Lectures 14-15: Galaxy Modeling I-II

The bulge distribution function (Kuijken & Dubinski 1995)

King model bulge DF:

$$f_{\rm bulge}(E) = \begin{cases} \rho_{\rm b}(2\pi\sigma_{\rm b}^2)^{-3/2}\exp[(\Psi_0 - \Psi_{\rm c})/\sigma_{\rm b}^2]\{\exp[-(E-\Psi_{\rm c})/\sigma_{\rm b}^2]-1\} & \text{if } E < \Psi_{\rm c}, \\ 0 & \text{otherwise.} \end{cases}$$

It depends on the three parameters: $\Psi_{\rm c}$ (the cutoff potential of the bulge), $\rho_{\rm b}$ (approximately the central bulge density, ignoring the effects of the DF truncation) and $\sigma_{\rm b}$, which governs the velocity dispersion of the bulge component. Ψ_0 is the gravitational potential at the centre of the model.

The density of the bulge component in a potential Ψ is obtained by integrating its DF over all velocities, resulting in

$$\rho_{\rm bulge}(\Psi) = \rho_{\rm b}\left[e^{(\Psi_0-\Psi)/\sigma_{\rm b}^2}\,\mathrm{erf}\left(\sqrt{(\Psi_{\rm c}-\Psi)/\sigma_{\rm b}^2}\right) - \pi^{-1/2}e^{(\Psi_0-\Psi_{\rm c})/\sigma_{\rm b}^2}\left(2\sqrt{(\Psi_{\rm c}-\Psi)/\sigma_{\rm b}^2} - \frac{4}{3}[(\Psi_{\rm c}-\Psi)/\sigma_{\rm b}^2]^{3/2}\right)\right]$$

where $\Psi < \Psi_{\rm c}$, and zero density elsewhere. $\mathrm{erf}(x) \equiv 2\pi^{-1/2}\int_0^x \exp(-t^2)\,dt$ is the usual error function.

In what follows, we will normally choose $\sigma_{\rm b} < \sigma_0$ and $\Psi_{\rm c} < 0$ to make the bulge more centrally condensed, and more radially confined, than the halo (the latter has a cutoff at zero energy).

The halo distribution function (Kuijken & Dubinski 1995)

$$f_{\rm halo}(E, L_z^2) = \begin{cases} [(AL_z^2 + B)\exp(-E/\sigma_0^2) + C][\exp(-E/\sigma_0^2) - 1] & \text{if } E < 0, \\ 0 & \text{otherwise.} \end{cases}$$

$$\begin{aligned} \rho_{\rm halo}(R, \Psi) \quad = \quad & \tfrac{1}{2}\pi^{3/2}\sigma_0^3(AR^2\sigma_0^2 + 2B)\,\mathrm{erf}(\sqrt{-2\Psi}/\sigma_0)\exp(-2\Psi/\sigma_0^2) \\ & +(2\pi)^{3/2}\sigma_0^3(C - B - AR^2\sigma_0^2)\,\mathrm{erf}(\sqrt{-\Psi}/\sigma_0)\exp(-\Psi/\sigma_0^2) \\ & +\pi\sqrt{-2\Psi}[\sigma_0^2(3A\sigma_0^2R^2 + 2B - 4C) + \frac{4}{3}\Psi(2C - A\sigma_0^2R^2)]. \end{aligned}$$

The halo DF has five free parameters: the potential well depth Ψ_0, the velocity and density scales σ_0 and ρ_1, the halo core radius R_c and the flattening parameter q (the last three of these contained within the parameters A, B, and C). For convenience, we have defined a characteristic halo radius R_a

$$R_a = \left(\frac{3}{2\pi G\rho_1}\right)^{1/2} \sigma_0 e^{\Psi_0/2\sigma_0^2}$$

it is roughly the radius at which the halo rotation curve, if continued at its $R = 0$ slope, would reach the value $2^{1/2}\sigma_0$.

Arbitrary amounts of rotation can be added to the halo model by splitting the DF into parts with positive and negative L_z.

The disc distribution function

In the construction of a realistic three-integral disc distribution function, the issue of a third integral cannot be evaded the vertical and radial dispersions are different, which is not possible in any DF that depends only on energy and angular momentum. The simplest approximate third integral in an axisymmetric disc system is the energy in the vertical oscillations, $E_z \equiv \Psi(R,z) - \Psi(R,0) + \frac{1}{2}v_z^2$. It is quite well conserved along nearly circular orbits which have no large radial or vertical excursions. We will use this quantity as third integral for the disc DF in our models.

$$f_{\rm disc}(E_{\rm p}, L_z, E_z) = \frac{\Omega(R_{\rm c})}{(2\pi^3)^{1/2}\kappa(R_{\rm c})} \frac{\tilde{\rho}_{\rm d}(R_{\rm c})}{\tilde{\sigma}_R^{\,2}(R_{\rm c})\tilde{\sigma}_z(R_{\rm c})} \exp\left[-\frac{E_{\rm p} - E_{\rm c}(R_{\rm c})}{\tilde{\sigma}_R^{\,2}(R_{\rm c})} - \frac{E_z}{\tilde{\sigma}_z^{\,2}(R_{\rm c})}\right]$$

Here, $E_{\rm p} \equiv E - E_z$ is the energy in planar motions, L_z is the specific angular momentum about the axis of symmetry, $R_{\rm c}$ and $E_{\rm c}$ are the radius and energy of a circular orbit with angular momentum L_z, and Ω and κ are the circular and epicyclic frequencies at radius $R_{\rm c}$. The density corresponding to this DF is obtained by integrating over the three velocity components. The v_R- and v_z-integrals are straightforward, leaving the v_ϕ-integral:

$$\rho_{\rm disc}(R,z) = \int_0^\infty \left\{ \left[{\rm d}v_\phi \equiv {\rm d}R_{\rm c} \left(\frac{R_{\rm c}\kappa(R_{\rm c})^2}{2R\Omega(R_{\rm c})} \right) \right] \frac{2\tilde{\rho}_{\rm d}(R_{\rm c})\Omega(R_{\rm c})}{(2\pi)^{1/2}\tilde{\sigma}_R(R_{\rm c})\kappa(R_{\rm c})} \times \right.$$
$$\left. \exp\left[-\frac{\Psi(R,0) - \Psi(R_{\rm c},0)}{\tilde{\sigma}_R^{\,2}(R_{\rm c})} - \left(\frac{R_{\rm c}^2}{R^2} - 1\right) \frac{v_{\rm c}^2(R_{\rm c})}{2\tilde{\sigma}_R^{\,2}(R_{\rm c})} - \frac{\Psi(R,z) - \Psi(R,0)}{\tilde{\sigma}_z^{\,2}(R_{\rm c})} \right] \right\}$$

In the $z = 0$ plane, this expression reduces to $\widetilde{\rho}_{\rm d}(R)$ with fractional error $O(\widetilde{\sigma_R}^2/v_c^2)$, and to the same order the radial velocity distribution is Gaussian with dispersion $\widetilde{\sigma_R}(R)$ The essence of the construction the replacement of the radius R (which is not an integral of motion) by the epicyclic radius R_c (which is a function of angular momentum, and therefore is conserved along orbits). In warm discs, in which excursions from circular orbits are small but not negligible, this parametrization still provides a good starting point for constructing a DF with given radial density and velocity dispersion profiles. The vertical structure of this disc is approximately isothermal, with the scale height set by the vertical velocity dispersion $\widetilde{\sigma_z}(R_c)$ and the vertical potential gradient.

In any gravitational potential, we can adjust the 'tilde' functions $\widetilde{\rho}$, $\widetilde{\sigma_R}$ and $\widetilde{\sigma_z}$ to the desired disc characteristics. we arrange for the disc density to be approximately radially exponential with scale length R_d and truncated at radius $R_{\rm out}$:

$$\rho_{\rm disc}(R,z) = \frac{M_{\rm d}}{8\pi R_{\rm d}^2 z_{\rm d}} {\rm e}^{-R/R_{\rm d}}\, {\rm erfc}\left(\frac{r - R_{\rm out}}{2^{1/2}\delta R_{\rm out}}\right) \exp\left[-0.8676\frac{\Psi_z(R,z)}{\Psi_z(R,z_{\rm d})}\right].$$

Here M_d is a parameter which is close to the mass of the disc unless the disc is severely truncated or the vertical structure is far from $\mathrm{sech}^2(z/z_d)$. $\delta R_{\rm out}$ governs the sharpness of the truncation. The vertical density of these discs is constructed to depend exponentially on the vertical potential $\Psi_z(R,z) \equiv \Psi(R,z) - \Psi(R,0)$, and to drop from the mid-plane value by a factor $\mathrm{sech}^2(1) \simeq \mathrm{e}^{-0.8676}$ at a height of z_d, similar to the behaviour of a constant-thickness isothermal sheet.

Given a total potential for the model, we then set the disc tilde functions in the disc DF as follows. In the limit of very small velocity dispersions these functions are the actual mid-plane density and velocity dispersions. We first choose the function $\widetilde{\sigma_R}(R_c)$, approximately determining the radial velocity dispersion in the disc. $\widetilde{\rho}$ and $\widetilde{\sigma_z}$ are then iteratively adjusted so that the densities on the mid-plane and at height $z = z_d$ agree with those of equation

Calculation of the combined potential

The distribution functions for the various galaxy components all imply a unique volume density in a given potential. To construct a self-gravitating model, we need to find the potential in which the combined density is also the one implied by Poisson's equation, i.e.

$$\nabla^2\Psi(R,z) = 4\pi G[\rho_{\rm disc}(R,\Psi,\Psi_z) + \rho_{\rm bulge}(\Psi) + \rho_{\rm halo}(R,\Psi)].$$

We can generate an N-body realization of a galaxy by randomly sampling from the DFs for each component. The bulge and the halo are straightforward to generate since the systems are nearly spherical and the velocity ellipsoids are nearly isotropic.

A particle's position is first determined by sampling from
the density distribution. With this position, one can find the local maximum of the DF (at $(v_x, v_y, v_z) = (0,0,0)$) and then use the acceptance–rejection technique to find a velocity. This involves selecting the three components of the velocity at random from a velocity sphere with radius equal to the escape velocity. A random value, $f_{\rm ran}$, of the DF is also chosen between 0 and the local maximum. If $f_{\rm ran}$ is less than the value of the DF at the chosen velocity then the velocity is accepted, otherwise it is rejected and another attempt is made.

Softening

In practice, N-body simulations employ a softened form of gravity in order to suppress two-body relaxation. Our models, as formulated above, will therefore not be in equilibrium under these modified forces. In principle, it is possible to solve a suitably modified Poisson equation to allow for the softening: a simply way to do this would be to smooth the density with the appropriate kernel before solving Poisson's equation. While possible, this extra smoothing step can be computationally expensive, and we have not implemented it in what follows. As will be seen, effects of this deficiency are small provided the smoothing length is smaller than relevant length scales in the model.

MODELS OF THE MILKY WAY

In Table 1, we present the parameters for generating a sequence of four models, MW-A, B, C, and D, which have mass distributions and rotation curves closely resembling those of the Milky Way within 5 scale radii. The disc and bulge mass distributions are the same for each model with mass and extent of the halo increasing through the sequence (Table 2). The haloes are all chosen with $q = 1.0$, though they are slightly squashed in the self-consistent galaxy models. Model MW-D has the halo with the largest mass and has the most realistic representation of the outer Galaxy. These models were found by trial and error and renormalized so that the flat portion of the rotation curve had $V_c \approx 1.0$. The contributions to the radial acceleration in the solar neighbourhood ($R = 1.8R_d$) from disc, bulge and halo are comparable in these models, as found by Kuijken & Gilmore (1989) in their study of the local disc surface density. The natural units for length, velocity, and mass for these dimensionless models are $R_d = 4.5$ kpc, $V = 220$ km s^{-1}, and $M = 5.1 \times 10^{10}$ M$_\odot$. The central velocity dispersion was chosen so that the observed radial velocity dispersion of 42 km s^{-1} at the solar radius ($R = 1.8R_d$) would be reproduced in the model. Figs 10 and 11 show the rotation curves out to 5 scale radii and 50 scale radii respectively.

Table 1. Galaxy model parameters.

Model	DISC					BULGE			HALO				
	M_d (1)	R_d (2)	R_t (3)	z_d (4)	δR_{out} (5)	Ψ_c (6)	σ_b (7)	ρ_b (8)	Ψ_0 (9)	σ_0 (10)	q (11)	C (12)	R_a (13)
Sample	1.00	1.0	5.0	0.15	0.3	-2.0	0.50	10.0	-4.0	1.00	0.9	0.1	0.5
MW–A	0.87	1.0	5.0	0.10	0.5	-2.3	0.71	14.5	-4.6	1.00	1.0	0.1	0.8
MW–B	0.87	1.0	5.0	0.10	0.5	-2.9	0.71	14.5	-5.2	0.96	1.0	0.1	0.8
MW–C	0.87	1.0	5.0	0.10	0.5	-3.7	0.71	14.5	-6.0	0.93	1.0	0.1	0.8
MW–D	0.87	1.0	5.0	0.10	0.5	-4.7	0.71	14.5	-7.0	0.92	1.0	0.1	0.8

(1) disc mass, (2) disc scale radius, (3) disc truncation radius, (4) disc scale height, (5) disc truncation width, (6) bulge cutoff potential, (7) bulge velocity dispersion, (8) bulge central density, (9) halo central potential, (10) halo velocity dispersion, (11) halo potential flattening, (12) halo concentration, $C = R_c^2/R_K^2$ (Kuijken & Dubinski 1994), (13) characteristic halo radius.

Table 2. Galaxy model properties.

Model	DISC			BULGE		HALO	
	M (1)	$\sigma_{R,0}$ (2)	R_e/R_d (3)	M (4)	R_e/R_d (5)	M (6)	R_e/R_d (7)
Sample	0.94	0.50	5.6	0.29	1.7	9.6	44.9
MW-A	0.82	0.47	6.0	0.42	1.0	5.2	21.8
MW-B	0.82	0.47	6.0	0.43	1.0	9.6	30.1
MW-C	0.82	0.47	6.0	0.43	1.0	19.8	44.0
MW-D	0.82	0.47	6.0	0.43	1.0	37.0	72.8

(1) disc mass, (2) disc central radial velocity dispersion, $\sigma_{R,0}$, (3) disc radial extent (radius where density drops to zero) in disc scale lengths, (4) bulge mass, (5) bulge radial extent in disc scale lengths, (6) halo mass, (7) halo radial extent in disc scale lengths.

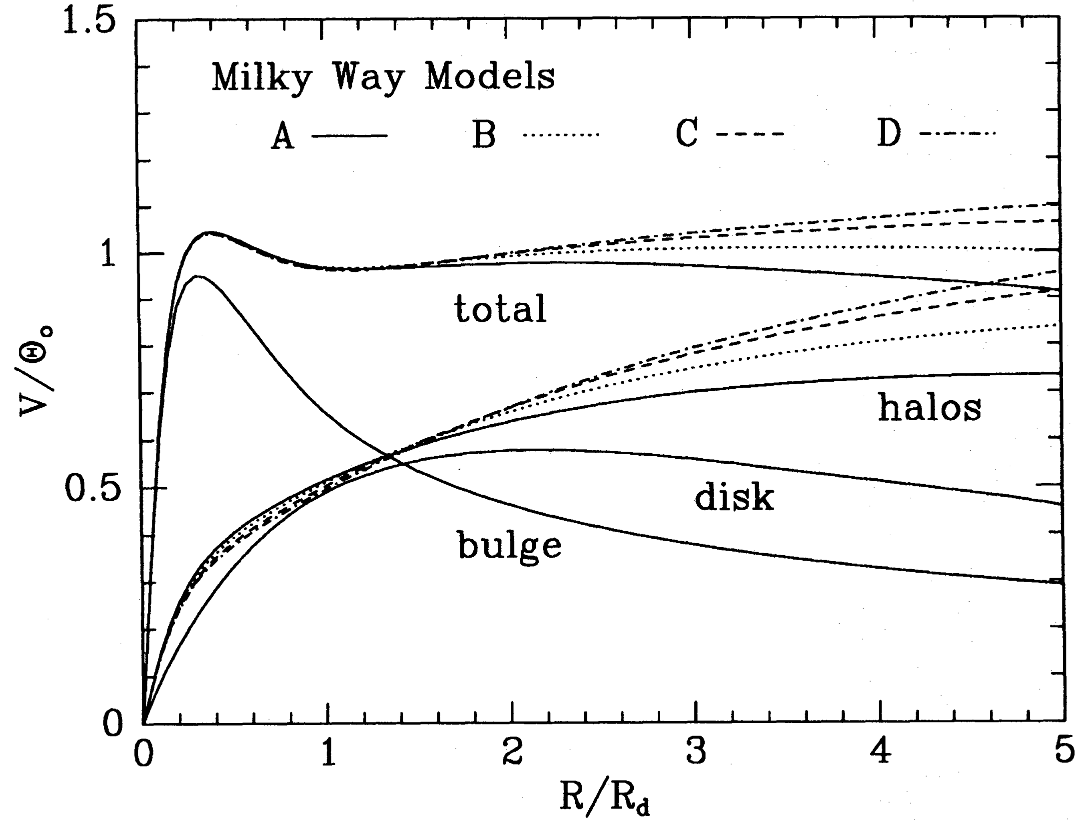

Figure 10. The rotation curves for the Milky Way models, MW-A, B, C, and D, showing contributions from the disc, bulge and halo in the inner regions within $R < 5R_d$.

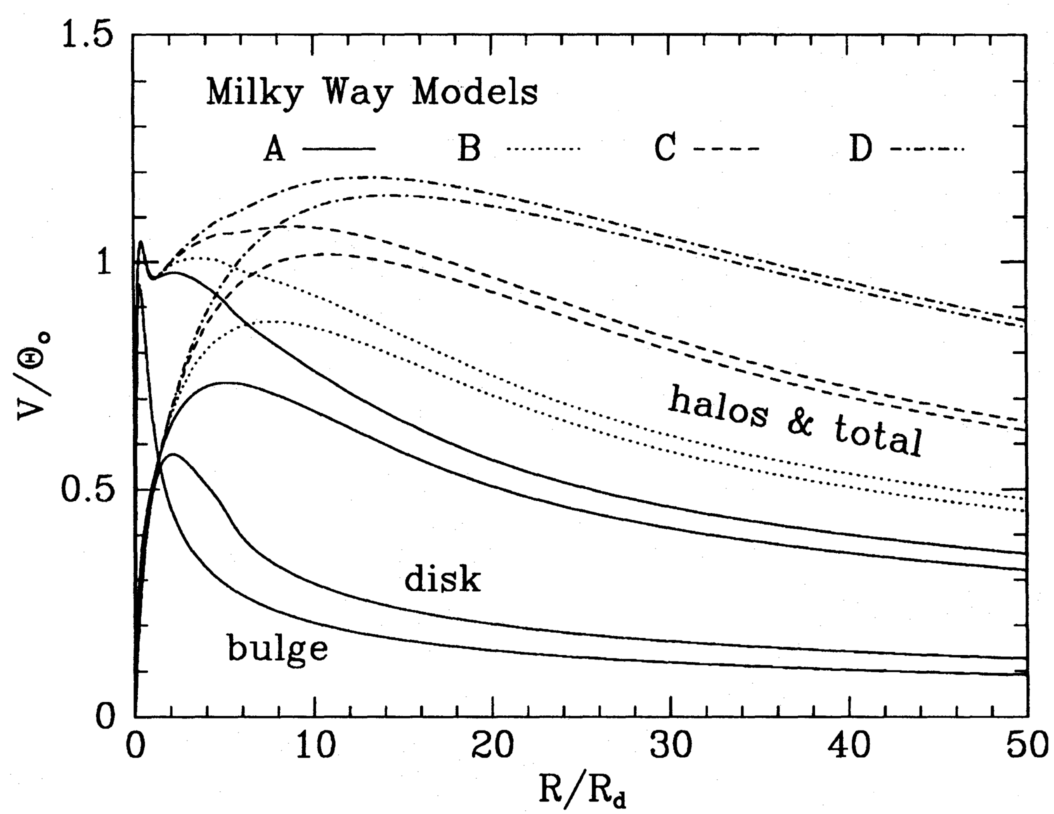

Figure 11. The rotation curves for the Milky Way models, MW-A, B, C, and D, showing contributions from the disc, bulge and halo in the outer regions out to $R = 50R_d$.

GalactICS – A Galaxy Model Building Package

First do the following:

1. cd src

2. type 'make all' – This builds the programs.

Note that some of the code is in fortran 77, using lines longer than 72 characters in some cases. The -e flag in the makefile allow for this for a Solaris f77 compiler. Other programs are written in C. Again, the linking between these routines works on solaris systems, but may need to be adjusted for other architectures. We have found that linking using f77 instead of ld will often automatically load the appropriate libraries.

The graphics output by some of the programs (dbh, plotforce, diskdf, plothalo) use the PGPLOT library. It can be found at http://astro.caltech.edu/~tjp/pgplot/. Alternatively, remove all calls to routines with names starting with "PG", as well as the -lpgplot flag in the Makefile, and the programs should still run fine.

3. type 'make install' – This copies programs to the directory bin/

Now you're set.

Test that the programs run:

Go into directory Milky_Way/A

Type "make galaxy" – this should build all the files and a small N-body galaxy – look at all of the in.* files to see the various parameters that go into building the models. And read below for more details.

The distribution contains the input files for Milky_Way/A through D, the models that were used in Kuijken & Dubinski 1995.

John & Konrad

This package contains a set of programs and subroutines for building galaxy
models including a disk, a bulge and halo. The details of the inner
workings of the code are described in Kuijken and Dubinski 1995.

There are 3 steps in building these models.

A. Calculating the potential.

B. Constructing a disk distribution function which will generate
the given potential.

C. And realizing each component with a self-consistent distribution of
particle orbits.

These actions are all performed by typing `make galaxy', which runs a
succession of programs to end up with a set of N-body particle masses,
positions and velocities representing your model.

Descriptions of the individual steps:

A. The Potential

Program: dbh
Sample input file: in.dbh
Output: dbh.dat - contains tabulated values of the harmonic coefficients
for the Legendre expansion of the density, potential and
radial force at the specified radii for the entire model
h.dat - same as above for halo only
b.dat - same as above for bulge only
mr.dat - gives mass and radial extent (or edge) of disk,bulge and halo

Parameters in in.dbh:

```
y                    #yes we want a halo (or no)
-6.0 1.32 1 .1 0.8   #psi0, v0, q, (rc/rk)^2, ra
y                    #yes we want a disk (or no)
.867 1 5 .1 .5       #M_d, R_d, R_outer, z_d, dR_trunc
y                    #yes we want a bulge (or no)
14.45 -3.7 .714      #rho_b, psi_cut, sig_b
.01 5000             #delta_r, nr
10                   #number of harmonics (even number)
dbh.ps/ps            # PGPLOT graphics device for the plots produced.
```

The program asks for parameters describing each of the components. You
have the option of including any combination of components (though I think
models without a halo won't work).

Halo Parameters:
psi0 - central potential - the smaller (the more negative) this parameter
the deeper the potential and the more extended the halo
v0 - v0 = sqrt(2.0)*sigma0 where sigma0 is the central velocity

dispersion. roughly the velocity where the halo rotation curve peaks
q - an optional flattening parameter for the potential - generally
0.7 < q < 1.05 - q=1.0 will give a nearly spherical halo
(rc/rk)^2 - a core smoothing parameter - ratio of the core radius to the
derived King radius for halo only models set this
to 1.0. For multicomponent models, this can be a smaller
number 0.0 to 0.1. I've found that with this parameter=0.0 the
program can crash.
Ra - a scaling radius for the halo -
The halo Ra radius is the radius at which the halo rotation curve, at its
initial slope ignoring cutoffs and the other components, reaches v0.

Disk Parameters:
M_d - mass of the exponential disk ignoring cutoffs
R_d - exponential scale length
R_outer - outer radius where we begin to truncate the disk density
z_d - disk scale height assuming a sech^2(z/zd) vertical density law
dR_trunc - truncation width - the disk density smoothly drops to zero in
the range R_outer < R < ~R_outer + 2*dR_trunc.

Bulge Parameters:
rho_b - bulge central density
psi_cut - bulge cut-off potential psi0 < psi_cut < 0.0
- energy cut-off for the bulge
sig_b - bulge central potential

```
Potential parameters:
dr       - the width of the radial bins used to calculate the
potential
nr       - number of radial bins - initially a guess since we don't
know
           the radial extent of the system
lmax     - the largest value in the potential harmonic expansion - use
          lmax=2 to get a quick look at the mass profile and lmax=10
for
          the final calculation of the model

Creating a galaxy model from these parameters is sort of a black art
since the halo and bulge models are not parameterized in terms of
their mass profiles but rather properties of their distribution
functions.  Changes in psi0, v0 etc. have weird but predictable
effects on the mass profile.

The halo is a flattened analogue of the King model so the

concentration
(R_tidal/R_core) is determined by the dimensionless central potential
$\psi_0/\sigma_0^2$.  The more negative the value the greater the
concentration.  The parameters $R_a$ and $v_0$, affect the scaling of
the
halo mass profile.

The effect of different bulge parameters is more predictable.
Decreasing the
central velocity dispersion will create a more centrally concentrated
bulge and decreasing the psi cut off will truncate the bulge and
decrease
its total mass.
```

The disk is parameterized directly by its mass profile so its effect
on the
rotation curve is predictable ahead of time.

Hit and miss seems to be a good strategy for finding a suitable
profile.
Generate a model to lmax=2 and then view the resulting rotation curve
by typing
'make vr.dat'. This uses the program to generate the file vr.dat
which
tells you the contributions to the total rotation curve. Another
useful
file is 'mr.dat' which tells you the mass and radial extent of the
disk
bulge and halo.

The program plotforce will also generate the rotation curves for you
directly from the dbh.dat, b.dat and h.dat files.

The potential is determined iteratively: starting from an initial guess at the potential, the density implied by the halo and bulge DFs is calculated, the disk density added, and the potential of that mass distribution is used as starting point for the next iteration. Initially only the monopole (l=0) components are calculated until the model converges, then one more harmonic is added per iteration up to the maximum requested, and once all harmonics are included the iterations are continued until the outer (tidal) radius of the halo is unchanged between iterations. At each iterations plots of the harmonic expansion coefficients are produced.
If the tidal radius reported is "outside grid" for a large number of iterations, increase the number of radial bins or increase their size. Sometimes infinite tidal radii are also reported: this happens when the total mass of the model using the current guess for the potential is insufficient to generate a potential well as deep as requested. If this persists over many iterations, again increase the number or size of the radial bins.

B. Disk distribution function

Program: getfreqs
Input files: dbh.dat h.dat b.dat
Output: freqdbh.dat

getfreqs tabulates various characteristic frequencies (omega, kappa etc.)
in the equatorial plane for use by diskdf below.

Program: diskdf
Input files: freqdbh.dat dbh.dat in.diskdf
Output files: cordbh.dat toomre.dat

The program diskdf iteratively calculates the correction functions for the disk distribution function. These functions are multiplicative corrections to the surface density and vertical velocity dispersion which appear to leading order in the Shu (1969) distribution functions. See KD95 for details. It requires the sample parameters:

```
.47 1.0   #central radial vel. dispersion, exponential scale length of
sig_r^2
50        #number of radial intervals for correction functions
10        #number of iterations
diskdf.ps/ps # PGPLOT device for plot of correction functions.
```

It also outputs the Toomre Q as a function of radius in the file toomre.dat.

C. Generating N-body realizations

Programs: gendisk, genbulge, genhalo
Input files: cordbh.dat dbh.dat

gendisk parameters:

```
4000        #number of particles
-1          #negative random integer seed
1           #1=yes we want to center 0=no we don't
dbh.dat     #multipole expansion data file
```

genbulge parameters:

```
0.5       #streaming fraction
1000      #number of particles
-1        #negative integer seed
1         #center the data 1=yes
dbh.dat   #harmonics file
```

genhalo parameters:

```
0.5      #streaming fraction
6000     #number of particles
-1       #negative integer seed for random number generator
1        #1=yes we want to center
dbh.dat  #multipole expansion data file
```

The streaming fraction, f, sets the fraction of orbits with L_z > 0. The remaining fraction, 1-f, have L_z<0. With this parameter you can therefore vary the rotation of the bulge and the halo. f=0.5 refers to the non-rotating case.

The N-body data are written to the stdout so the programs should be run as:

```
gendisk < in.disk > disk
genbulge < in.bulge > bulge
genhalo < in.halo > halo
```

Format is ascii with data arranged as:

```
N_bodies time
m_1 x_1 y_1 z_1 vx_1 vy_1 vz_1
m_2 x_2 y_2 z_2 vx_2 vy_2 vz_2
m_3 x_3 y_3 z_3 vx_3 vy_3 vz_3
.
.
.
etc.
```

There is a shell script 'mergerv' which can merge the disk, bulge and halo
files into a single N-body file.

The program tobinary turns the ascii files into a simple binary format, listing first the number of particles, then all their masses, then the time, and finally the x,y,z,vx,vy,vz coordinates for each particle.