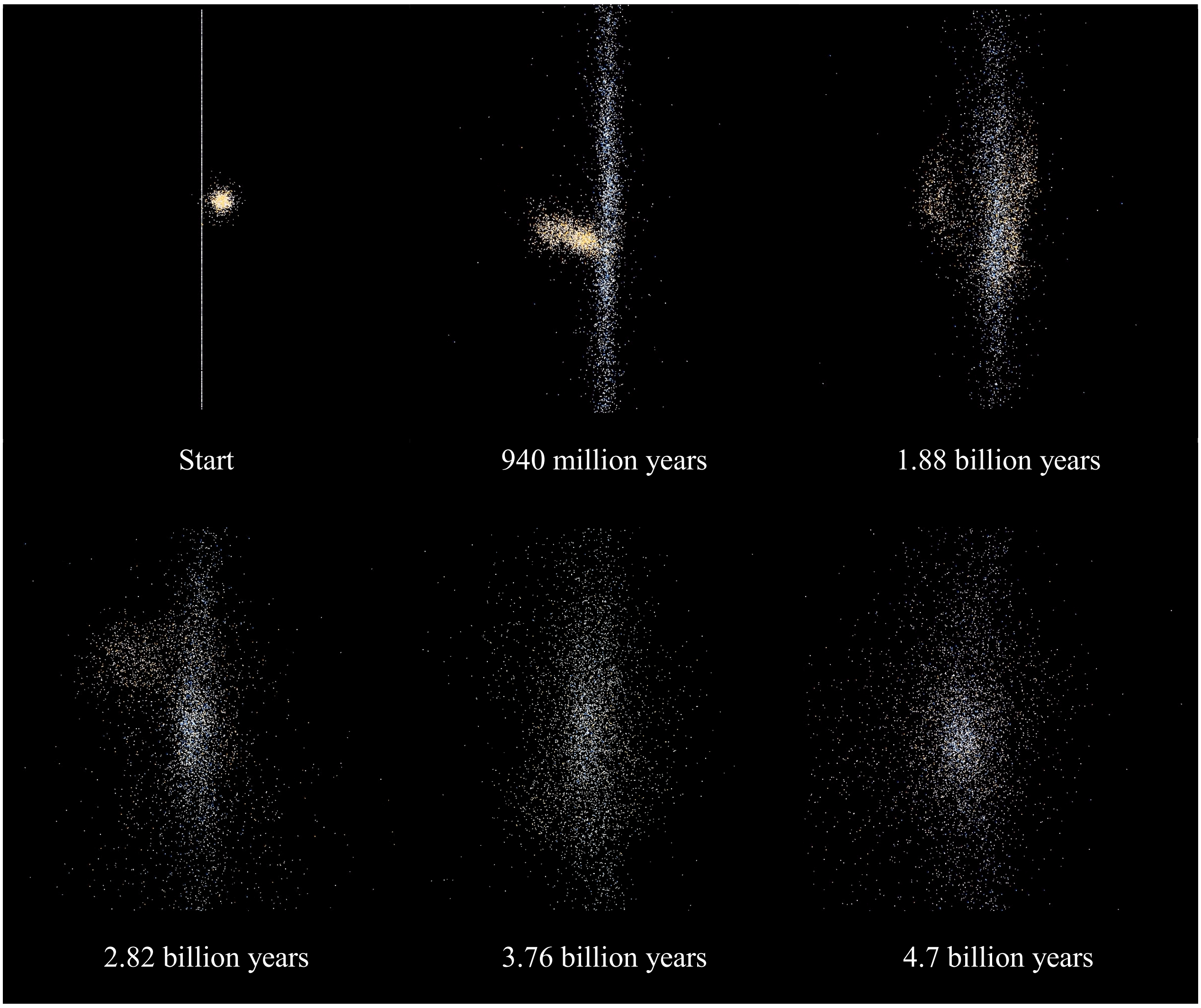


Figure 6–14: A time sequence of the SagDEG evolution in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm.

Future of the Milky Way

Recent discoveries of new dwarf spheroidal galaxies (dSph) around the MW (Willman et al. 2005, Zucker et al. 2006, Belokurov et al. 2006, Belokurov et al. 2007) with more satellites waiting to be discovered are posing a question about a future evolution of the MW. It is now known that dwarf spheroidals are the most common type of galaxies in the universe.

We will create *N*-body model of the whole MW system to see its future. Three-dimensional Cartesian positions of satellite galaxies relative to the center of the MW were calculated from given papers (see Table 6–10) as follows:

No	Name	l [°]	b [°]	r [kpc]	Source	x [kpc]	y [kpc]	z [kpc]
1	LMC	–	–	–	Kroupa and Bastian (1997)	-1.0	-40.7	-26.3
2	SMC	–	–	–	Kroupa and Bastian (1997)	14.8	-36.1	-41.9
3	SagDEG	8.8	-21.5	16.0	Lee and Kim (2000)	6.2	2.3	-5.9
4	Ursa Minor	105.8	44.8	66.0	Mateo (1998)	-21.3	45.1	46.5
5	Sculptor	287.5	-83.2	79.0	Mateo (1998)	-5.7	-8.9	-78.4
6	Draco	86.4	34.7	82.0	Mateo (1998)	-4.3	67.3	46.7
7	Sextans	243.5	42.3	86.0	Mateo (1998)	-36.9	-56.9	57.9
8	Carina	260.1	-22.2	101.0	Mateo (1998)	-24.6	-92.1	-38.2
9	Fornax	237.1	-65.7	138.0	Mateo (1998)	-39.3	-47.7	-125.8
10	Leo I	226.0	49.1	250.0	Mateo (1998)	-122.2	-117.7	189.0
11	Leo II	220.2	67.2	205.0	Mateo (1998)	-69.2	-51.3	189.0
12	CVn	86.9	80.2	219.8	Lee and Kim (2000)	-6.5	37.4	216.6
13	Ursa Major	162.0	50.8	104.9	Willman et al. (2005)	-71.6	20.5	81.3
14	Boötes	358.1	69.6	60.0	Belokurov et al. (2006)	12.4	-0.7	56.2
15	Canes Venatici I	74.3	79.8	220.0	Zucker et al. (2006)	2.0	37.5	216.5
16	Coma Berenices	241.9	83.6	44.0	Belokurov et al. (2007)	-10.8	-4.3	43.7
17	Canes Venatici II	113.6	82.7	150.0	Belokurov et al. (2007)	-16.1	17.5	148.8
18	Hercules	28.7	36.9	140.0	Belokurov et al. (2007)	89.7	53.8	84.1
19	Leo IV	265.4	56.5	160.0	Belokurov et al. (2007)	-15.6	-88.0	133.4

Table 6-9: Dwarf satellites of the Milky Way and their corresponding Galactic coordinates $\left(x, y, z\right)$.

$$
\begin{aligned}
x &= r \cdot \cos b \cdot \cos l - R_{\odot} \\
y &= r \cdot \cos b \cdot \sin l \\
z &= r \cdot \sin b,
\end{aligned}
\tag{6.1}
$$

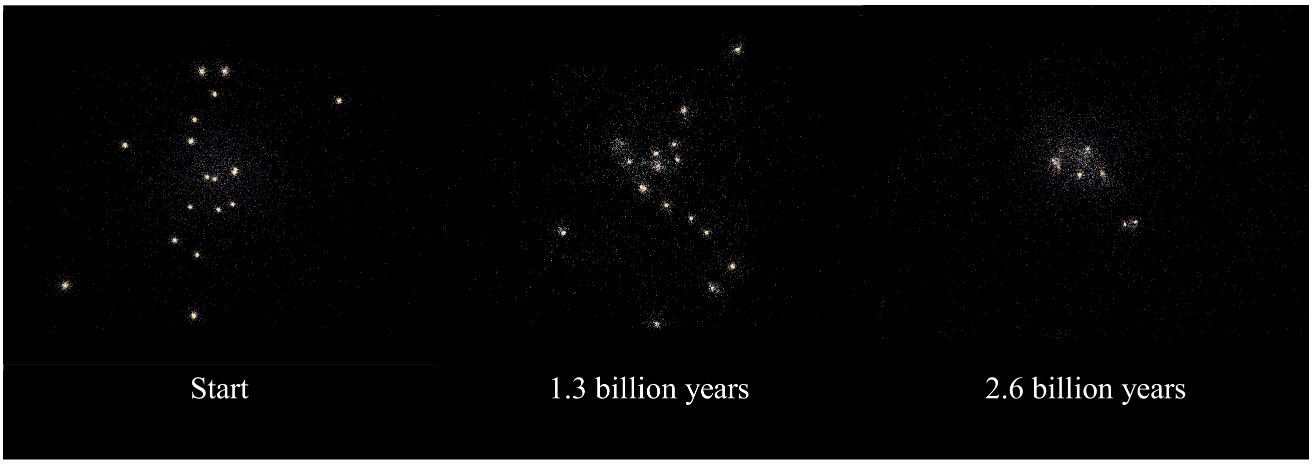
where r denotes the observed distances from the MW, (l,b) are the Galactic longitude and latitude, and $R_{\odot} = 8.5\ \mathrm{kpc}$ is the distance between the Sun and the center of the MW.

Difficulties faced by observational astronomers who derive orbital parameters of real galaxies are enormous. As the result, only radial velocities of galaxies are usually known and no other orbital and dynamical parameters are tabulated with an adequate precision. I assigned to every individual MW's satellite random velocity in the range of $\left(\frac{1}{2} \div 1\right)$ of its circular velocity.

Satellite galaxies were set-up as Plummer's spheres. Parameters of model galaxies are given in Table 6-10. Initial conditions of interaction are evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The timestep is equal to $34.1 \cdot 10^3\ \mathrm{yr}$ and the overall simulation covers $6.4 \cdot 10^9\ \mathrm{yr}$.

Milky Way		Satellites		Evolution	
$M_{central}$	10.0	M_{sat}	0.001	Timestep	0.0005
$R_{central}$	3.0	R_{sat}	0.1	Steps	187,000
Q	1.5				
$N_{central}$	3,000	N_{sat}	400		
Kuzmin		Plummer			

Table 6-10: Model parameters of the MW/satellites interaction.



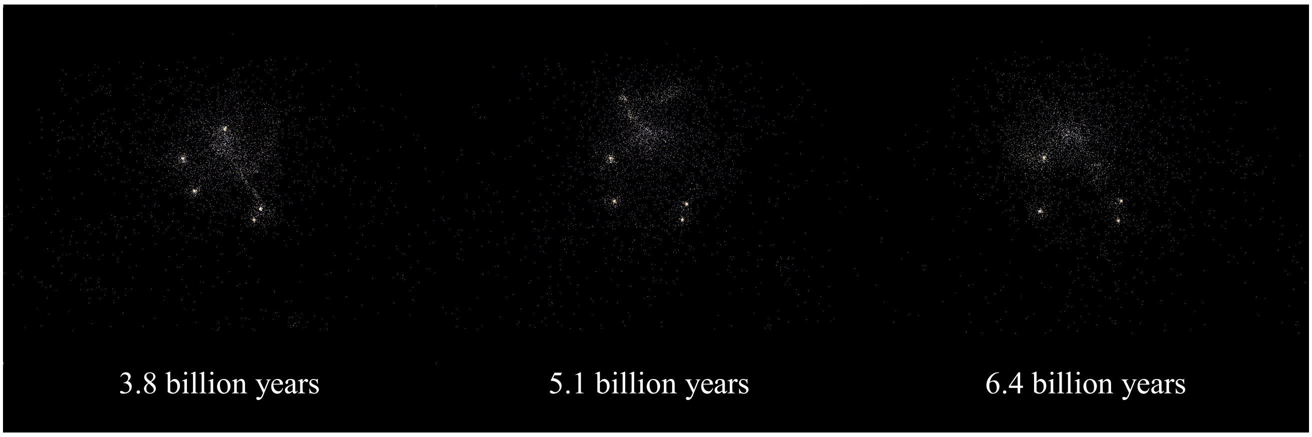


Figure 6–15: A time sequence for the evolution of the Milky Way and her satellite galaxies in *xy* plane evolved with the Barnes-Hut *N*-body algorithm.

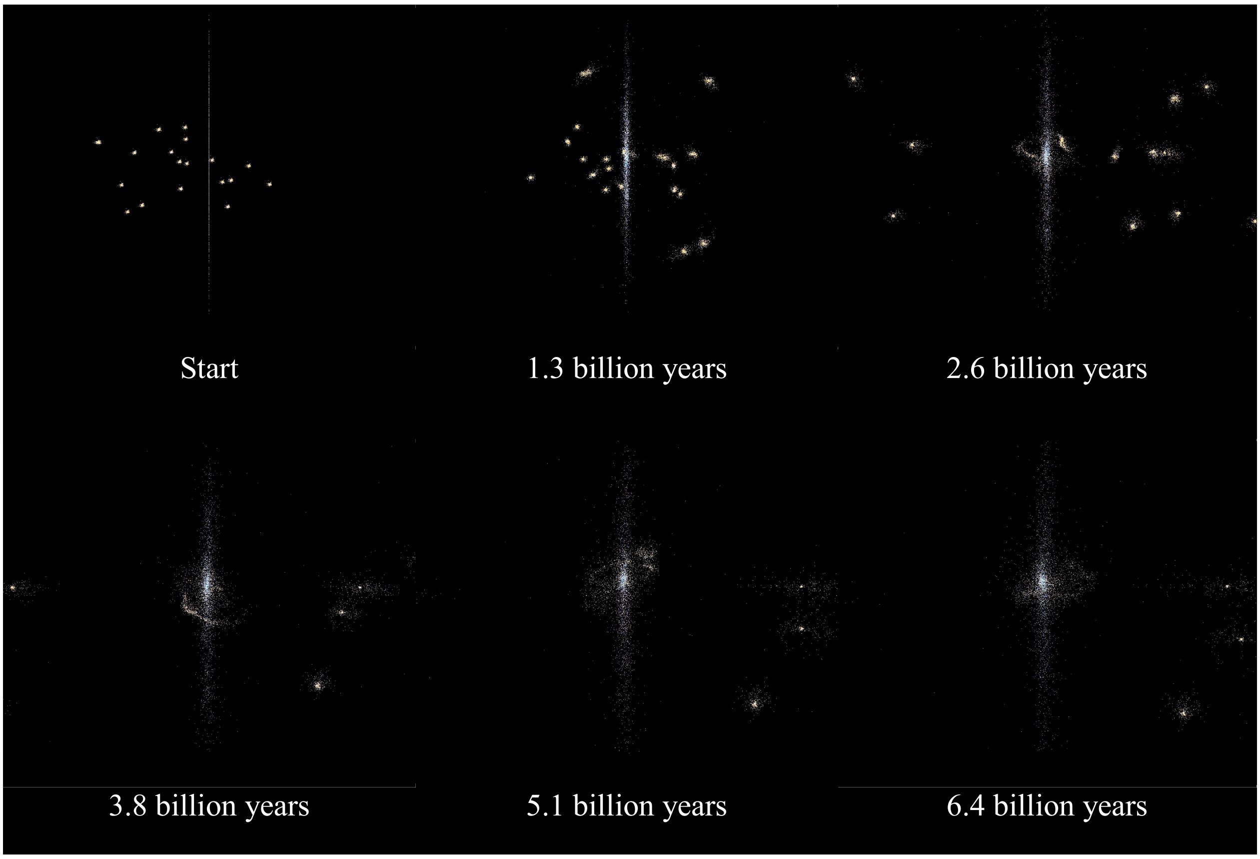


Figure 6–16: A time sequence for the evolution of the Milky Way and her satellite galaxies in the *yz* plane evolved with the Barnes-Hut *N*-body algorithm.

We can see that some galaxy satellites might merge with the Milky Way galaxy in several billions years. The process of building larger galaxies from smaller protogalaxies through a gravitational assembly can be seen. We might imagine that the whole Milky Way system will merge after all.

6.4.3 Mergers of two disk galaxies

Mergers of two large, roughly equal-mass disk galaxies are called *major mergers* (mass ratios of involved galaxies is in the range of 1/1 to about 1/4). These major mergers are less common than minor mergers. During the close encounter is the orbital energy converted into internal energy and causes two progenitor systems to sink together into an oblate galaxy with random stellar motions. Major mergers remnants are typically pressure supported and resembles elliptical galaxy. Original disks are usually completely destroyed.

Tidal forces

Galaxies are extremely large objects so that the gravitation is not influencing all parts of galaxies with the same force during a close encounter. When one galaxy (A) is directly over a given point of another galaxy (B), it exerts a powerful pull on stars at that point, and the bridge of the stars is pulled out of the galaxy (B) to the galaxy (A). At the same time, the material on the opposite side of the galaxy (B) bulges outward and forms a tail because of the centrifugal force of the revolving A-B galaxy system. The orbital energy is transferred to internal motions of stars within galaxies. Thus, there is always one bridge and one tail in the galaxy (B) at any given time: the area closest to the galaxy (A) and the area opposite the galaxy (A). The same is doing the galaxy (B) to the galaxy (A). This differential gravitation (tides) stretches the system radially. In the same manner, the Moon raises bulges on the surface of the Earth, and is responsible for our oceanic tides.

Fate of the Local Group

The Local Group of galaxies is an ensemble of two main galaxies, the Milky Way galaxy and the Andromeda galaxy, their satellites and some other galaxies bound to the whole Local Group system. We already realized that small companions will merge with their central galaxy through the process of minor merger.

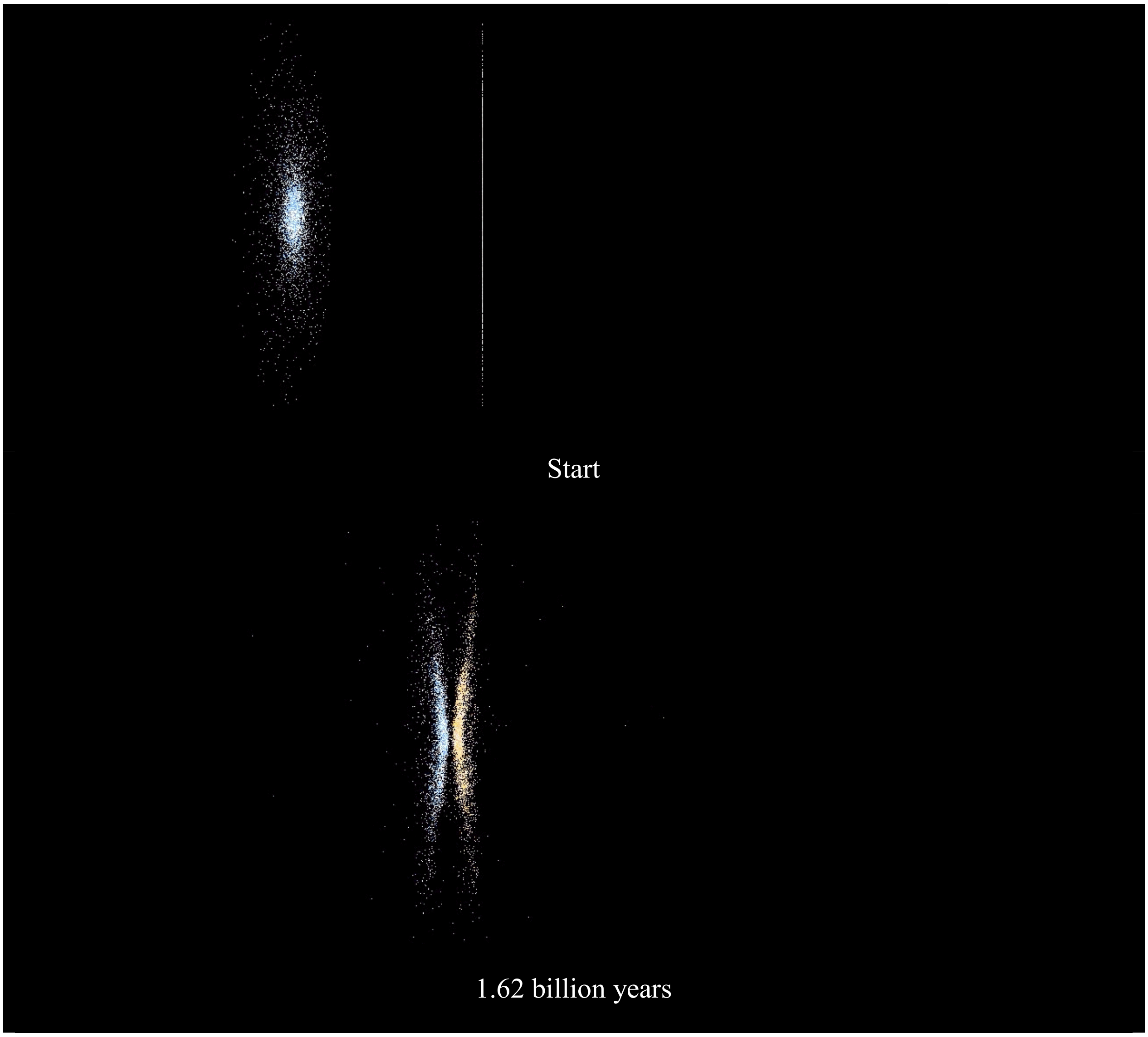
Our objective will be to model the future evolution of the MW and Andromeda galaxies. Andromeda galaxy Messier 31 (M31) is the nearest major spiral galaxy to the MW. Recent discoveries revealed that its disk radius is 60 kpc, total galaxy visible mass $M = 1.1 \cdot 10^{12}\ M_{\odot}$ (Brunthaler et al., 2006), bulge radius 30 kpc, and halo radius 165 kpc (Gilbert et al., 2006).

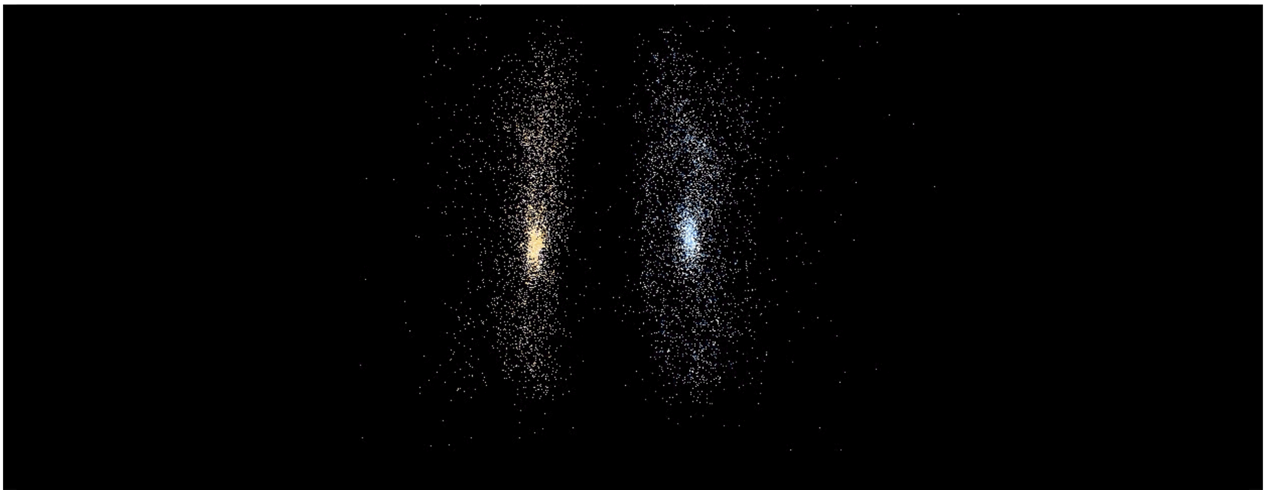
The Andromeda galaxy is moving toward the MW with a velocity of about 130 km/s (Rubin, 1998). It is now 730 kpc from us and is probably going to crash into the MW in about 10^{10} years. It is unclear, whether the collision will be central or off-central. I simulated their interaction as the direct collision of two same coplanar disk.

Milky Way		Andromeda		Direct collision		Evolution	
M_{MW}	1.0	M_{And}	1.0	d_z	10.0	Timestep	0.0005
R_{MW}	1.0	R_{And}	1.0	d_x	0.0	Steps	238,000
Q_{MW}	1.5	Q_{And}	1.5	v_{comp}	0.5		
N_{MW}	5,000	N_{And}	5,000				
Kuzmin		Kuzmin					

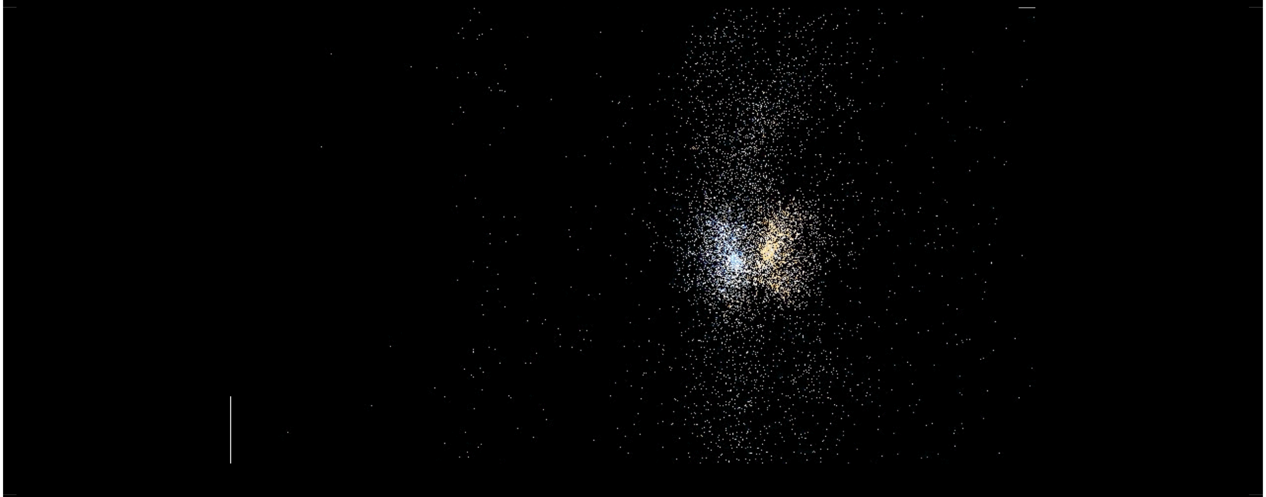
Table 6-11: Parameters of the Milky Way galaxy and Andromeda galaxy collision.

Parameters of model galaxies are given in Table 6–11. Initial conditions of interaction are evolved with the Barnes-Hut N-body simulation code with the opening angle $\theta = 1.0$. The timestep is equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $8.1 \cdot 10^9$ yr.





3.24 billion years



4.86 billion years

6.48 billion years

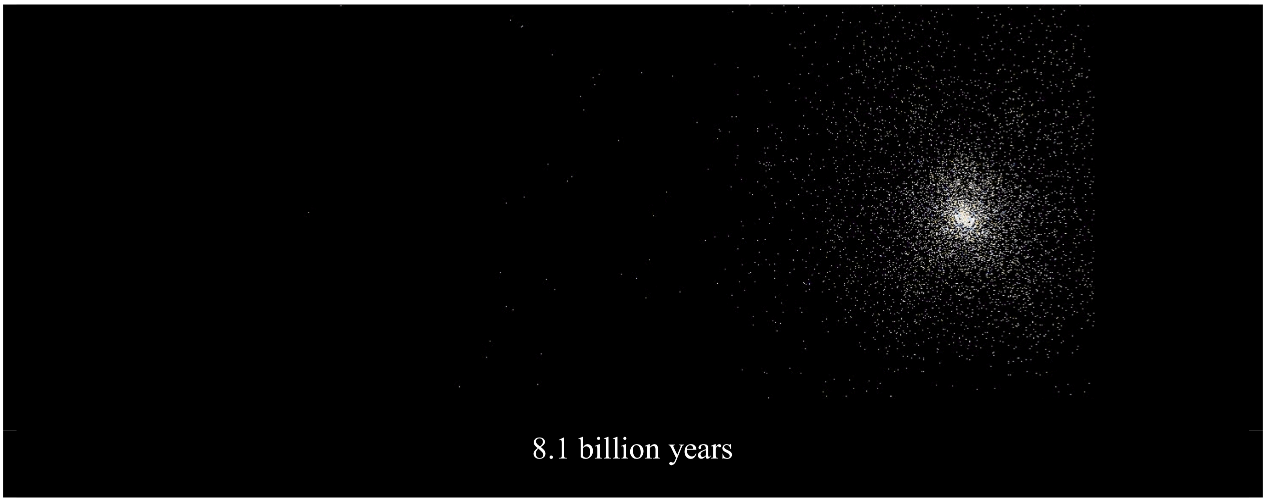


Figure 6–17: A time sequence for the merger of two same mass galaxies in the *xy* plane evolved with the Barnes-Hut *N*-body algorithm.

6.5 Modeling galaxy harassment

Mergers are occurring between galaxies at relatively low velocities. Galaxy harassment is a galaxy encounter at velocities greater than are circular velocities of stars in galaxies. These rapid frequent high-velocity fly-by encounters came about in regions with a high density of galaxies at relative velocities of 1,500 km/s (Dressler et al., 1994). In the area comparable in extent to the distance between the MW and Andromeda galaxies were observed hundreds of galaxies moving with relative speeds of several thousand km/s (Moore et al., 1996).

We have modeled galaxy harassment with one disk galaxy and seventy-five dwarf spherical galaxies that may have origin in initial gravitational collapses of galaxy protoclouds. We distributed dwarf galaxies in the cubic volume of 120 kpc one side around the central disk galaxy. Velocities ranged up to 4,000 km/s. Galaxy models are given in Table 6–12.

Disk galaxy		dSphs		Evolution	
$M_{central}$	1.0	M_{dSph}	0.1	Timestep	0.0005
$R_{central}$	1.0	R_{dSph}	0.1	Steps	30,000
Q	1.5				
$N_{central}$	4,000	N_{dSph}	70		
Kuzmin		Plummer			

Table 6-12: Parameters of disk galaxy, dwarf spherical galaxies and evolution in the galaxy harassment simulation.

Initial conditions of interaction are evolved with the Barnes-Hut *N*-body simulation code with the opening angle $\theta = 1.0$. The timestep is equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $1.02 \cdot 10^9$ yr.

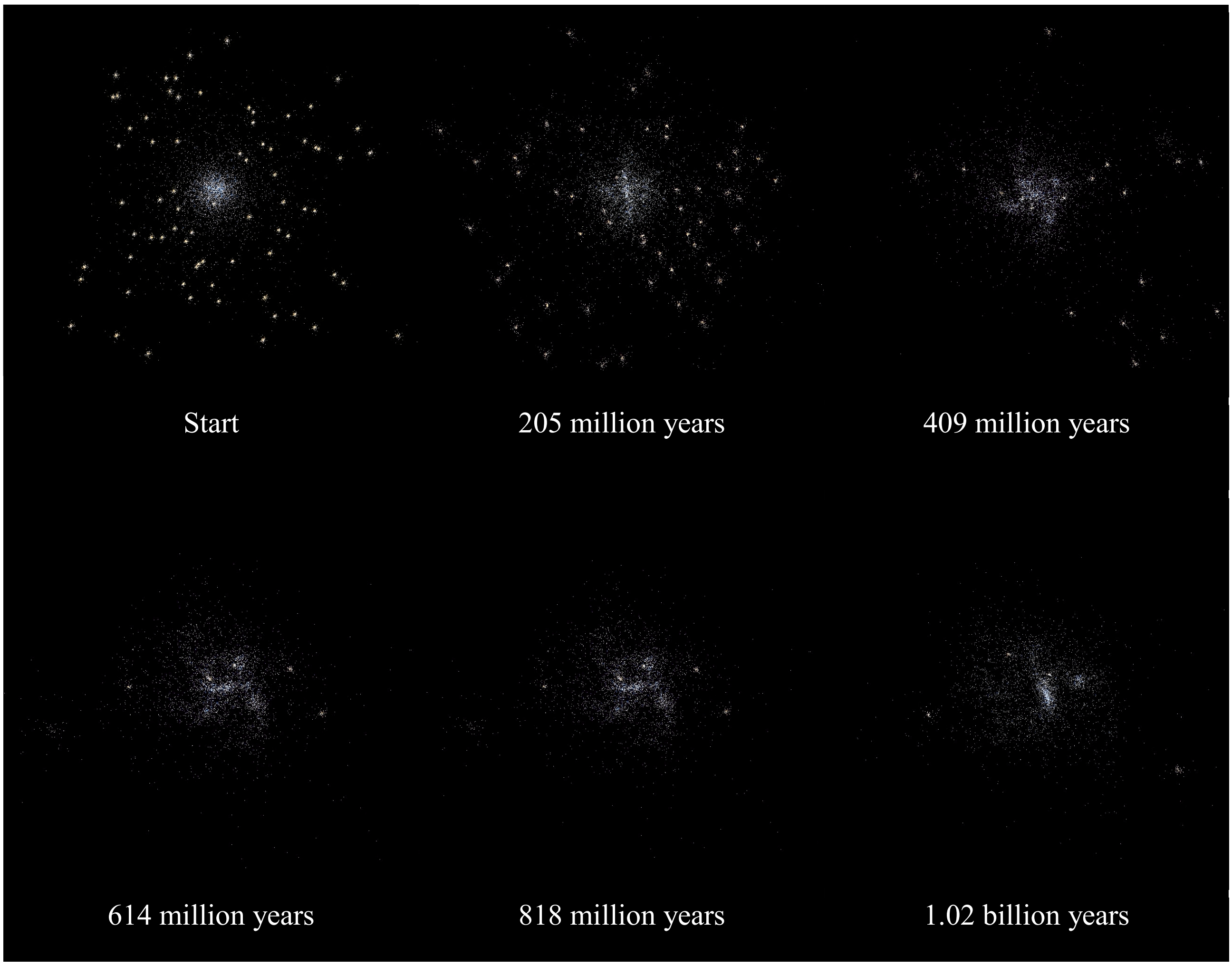


Figure 6–18: A time sequence for the galaxy harassment evolution in the *xy* plane simulated with the Barnes-Hut *N*-body algorithm.

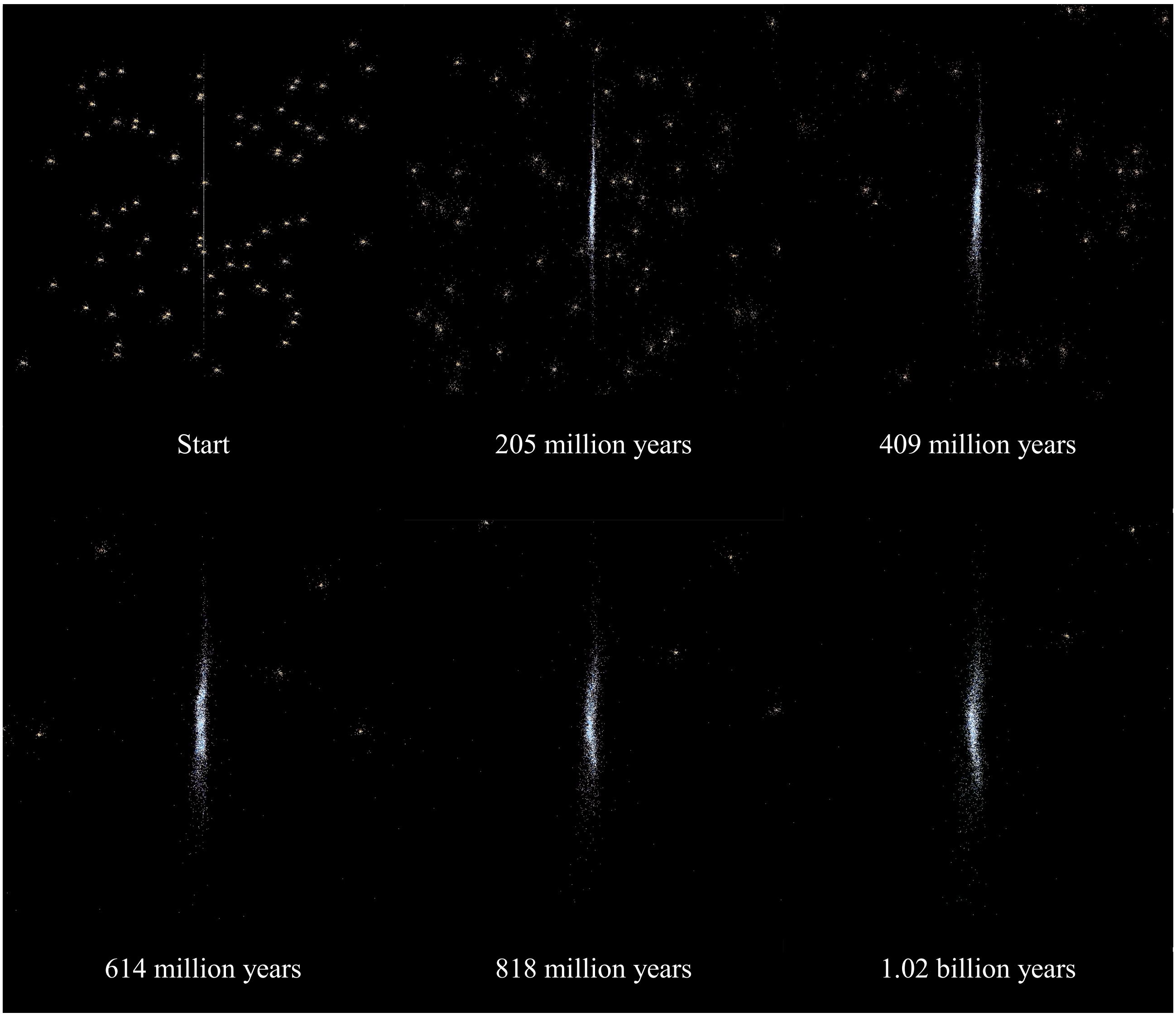


Figure 6–19: A time sequence for the galaxy harassment evolution in the *yz* plane simulated with the Barnes-Hut *N*-body algorithm.

It can be seen that the evolution of galaxy exposed to the galaxy harassment is chaotic and violent. Galaxy harassment warms galaxy disk and multiple small interactions over time strongly affect the disk of central galaxy (Figure 6-19).

6.6 Concluding remarks

In Chapter 6, we have demonstrated how to study galaxy collisions and mergers with computer simulations. Interacting galaxies are very complex and highly dynamic systems. With modest computational resources, we performed computer simulations of galaxy interactions that are in excellent agreement with observations. These simulations provide an accurate and entertaining insight into galaxy collisions and mergers.

However, model results were not without their shortcomings. Our aim was not to study interactions in detail, but to show how such study can be done with all details provided. We studied the evolution of spherical galaxy interactions, minor and major mergers, and galaxy harassment.

We simulated the evolution of the Milky Way galaxy, the Large and the Small Magellanic Clouds and all 19 known satellite galaxies of the Milky Way. We have simulated the future evolution of the Local Group in the collision of two disk galaxies representing the Andromeda galaxy (M31) and the Milky Way galaxy. Models were evolved for up to 8.1 billion years and movies from all computations were produced.

"By about 1880 physics was serene; most phenomena could be explained by Newtonian mechanics, Maxwell's electromagnetic theory, thermodynamics, and Boltzmann's statistical mechanics. Only a few problems appeared unsolved."
— Encarta Encyclopedia (2006) —

7

MODIFICATION OF GRAVITY MODEL

In this chapter, we will show how to prepare our simulation for alternative gravity model. Explaining rotation curves at large galactic radii remains an open question in galaxy dynamics. Visible matter itself under the influence of Newtonian gravity and dynamics cannot explain the rotation of visible material in the galaxy. A lot of dark matter (DM) must be introduced into calculations to match the observational evidence of flat rotation curve in disk galaxies. This nonluminous material cannot be directly detected by observing any form of electromagnetic radiation and its existence is suggested by theoretical considerations.

7.1 Standard model

The Newtonian theory and Einstein's theory of General Relativity (GR) had been well tested on a solar system scale and form the part of established physics. Experimentally known forms of matter under the assumption of the validity of established physics on scales greater than solar system cannot explain

- a disk and some elliptical galaxy rotation (Romanowsky, 2006),
- galaxies grouped together into clusters (large scale structure of the universe, LSS),
- the extent of the X-ray-emitting region of gas and the temperature of the gas in galaxy clusters,

- an amount of light from a background cluster of galaxies bended by another cluster of galaxies in the foreground (through gravitational lensing) and
- the shape of anisotropy spectrum of cosmic microwave background (CMB) radiation.

These problems can be roughly explained by introducing an additional matter that does not radiate. The dark matter is partially composed of gas, dust, planets, brown dwarfs, black holes, and other ordinary (baryonic) matter that is very low-radiating and is bellow detection abilities of observational astronomy. However, cosmology constrains the amount of this baryonic matter in the universe to about 20 % of overall mass content of the universe. Instead, according to the standard model, the most of the matter in the universe must be of non-baryonic origin. In the following text, I will use the term "dark matter" as substitution for the "non-baryonic dark matter".

Several issues complicate a picture with the (non-baryonic) dark matter on galactic scales (CDM crisis). Computer simulations with the dark matter (e.g. Springel et al., 2006) show in galaxies

- steep density cores (density grows in a cusp towards infinity as one approaches the halo center),
- hundreds of satellite galaxies (dark galaxies) and
- small disk sizes.

On contrary, observations show

- shallow density profiles,
- tens of satellite galaxies (missing satellites problem) and
- large disk sizes.

Moreover, the distribution of known MW satellites, including recently discovered ones, was found to be inconsistent with an isotropic or prolate dark matter halo distribution at a 99.5 % level (Kroupa et al., 2005). On super-galactic scales, the predictions of ΛCDM model simulations match observations perfectly.

Theoreticians motivated primarily by explaining nature on small scales predict the existence of dark particles. The leading candidate for the dark matter is the neutralino from super-symmetric (SUSY) scenarios. Dark matter particle can collide with baryons and reveal its presence in laboratory detectors. Experiments at the Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) will look for indirect evidence of such particles once finished in 2007. Until now, however, all projects trying to detect the dark matter particles directly or indirectly failed to proof the existence of dark particles that could be able to explain 80 % of missing matter[62].

[62] Neutrinos has been found to have non-zero mass. However, they can account only to 0.3 % of total mass-energy content of the universe.

Since there is no experimental evidence for the dark matter, reasonable alternative ways should be explored by physicists. It is possible that what we have interpreted as the evidence for the dark matter is, in fact, the evidence for the breakdown of Newton's (and Einstein's) gravity and dynamics. We should be cautious about accepting the idea of dark matter and remain open minded to other possibilities.

The Newtonian theory appears to have several limitations. An example of this limitation is the orbit of the planet Mercury. This planet follows an elliptical orbit, where every completed rotation around the Sun is shifted with an constant factor from the preceding trajectory. This can be fully explained by Newton's parent theory: Einstein's theory of General Relativity. However, even GR is not the final theory and has known limitations on quantum scales and may have its limitations on galaxy scales as well. GR is considered to be a low energy approximation of some other parental theory (e.g. string/M-theory).

When there is a discrepancy between the theory and observations in physics then it is necessary to start looking for a new theory, which as the (small) subset contains properties of previous theories, which is in correspondence with new observations and which adds new predictions. Hence, there is a possibility that observed inconsistency comes from our imperfect models of gravity and dynamics, so that Newton's law of gravity is not valid on large scales.

7.2 Modified Newtonian Dynamics

Perhaps the most interesting alternative model is the Modified Newtonian Dynamics (MOND) introduced by Milgrom (1983). Milgrom suggested a phenomenological explanation of the flat rotation curve through the modification of Newton's laws without the need of DM. MOND's assumption is that galaxies do not have significant DMH. MOND essentially says that in order to reproduce the flat rotation curves, the real acceleration of bodies (stars) must be significantly greater for small accelerations than that predicted by the Newtonian theory of gravity.

MOND assumes that there is a characteristic acceleration a_0, under which the gravity is more powerful than is predicted by Newton's theory. If a Newtonian acceleration a_N is smaller than a_0, gravity is stronger than Newton's theory predicts. Real acceleration is then $a = \sqrt{a_N \cdot a_0}$. The value of a_0 can be experimentally determined from the rotation curve to fit observations[63]. If acceleration a_N is greater than a_0, Newton's value of acceleration is preserved. MOND is therefore modification for small accelerations. It is very hard to test MOND in the laboratory since the acceleration on the surface of the Earth is always much larger than a_0, thus always entering Newtonian regime.

[63] Favored value of characteristic acceleration a_0 is $a_0 = 1.2 \cdot 10^{-10}\ \mathrm{m} \cdot \mathrm{s}^{-2} \approx \frac{c \cdot H_0}{2 \cdot \pi} \approx c \cdot \left(\frac{\Lambda}{3}\right)^{\frac{1}{2}}$, where c, H_0 and Λ are the speed of light, current Hubble constant and current cosmological constant, respectively.

As the notion suggests, MOND is the modification of Newton's second law and is mathematically expressed for bodies with the constant mass as

$$\frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \frac{\mathrm{d}\left(m \cdot \vec{v}\right)}{\mathrm{d}t} = \overrightarrow{F_N} = m \cdot \overrightarrow{a_N}\,, \tag{7.1}$$

where m is a mass of a body, $\overrightarrow{a_N}$ is the Newtonian acceleration and $\overrightarrow{F_N}$ is the Newtonian force. MOND changes Equation 7.1 into a new form (Milgrom, 1983)

$$\vec{F} = m \cdot \mu\left(\frac{\left|\vec{a}\right|}{a_0}\right) \cdot \vec{a}\,, \tag{7.2}$$

where a_N was substituted with

$$a_N = \mu\left(\frac{\left|\vec{a}\right|}{a_0}\right) \cdot \vec{a}\,. \tag{7.3}$$

Here a is MONDified ("more accurate") acceleration and μ is an interpolation function. The interpolation function is chosen in a way that acceleration becomes

$$\begin{aligned} &a = a_N \text{ for } a_N \gg a_0 \text{ and} \\ &a = \sqrt{a_N \cdot a_0} \text{ for } a_N \ll a_0\,. \end{aligned} \tag{7.4}$$

In everyday life is $a_N \gg a_0$, therefore $\mu(a/a_0) = 1$ and Equation 7.2 is reduced back to the classical Newtonian form (Eq. 7.1). Between these two extreme cases, some interpolated behavior is applied. Milgrom (1983) suggested between these regimes interpolation function

$$\mu(x) = x \cdot \left(1 + x^2\right)^{-\frac{1}{2}}, \tag{7.5}$$

where

$$\mu(x) \to \begin{cases} 1 & for\ x \gg 1 \\ x & for\ x \ll 1. \end{cases} \tag{7.6}$$

7.3 Rotation curve in MOND

Initial motivation for introducing MOND was the flat rotation curve of disk galaxies. We can start with Newton's law of gravity in classical formulation and with the MOND's modification of Newton's second law of motion

$$G \cdot \frac{M \cdot m}{r^2} = m \cdot \mu\left(\frac{a}{a_0}\right) \cdot a$$
$$G \cdot \frac{M}{r^2} = \mu\left(\frac{a}{a_0}\right) \cdot a. \tag{7.7}$$

When we take into account that $a \ll a_0$ and therefore $\mu(a / a_0) = a / a_0$ then we get

$$G \cdot \frac{M}{r^2} = \left(\frac{a^2}{a_0}\right), \tag{7.8}$$

from which can be expressed

$$a = \left(\frac{G \cdot M \cdot a_0}{r^2}\right)^{\frac{1}{2}} = \frac{\sqrt{G \cdot M \cdot a_0}}{r}. \tag{7.9}$$

For a circular motion is a centripetal (centrifugal) force $F = m \cdot a = m \cdot (v_c^2 / r)$, so that $a = v_c^2 / r$. Then

$$a = \frac{v_c^2}{r} = \frac{\sqrt{G \cdot M \cdot a_0}}{r} \tag{7.10}$$

and the circular velocity is finally

$$v_c = \sqrt[4]{G \cdot M \cdot a_0}. \tag{7.11}$$

It can be seen that the circular velocity is independent on the distance r from the center. This consequently leads to a conclusion that rotational curve for $a_N \ll a_0$ must be slightly increasing. We can also immediately determine the characteristic acceleration a_0 from the observed circular velocity.

Simulations and observations of Bissantz et al. (2004) referred about the absence of the DM in the Milky Way's galactic bulge, where is $a_N \gg a_0$ and Newtonian gravity applies. At larger distance, acceleration is smaller and enters into the MOND regime.

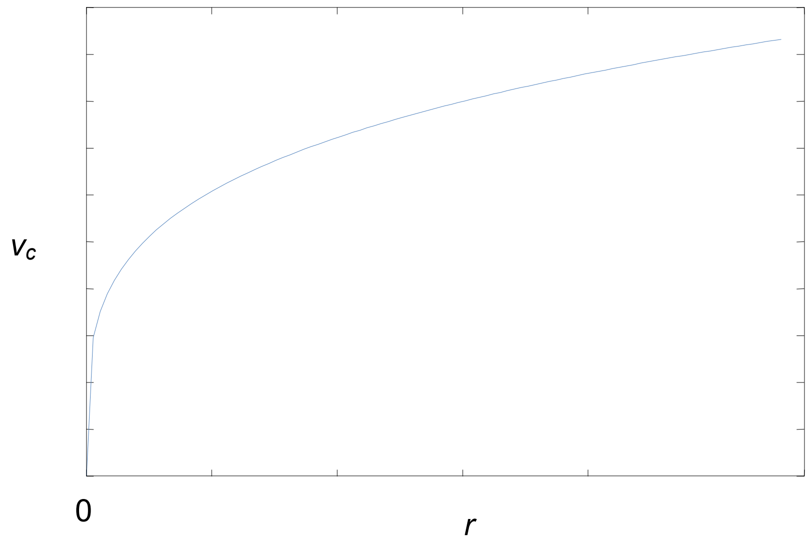


Figure 7–1: The rotation curve in MOND.

Rotation curves of disk galaxies are not the only evidence of the dark matter. MOND should be able to explain all problems listed in Chapter 7.1 consistently. However, MOND is not without difficulties.

- An original formulation of MOND (Milgrom, 1983) violates basic principles underlying physics (Felten, 1984): The Newton's third law and linear momentum are not conserved (an acceleration is not inversely proportional to a mass, there is no symmetry between a galaxy and a test mass); Milgrom and Bekenstein (1984) explained these shortcomings.
- MOND was introduced to relief the universe of the dark matter; however, it cannot explain the motion of galaxies in rich clusters, where a lot of the dark matter is needed anyway.
- Minchin et al. (2005) claim to have detected an HI source in the Virgo Cluster that could be a dark halo without shining stellar galaxy inside. Others claim that this detection is little more than tidal debris, originating from the nearby galaxy NGC 4254.

MOND was also for a long time seen as an isolated theory. However, Bekenstein (2004) invented its parent theory. Bekenstein published a relativistic version of MOND called tensor-vector-scalar theory (TeVeS), essentially an extension of GR. TeVeS reduces to Einstein's theory for high speeds and large accelerations, to Newtonian gravity for low speeds and small accelerations and to MOND when accelerations are even smaller. It is possible that some unknown fundamental physics (e.g. string/M-theory) is reducing to TeVeS in a suitable limit. Both hypotheses, dark matter and gravity modification, have their pros and cons. Future experiments will shed more light into this darkness.

7.4 Modification to gravity model

We now want to find a relationship between the Newtonian and MONDian acceleration so that modifications of our simulation code can be made. Equation 7.3 can be manipulated as follows

$$\begin{aligned}
a_N &= \frac{a}{a_0} \cdot \left[1 + \left(\frac{a}{a_0}\right)^2\right]^{-\frac{1}{2}} \cdot a \\
a_N^2 &= \frac{a^4}{a_0^2} \cdot \left[\frac{a_0^2 + a^2}{a_0^2}\right]^{-1} \\
a_N^2 &= \frac{a^4}{a_0^2} \cdot \left(\frac{a_0^2}{a_0^2 + a^2}\right) \\
a_N^2 &= \frac{a^4}{a_0^2 + a^2} \\
a^4 &= a_N^2 \cdot \left(a_0^2 + a^2\right).
\end{aligned} \tag{7.12}$$

One now only needs to solve

$$a^2 - a_N \cdot \sqrt{a_0^2 + a^2} = 0 \tag{7.13}$$

to get a as a function of a_N. Relationship between the Newtonian a_N and MONDian acceleration a is then (e.g. Read and Moore, 2005)

$$a = \frac{a_N}{\sqrt{2}} \cdot \left[1 + \sqrt{1 + \left(2 \cdot \frac{a_0}{a_N}\right)^2}\right]^{\frac{1}{2}}. \tag{7.14}$$

Equation 7.14 can be used in *N*-body modeling of gravity interaction instead of Newtonian gravitational acceleration from Chapter 4. The generation of initial conditions of isolated galaxies from Chapter 5 should also take into account the modification of Poisson's equation. However, for initial investigations, we can leave it without changes.

We will create a Kuzmin's disk with our classical initial conditions generation program. We will evolve initial conditions with the Barnes-Hut *N*-body simulation code in MONDified gravity mode with the opening angle $\theta = 0.75$ and the softening length $\varepsilon = 0.0001$.

Model	Q	Resolution (star particles)	Timestep
Kuzmin	1.5	5,000	0.0005

Table 7–1: Simulation parameters of the Kuzmin's model.

The timestep was set equal to $34.1 \cdot 10^3$ yr and the overall simulation covers $1.02 \cdot 10^9$ yr.

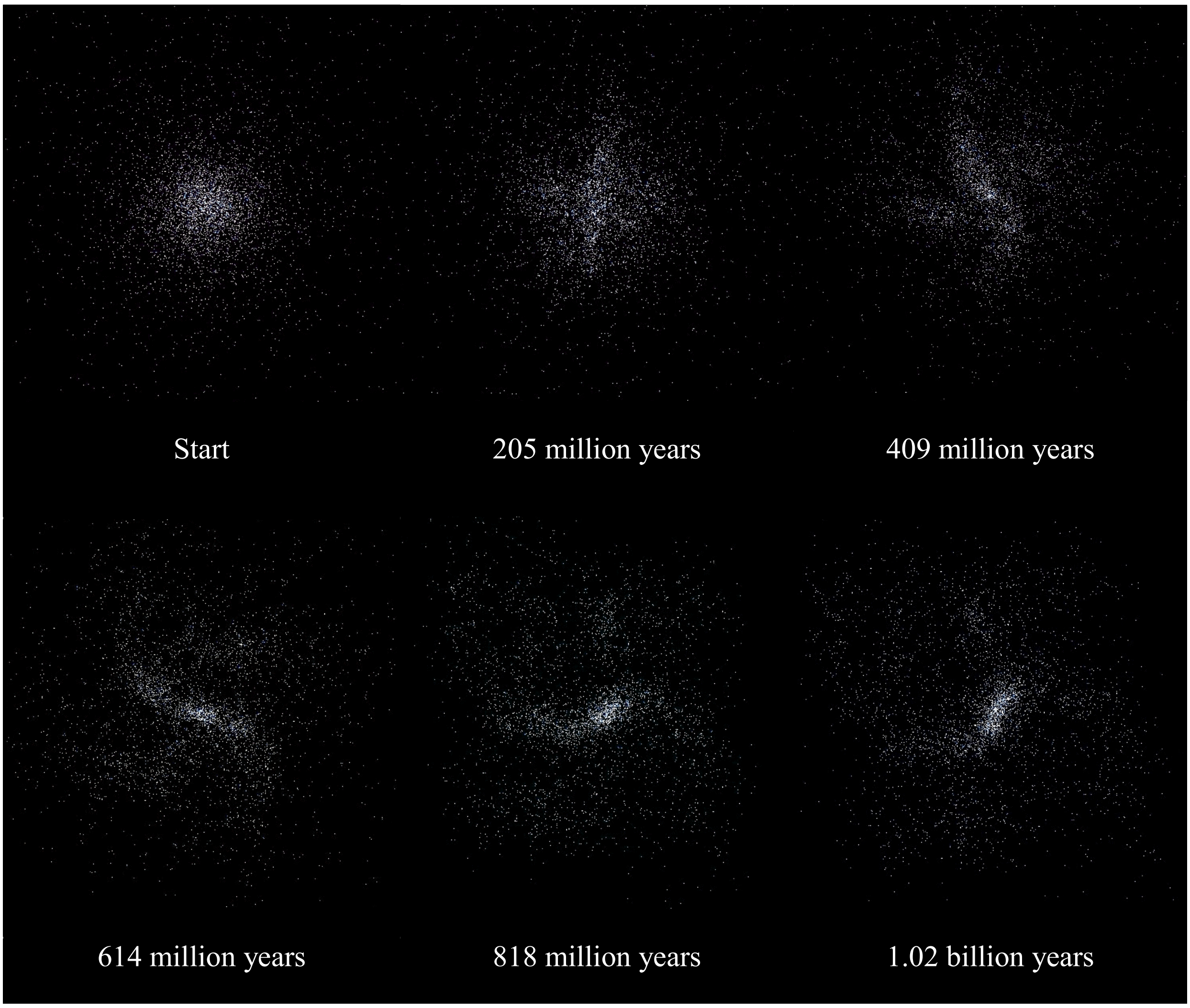
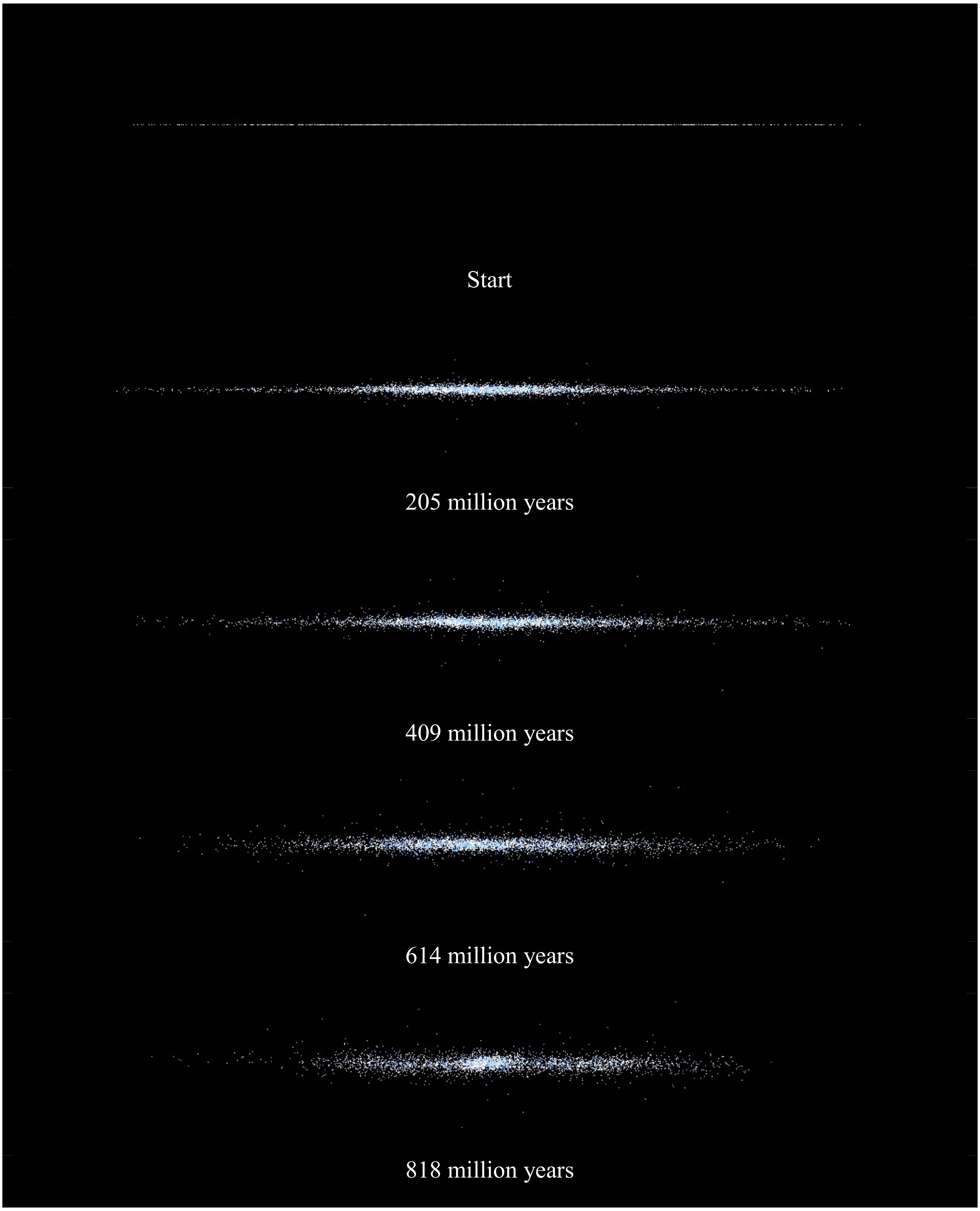


Figure 7–2: A time sequence for the hot Kuzmin's disk in the *xy* plane evolved with the MONDified Barnes-Hut *N*-body algorithm.


Start
205 million years
409 million years
614 million years
818 million years

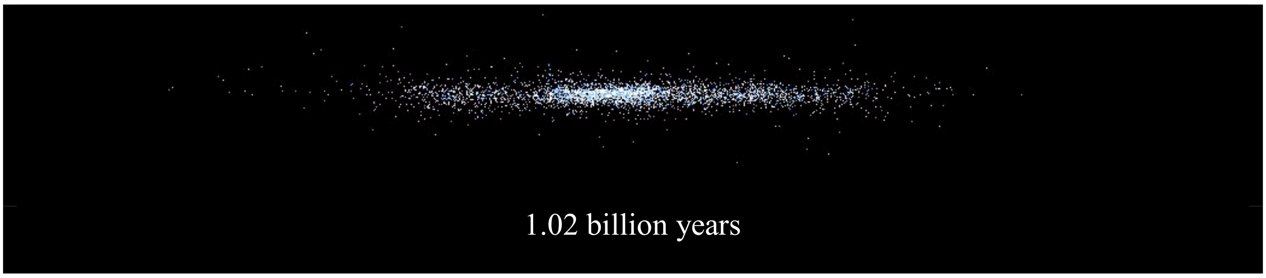


Figure 7–3: A time sequence for the hot Kuzmin's disk in the *yz* plane evolved with the MONDified Barnes-Hut *N*-body algorithm.

The shrinking of the disk can be seen on Figure 7–3 as the result of a more powerful gravity in Milgrom's MOND.

7.5 Concluding remarks

In Chapter 7, we have shown how to prepare our simulation for the alternative gravity model. We have learned that the simulation of a galaxy in Modified Newtonian Dynamics (MOND) theory can be performed with at least the same result as the simulation in Newton's theoretical framework. Cosmological large-scale dark matter computer simulations performed by other authors agree with the observations, while the results on galaxy scales are inconsistent. These simulations with dark matter may miss some important small scale physics of both baryonic and non-baryonic matter that is not resolved with a current resolution of cosmological simulations and computer models, while the standard model can be accurate.

One should be cautious, however, as the theory is stretched and adapted to fit the evidence, or facts are carefully selected to fit the theory. We learned from the history of physics that models of nature usually comprehended a lot of things accurately, but also usually missed important big ones. Mordehai Milgrom and others has done interesting work that healthy competes with dark matter theory. In any case, even if MOND should be revealed as an incorrect theory, it serves as a good exercise for galaxy modeling. We should keep in mind that "laws of physics" are not an accurate description of nature.

"A picture is worth a thousand words."
"An animation is worth a million words."

8

SIMULATION SOFTWARE

In this chapter, we will present the main features of simulation programs and how to use them.

8.1 Introduction

This thesis was written in the department of physics, so I wanted to be clear as much as it was possible in a computational side of the thesis. Therefore, I decided not to use a massively parallel implementation as I did in previous work (Schwarzmeier, 2004). My source codes are written in C++ computer language, because it bears both qualities that I require on the computer language and compiler for the galaxy dynamics educational-research work: portability (people in education usually work in Microsoft Windows environment, while people in research on Linux workstations) and high processing speed of compiled code[64]. In any case, anyone is free to choose the favorite programming language and platform according to her or his choice, while guided by a physical description in the text and numerical description in the source codes.

All source codes[65] are written in ISO/ANSI Standard C++ with Standard C++ library and should compile with any C++ compiler conforming to these standards. I used freely available Microsoft Visual C++ 2005 Express Edition[66] for the Microsoft Windows platform and GNU C++[67] for the Linux platform. The source codes utilize two widely used third-party libraries: GNU Scientific

[64] I also considered the usage of popular languages like JAVA and C#. After initial speed tests, however, I have found that an executable code produced by compilers of these languages was far too slow for galaxy dynamics simulations.

[65] Available for download from http://www.kof.zcu.cz/st/dis/schwarzmeier/galdyn.zip

[66] http://msdn.microsoft.com/vstudio/express/

[67] http://gcc.gnu.org/

Library (GSL)[68] and LIBXML2[69]. After unpacking the code, it must be compiled. Solutions and makefiles are in the `/amon/_make/` and `/ics/_make/` subdirectories. Hopefully, the compilation should be successful.

The code created for this thesis may be used by a student in two ways. The code can be used as a finished program that can be executed with command-line parameters in order to create galaxy models and evolve them. The second and more desirable way to use this software is to extend it. I have used an object-oriented design that enables the extensibility and reuse of existing simulation classes through the addition of new physics without the need to rewrite the original code. A student, who makes extensions to the existing simulation code, will reach the top of the educational pyramid presented in Chapter 2: New Pedagogy.

8.2 Initial routines

All calculation using the AMON-2 (Astronomical Modeling with *N*-bodies) code are run using the executable `amon-main` in the `/amon/bin/` subdirectory, but you will need different input files to tell the code to do different things. We begin with a description of routines used to start a new calculation from scratch.

Generator of Initial Conditions (GENICS) is, in fact, a set of programs and classes located in the `/ics/` directory. Executables should be located in the `/ics/bin` subdirectory after the successful compilation. All executables are console (command line) applications[70].

8.2.1 Spherical galaxies

Let us suppose that we want to create a spherical galaxy called "Model A" consisting of 5,000 bodies with the Hernquist's distribution function. It can be done by executing the file `ics-spheric-main` in the command-line window of the operating system as follows

```
> ics-spheric-main h model_A 5000
```

The first switch behind the executable name can be `h` (Hernquist), `p` (Plummer), `j` (Jaffe), `t` (Isothermal) or `u` (Uniform). The `ics-spheric-main` program will generate two files in the current directory: `model_A.xml` and `model_A_ics.bin`. The file `model_A.xml` contains XML (Extensible Markup Language)[71] description of the simulation binary file

[68] http://www.gnu.org/software/gsl/ and http://gnuwin32.sourceforge.net/packages/gsl.htm

[69] http://xmlsoft.org/

[70] Graphical User Interface (GUI) cannot be used because of Application Programming Interface (API) diversity of individual GUIs across different operating systems.

[71] XML is a metalanguage to design markup languages using extra markup information enclosed between angle brackets. HTML (HyperText Markup Language) is the most well-known markup language.

`model_A_ics.bin`. The simulation binary file will contain the generated simulation data of Hernquist's model, i.e. masses, positions and velocities of individual bodies.

To verify the validity of numerically calculated functions, it is useful to produce their plots. It can be done by plugging a `p` (plot) parameter at the end of the command line:

```
> ics-spheric-main h model_A 5000 p
```

With the `p` parameter, the program will create a series of files with `txt` extension that are suitable for plotting with `gnuplot`[72]. When Microsoft Excel is installed on a computer, `ics-spheric-main` also creates a new instance of Microsoft Excel through the Microsoft Windows' Component Object Model (COM), inserts tabulated values of numerically calculated functions into an Excel sheet and automatically generates an Excel chart for each of these functions.

8.2.2 Disk galaxies

The generation of disk (flat) models can be done in a similar way. Let us suppose that we want to create a realization of Kuzmin's model called "Disk A" with 6,000 bodies and plot functions numerically calculated by the program:

```
> ics-flat-main u disk_A 6000 p
```

For that purpose, we have used `ics-flat-main` executable that recognizes the following models: `u` (Kuzmin), `k` (Kepler), `e` (Exponential). Remaining parameters are the same as for the generator of spherical models.

8.2.3 Rotation

Every galaxy created by the GENICS can be optionally rotated. E. g., let us suppose that we make preparations for the collision of counter-rotating disks. Disks are generated by the `ics-flat-main` in the *xy* plane. So we must rotate such a disk around the *y*-axis by 180 degrees to obtain the desired counter-rotation:

```
> ics-rotate-main disk_A.xml disk_B 0.0 180.0 0.0
```

This essentially says to the `ics-rotate-main` program: take a galaxy described by the `disk_A.xml` file, transform it by rotations of 0.0 degrees about the *x*-axis, 180.0 degrees about the *y*-axis and 0.0 degrees about the *z*-axis and save it to `disk_B` binary and XML files.

8.2.4 Conversion of units

Now imagine that we want rescale units of a generated galaxy, i.e., change its gravitational constant, scale length and mass. We can do that by executing the following command

[72] http://www.gnuplot.info/

```
> ics-units-convertor-main model_A.xml model_B 1.0 0.1 0.001
```

This command will execute `ics-units-convertor-main` that will read a galaxy described by the `model_A.xml` file and rescale it, while leaving the gravitational constant the same (`1.0`), multiply its scale length with `0.1` and its mass unit with `0.001`.

8.2.5 Direct collision

To set galaxies on a direct collision course, the `ics-encounter-main` program with the `d` parameter must be evoked. Consider as an example the following command-line:

```
> ics-encounter-main d galaxy_A.xml galaxy_B.xml collision 10.0
0.5 0.1
```

The listed command will set galaxies described by the `galaxy_A`.xml (central galaxy) and `galaxy_B.xml` (companion galaxy) files on the direct collision course with the *z*-axis distance equal to 10.0, *z*-velocity equal to 0.5 and *x*-axis shift equal to 0.1. This collisional configuration will be written to `collision.xml` and `collision_ics.bin` files.

8.2.6 Keplerian encounter

Galaxies can be set on Keplerian orbit as follows

```
> ics-encounter-main k galaxy_A.xml galaxy_B.xml collision 10.0
0.8 25.0
```

Notice that we have changed the first parameter of `ics-encounter-main` to `k` (Keplerian). The meaning of numerical parameters is now following: a semi-major axis is equal to 10.0, numerical eccentricity equals to 0.8 and true anomaly to 25 degrees.

8.2.7 Extensibility of GENICS

Not all features of the GENICS, however, are accessible from the command-line. Anyone can come with a new mass density distribution function of a galaxy and try to test its evolution. This can be simply done through deriving a descendant class from the CSphericalModel class and overriding the *rho* virtual function as is done, for example, in the Hernquist's model:

```
#include "sphericalmodel.h"

//  derive descendant class
class CHernquistModel: public CSphericalModel

{

public:

        CHernquistModel()

        {
```

```
                m_name = "HernquistModel";
            }
    protected:
            //  override with a new mass density distribution function
            virtual double rho( double r )
            {
                if (r == 0) return 0;
                return ( m_totalMass / ( 2.0 * M_PI * pow(m_a,3) ) ) *
                ( ( pow( m_a, 4 ) / ( r * pow( r + m_a, 3) ) ));
            }
    };
```

A full hierarchy chart of GENICS can be seen on Figure 8–1.

8.3 Running calculations

AMON-2 is a tree *N*-body simulation code for evolution of three-dimensional self-gravitating collisionless systems. It is based on the Barnes-Hut algorithm. Executable `amon-main` should be located in the `/amon/bin` subdirectory after a successful compilation.

Syntax is following:

```
amon-main [algorithm] [input_file] [timesteps] [theta] [epsilon]
      [group_size] [timestep] [gconst]
```

Parameters:

`algorithm`

Specifies which algorithm will be used: `b` Barnes-Hut, `d` direct (brute-force) calculation.

`input_file`

Specifies the simulation XML file.

`timesteps`

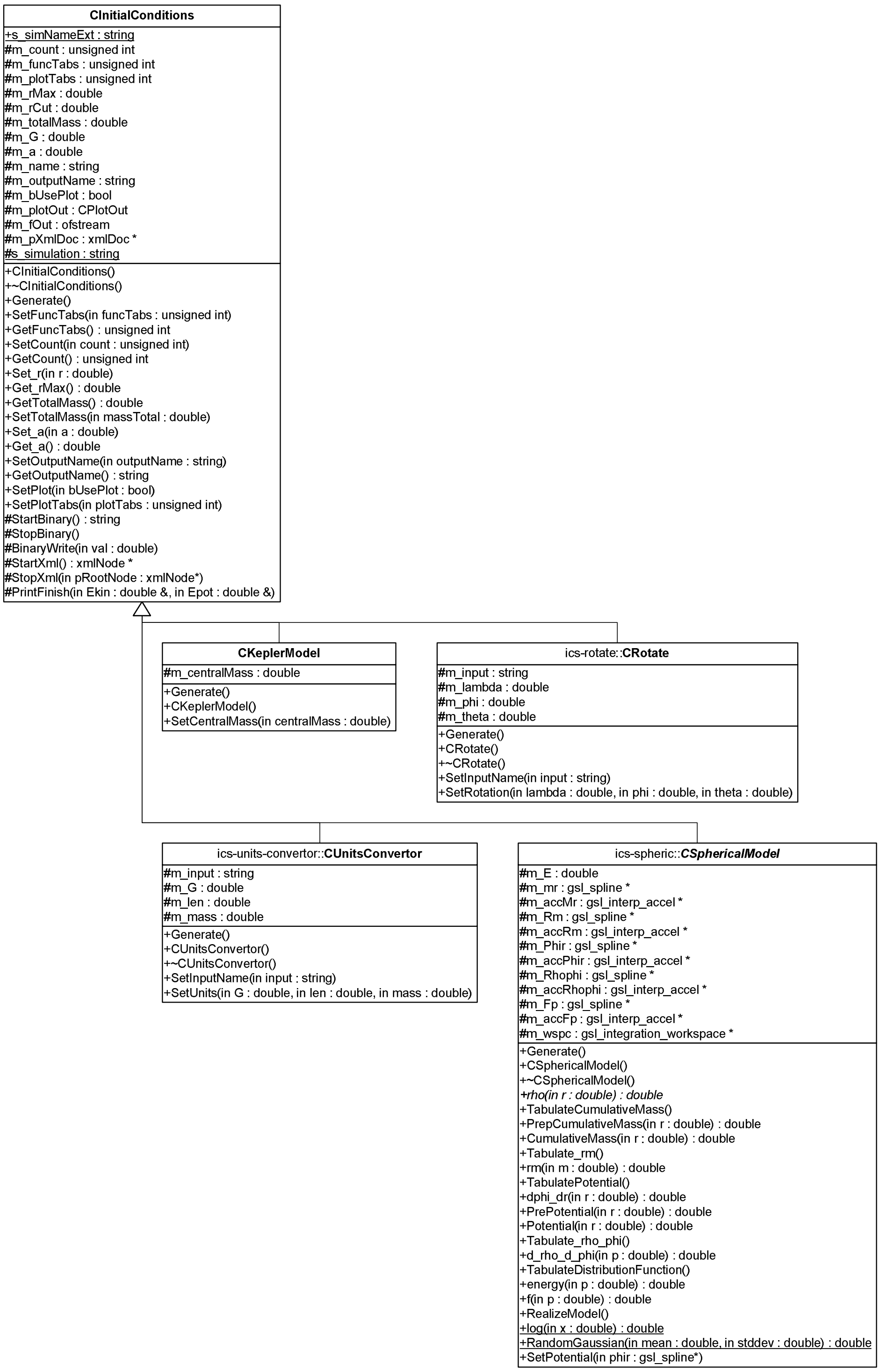
A total number of timesteps.

`theta`

Parameter θ from the Barnes-Hut algorithm.

`epsilon`

Gravitational softening ε.


CInitialConditions
+s_simNameExt : string
#m_count : unsigned int
#m_funcTabs : unsigned int
#m_plotTabs : unsigned int
#m_rMax : double
#m_rCut : double
#m_totalMass : double
#m_G : double
#m_a : double
#m_name : string
#m_outputName : string
#m_bUsePlot : bool
#m_plotOut : CPlotOut
#m_fOut : ofstream
#m_pXmlDoc : xmlDoc *
#s_simulation : string
+CInitialConditions()
+~CInitialConditions()
+Generate()
+SetFuncTabs(in funcTabs : unsigned int)
+GetFuncTabs() : unsigned int
+SetCount(in count : unsigned int)
+GetCount() : unsigned int
+Set_r(in r : double)
+Get_rMax() : double
+GetTotalMass() : double
+SetTotalMass(in massTotal : double)
+Set_a(in a : double)
+Get_a() : double
+SetOutputName(in outputName : string)
+GetOutputName() : string
+SetPlot(in bUsePlot : bool)
+SetPlotTabs(in plotTabs : unsigned int)
#StartBinary() : string
#StopBinary()
#BinaryWrite(in val : double)
#StartXml() : xmlNode *
#StopXml(in pRootNode : xmlNode*)
#PrintFinish(in Ekin : double &, in Epot : double &)
CKeplerModel
#m_centralMass : double
+Generate()
+CKeplerModel()
+SetCentralMass(in centralMass : double)
ics-rotate::CRotate
#m_input : string
#m_lambda : double
#m_phi : double
#m_theta : double
+Generate()
+CRotate()
+~CRotate()
+SetInputName(in input : string)
+SetRotation(in lambda : double, in phi : double, in theta : double)
ics-units-convertor::CUnitsConvertor
#m_input : string
#m_G : double
#m_len : double
#m_mass : double
+Generate()
+CUnitsConvertor()
+~CUnitsConvertor()
+SetInputName(in input : string)
+SetUnits(in G : double, in len : double, in mass : double)
ics-spheric::CSphericalModel
#m_E : double
#m_mr : gsl_spline *
#m_accMr : gsl_interp_accel *
#m_Rm : gsl_spline *
#m_accRm : gsl_interp_accel *
#m_Phir : gsl_spline *
#m_accPhir : gsl_interp_accel *
#m_Rhophi : gsl_spline *
#m_accRhophi : gsl_interp_accel *
#m_Fp : gsl_spline *
#m_accFp : gsl_interp_accel *
#m_wspc : gsl_integration_workspace *
+Generate()
+CSphericalModel()
+~CSphericalModel()
+rho(in r : double) : double
+TabulateCumulativeMass()
+PrepCumulativeMass(in r : double) : double
+CumulativeMass(in r : double) : double
+Tabulate_rm()
+rm(in m : double) : double
+TabulatePotential()
+dphi_dr(in r : double) : double
+PrePotential(in r : double) : double
+Potential(in r : double) : double
+Tabulate_rho_phi()
+d_rho_d_phi(in p : double) : double
+TabulateDistributionFunction()
+energy(in p : double) : double
+f(in p : double) : double
+RealizeModel()
+log(in x : double) : double
+RandomGaussian(in mean : double, in stddev : double) : double
+SetPotential(in phir : gsl_spline*)

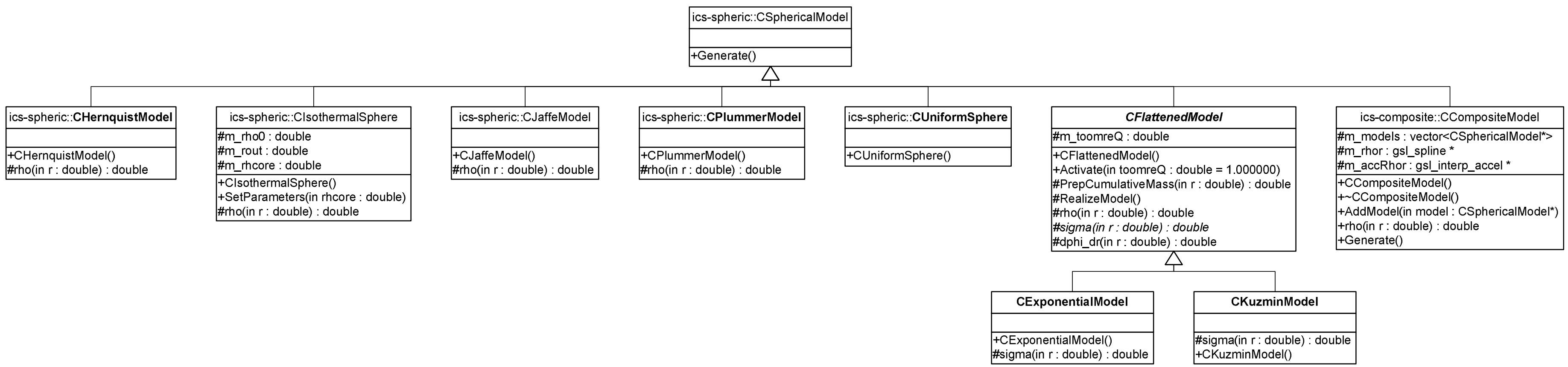


Figure 8–1: UML (Unified Modeling Language) diagram of the Generator of Initial Conditions (GENICS).

`group_size`

The size of a group in the grouping strategy for the gravitational calculation.

`timestep`

The length of timestep.

`gconst`

Gravitational constant.

Let us suppose that we have generated initial conditions with GENICS to the `collision.xml` file. We want to evolve it with the Barnes-Hut algorithm for $30,000$ timesteps, with $\theta = 0.75$, $\varepsilon = 0.0001$, group size equal to 32, timestep equal to 0.0005 and gravitational constant $G = 1$. Then the command line should be

```
> amon-main b collision.xml 30000 0.75 0.0001 32 0.0005 1
```

To execute AMON-2 in MONDian mode, preprocessor definition `MOND` must be defined during compilation.

8.4 Visualization

Computer simulations are useless without accompanying tools that analyze and visualize results. During my experiments, AMON-2 produced prodigious amounts of simulated data reaching 1.5 TB. Output from such computer simulations of galaxy dynamics in the form of data sets must be presented for both public outreach and scientists in an easy way. These galaxy dynamics simulations contain higher-dimensional data – phase space's configuration and velocity spaces. Scientific visualization shows these complex information in a way that makes them easy to understand and study.

To analyze and interpret the data generated by the computer simulations of galaxy dynamics, I have created the Digital Galaxy Explorer (DIGALEX). This sophisticated program renders different particles with different colors, allows a scene to be rotated, viewed from an arbitrary position and under any angle in a virtual world. From a set of user defined cameras (eye-views) and time positions in the simulation, DIGALEX creates smooth fly-through (dynamic camera) animations. This output of numerical simulations is a useful way to communicate scientific results to public.

The most straightforward way to visualize a set of bodies from a simulation is to project the positions of the bodies onto a viewing plane. Colors of bodies are based on their individual masses to distinguish different groups of bodies (disk, bulge, halo). The simulation exists in a three-dimensional configuration space, but a viewing screen is two-dimensional, so the positions of bodies must be projected onto a plane. I have used OpenGL (Open Graphics Library) industry standard for visualization.

I have created animations with a static camera at least in two perpendicular planes for all simulations. For public outreach, I have created dynamic camera animations. These fly-through movies were produced in the natural aspect ratio 16:9, MPEG-2 video compression and resolution of 1280x720 pixels, thus compatible with the High Definition Television (HDTV) format.

8.5 Concluding remarks

In Chapter 8, we have presented the main features of simulation programs and how to use them. I developed several software tools for this thesis that are available publicly to the community. GENICS (Generator of Initial Conditions) and AMON-2 (Astronomical Modeling with *N*-bodies) contain together over 9,000 lines of C++ source codes. DIGALEX (Digital Galaxy Explorer) contains over 5,500 lines of C++ source codes. All of the source code of software is available at http://www.kof.zcu.cz/st/dis/schwarzmeier/ or on the companion Digital Versatile Disk (DVD), and is released under the GNU General Public License (GPL).

I have created 70 animations that show simulated *N*-body systems described in the thesis. These animations are also available on mentioned internet website and on the companion DVD. I invite you to visit this website and explore all of these animations.

9 RESULTS

9.1 Summary

What follows is a chapter-by-chapter summary of the main results.

- In Chapter 1, we have described thesis objectives and methodology used in the thesis.
- In Chapter 2, we have shown that physics education is an important and crucial element for human society. Students should be more motivated by their teachers with less importance on learning and more emphasis on differentiation, individualization and self-teaching. It is for this purpose that the formation of self-teaching projects is suggested. Together with advancement in science and technology, an early connection of education and research should be made. Self-teaching educational-research projects created by specialists in their fields should be made freely available on the Internet as a service to society. A research method of education can develop student's abilities in a complex way. Computer models and simulations of nature's behavior are acknowledged as useful, providing connections between various fields of science education. A scheme incorporating these approaches is suggested in the "four-level educational architecture". Surely, education is a complex system and this concept may not be valid for every student.
- In Chapter 3, we have sketched our basic understanding of nature, laws of physics, models, simulations and confusion among them.
- In Chapter 4, we have shown how to simulate the effect of the gravitational field and of Newton's laws of motion to move the stars around. I described my implementation of Barnes-Hut algorithm for many-body simulation and novel geometry-based construction of the 3-dimensional Hilbert's curve. Simulation code works in four steps. First, a tree is constructed by space decomposition from a list of bodies that form the simulated system. Space is divided utilizing Hilbert's self-similar space-filling curve. Groups of close bodies are created. Second, centers of mass of individual nodes are computed. Third, accelerations are computed with the Barnes-Hut algorithm. Fourth, new positions and velocities are computed. Thanks to this

algorithm, all simulations will be fully self-consistent, i.e. no rigid potentials will be employed.

- In Chapter 5, we have shown how to create a computer model of a galaxy in order to study galaxy dynamics. We found that the construction of galaxy in a controllable way is difficult. Due to immense complexity, all models are very artificial in comparison to real galaxies. In spite of that, the initial density distribution function of models are in good agreement with observations of real galaxies. The galaxy models created were single or multicomponent systems in stable dynamical equilibrium.

  Initial conditions were generated as follows. Specifying the mass density distribution function, we first calculate the model's cumulative mass distribution function and corresponding gravitational potential. Then the mass density distribution function is expressed as a function of gravitational potential. The phase-space distribution function is calculated on the fly using a numerical formulation of Eddington's formula. Once the phase-space distribution function has been calculated, one can start to randomly sample particles from the distribution function. If the use of another mass density profile is requested, all that is necessary, is to override a virtual function "rho" according to the chosen mass density profile. Through this approach, many kinds of models may be constructed. Models created in this way are quickly getting into equilibrium.

  We created realizations of an elliptical galaxy from various spherical models. The spherical models were in equilibrium from the beginning. We have created disk models that showed continuous evolution. We saw that dynamically cold disk without a dark halo spontaneously formed features resembling galactic bar and spiral arms. It has been shown that a self-gravitating disk system is unstable unless a certain velocity dispersion and dark halo were included. All models were evolved for more than 1 billion years and movies from all computations were produced.

- In Chapter 6, we have demonstrated how to study galaxy collisions and mergers with computer simulations. Interacting galaxies are very complex and highly dynamic systems. With modest computational resources, we performed computer simulations of galaxy interactions that are in excellent agreement with observations. These simulations provide an accurate and entertaining insight into galaxy collisions and mergers.

  However, model results were not without their shortcomings. Our aim was not to study interactions in detail, but to show how such study can be done with all details provided. We studied the evolution of spherical galaxy interactions, minor and major mergers, and galaxy harassment.

  We simulated the evolution of the Milky Way galaxy, the Large and the Small Magellanic Clouds and all 19 known satellite galaxies of the Milky Way. We have simulated the future evolution of the Local Group in the collision of two disk galaxies representing the

Andromeda galaxy (M31) and the Milky Way galaxy. Models were evolved for up to 8.1 billion years and movies from all computations were produced.

- In Chapter 7, we have shown how to prepare our simulation for the alternative gravity model. We have learned that the simulation of a galaxy in Modified Newtonian Dynamics (MOND) theory can be performed with at least the same result as the simulation in Newton's theoretical framework. Cosmological large-scale dark matter computer simulations performed by other authors agree with the observations, while the results on galaxy scales are inconsistent. These simulations with dark matter may miss some important small scale physics of both baryonic and non-baryonic matter that is not resolved with a current resolution of cosmological simulations and computer models, while the standard model can be accurate.

  One should be cautious, however, as the theory is stretched and adapted to fit the evidence, or facts are carefully selected to fit the theory. We learned from the history of physics that models of nature usually comprehended a lot of things accurately, but also usually missed important big ones. Mordehai Milgrom and others has done interesting work that healthy competes with dark matter theory. In any case, even if MOND should be revealed as an incorrect theory, it serves as a good exercise for galaxy modeling. We should keep in mind that "laws of physics" are not an accurate description of nature.

- In Chapter 8, we have presented the main features of simulation programs and how to use them. I developed several software tools for this thesis that are available publicly to the community. GENICS (Generator of Initial Conditions) and AMON-2 (Astronomical Modeling with *N*-bodies) contain together over 9,000 lines of C++ source codes. DIGALEX (Digital Galaxy Explorer) contains over 5,500 lines of C++ source codes. All of the source code of software is available at http://www.kof.zcu.cz/st/dis/schwarzmeier/ or on the companion Digital Versatile Disk (DVD), and is released under the GNU General Public License (GPL).

  I have created 70 animations that show simulated *N*-body systems described in the thesis. These animations are also available on mentioned internet website and on the companion DVD. I invite you to visit this website and explore all of these animations.

Shortened version of this thesis was presented at the conference "Moderní trendy v přípravě učitelů fyziky 3", Srní, Czech Republic, April 2007 and was accepted for publication in conference proceedings (Rauner, 2007). Parts were presented on two monthly meetings of our department and on the annual meeting of Ph.D. students of "Theory of Education in Physics".

9.2 Conclusions and future prospects

For the first time, a complete educational description of computer simulations of galaxy dynamics, from initial conditions generation to visualization is described in detail. It was framed into the self-teaching educational-research project. All parts of the thesis (both printed and electronic) are available for all interested on the internet website of the Department of General Physics.

The understanding of galaxy formation, evolution and interaction is obscured with complexity and uncertainty in the modeling of physical phenomena involved in galaxies. Future improvements of models should come with additional physics, and more of such self-teaching educational-research projects should be created and made publicly available.

LIST OF ABBREVIATIONS

AMON Astronomical Modeling with *N*-bodies

BH Black Hole

cD central Dominant

DIGALEX Digital Galaxy Explorer

DM Dark Matter

DMH Dark Matter Halo

GENICS Generator of Initial Conditions

GR General Relativity

ISM Interstellar Matter

LSS Large Scale Structure of the universe

MOND MOdified Newtonian Dynamics

ODE Ordinary Differential Equations

PDF Probability Distribution Function

SFR Star Formation Rate

SMBH Supermassive Black Hole

SPH Smoothed Particle Hydrodynamics

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This research made use of the brilliant NASA's Astrophysics Data System Abstract Service. All the simulations described in this thesis were performed using my code AMON-2 on computers in the Department of General Physics of the University West Bohemia in Pilsen. All code for this thesis was developed with magnificent Microsoft Visual C++ 2005 environment and compiler.

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RELATED PUBLICATIONS

Schwarzmeier J. (2004): *Computer Modelling of Astronomical Objects Dynamics*, Astronomische Nachtrichten 325, 121.

Schwarzmeier J. (2005): *Mnohatělesové simulace a jejich využití při studiu fyziky*, Conference proceedings "Moderní trendy v přípravě učitelů fyziky 2", Západočeská univerzita v Plzni, Rauner K. (Ed.) – in Czech.

Schwarzmeier J. (2007): *On Simulations of Galaxy Dynamics and Their Application to Physics Education,* Conference proceedings "Moderní trendy v přípravě učitelů fyziky 3", Západočeská univerzita v Plzni, Rauner K. (Ed.)

CURRICULUM VITAE

Personal data

Name: Jakub Schwarzmeier, born in Pilsen, Czech Republic
Date of birth: March 5, 1981
Nationality: Czech

Education

2004-2007

University of West Bohemia, Ph.D. in "Theory of Education in Physics",
Topic: "On Simulations of Galaxy Dynamics and Their Application to Physics Education"

December 19, 2006

University of West Bohemia, Rigorous (PhDr.) Exam in Physics Education,
Topic: "Computer Simulations of Astronomical Objects Dynamics"

1999-2004

University of West Bohemia, Master (Mgr.) of Physics and Computer Science Education,
Diploma in Physics: "Computer Modeling of Astronomical Objects Dynamics"

1995-1999

High School of Electro-Engineering, Pilsen, specialization in Computer Science and Technology

Teaching Experience

2006-2007

Teaching assistant for Cosmology

2005-2006

Teaching assistant for Astronomy for Everyone
Teaching assistant for Cosmology
Teaching assistant for Development of Measurement Units and Dimensional Analysis

2004-2005

Teaching assistant for Laboratory experiments in physics 1 & 2
Teaching assistant for Mathematics 1

Honors

Merit Scholarship	September 2003 – May 2004
1st Place in Diploma Thesis Competition on Faculty	May 2004
Scholarship for Bonn University visit	November 2004
2nd Place in National Diploma Thesis Competition	June 2005

Languages

Czech: native language, English: advanced