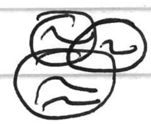
Polymer Drag Reduction, cont'd

N.B.: Separate Module Section (below 2D/GFD) on "Handouts".

Recall: Caveats

→ Phenomenology (~~Large~~ Largely Pipe) (cf FD NY pic)
- drag reduction happens, few ppm sufficient, affects turbulent state
- critical τ_w, related N
- buffer layer forms — "transport barrier"
- C_p dependence.

→ Polymers  not  [simple hydro interaction!?]
- SAW, $R \sim a N^{.6}$
- 'elastic shape' model — better than Rouse due hydro interactions/interference
- relaxation rate $\boxed{1/\tau_Z \sim T/6\pi\eta_s R_c^3}$ Zimm Kis/Yst.
- no need venture to gels, reptation, etc.

→ Questions re: Drag Reduction

- activation — critical wall stress
  ($v(l)/l \gtrsim \frac{1}{\tau_Z}$)

  for $l > l_d$.

- how understand interaction of polymers with fluid turbulence?
  - i.e. enhanced viscosity due stretching (Lumley) ?

e.g. $\nu = \nu\left[1 + \# C R_{eff}^3\right]$

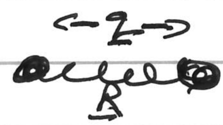
and or ?

  - elasticity / elastodynamics (de Gennes)

N.B. Need not be mutually exclusive

[Favor de Gennes, as coil-stretch transition debatable for stochastic environment of turbulent flow

→ Model — Continuum Dumbbell { variant of Oldroyd-B; NL spring - FENE

$\underline{R}, \underline{q}$ 

$Q_{ij} = \int q_i q_j f(q)\, d^3 q$ ⇒ elastic energy field

stretching

$$\partial_t \underline{\underline{Q}} + \underline{V} \cdot \underline{\nabla}\, \underline{\underline{Q}} = Q_{i\gamma} \partial_\gamma V_j + Q_{i\gamma} \partial_j V_\gamma$$

$$- 4\, \omega_Z Q_{ij} + D_0 \nabla^2 Q_{ij} + \frac{4 k_B T}{\gamma} \delta_{ij}$$

Zimm damping

N.B. Activation criterion emerges from

$|\underline{\nabla}\underline{V}| > \omega_Z$

$\Rightarrow \underbrace{D}_{\text{Deborah \#}} \sim \tilde{\tau}_{Relax} / \tilde{\tau}_{dynamic} > 1$

$D > 1 \Rightarrow$
- activated polymer regime
- polys extend
- pull back on flow

and

$$\rho(\partial_t \underline{V} + \underline{V}\cdot\underline{\nabla}\,\underline{V})_i = -\underline{\nabla}_i P + \eta \nabla^2 \underline{V}_i + \partial_j (\underbrace{c_p}_{\text{concen}} \underbrace{k_T}_{\text{entropic spring}} Q_{ij}) + \underline{f}_i$$

cons. Laws

Viscoelastic

obvious analogy $\rightarrow$ MHD

$$\partial_t \underline{B} + \underline{V}\cdot\underline{\nabla}\,\underline{B} = \underline{B}\cdot\underline{\nabla}\,\underline{V} + \eta_R \nabla^2 \underline{B}$$

($R_m \gg 1 \Rightarrow \underline{B}$ "Frozen into" $\underline{V}$)

and

$$\rho(\partial_t \underline{V} + \underline{V}\cdot\underline{\nabla}\,\underline{V}) = -\underline{\nabla} p^* + \frac{\underline{B}\cdot\underline{\nabla}\,\underline{B}}{4\pi} + \eta \nabla^2 \underline{V} + \underline{f}$$

cf. Fermi: "magneto-elastic wave"

$Q_{ij} \leftrightarrow \dfrac{\underline{B}\,\underline{B}}{4\pi} \Rightarrow$ Maxwell stress.

<u>TBC</u>

→ Drag Reduction – <u>Scaling Concepts</u>
(follows deGennes '86 et seq.)
→ useful to have continuum model in mind.

– Polymer + Cascade (i.e. homogeneous turbulence)

How does polymer interact with K41?

– identify $\underline{3}$ scales l^*, l^{**}, l_d

need $\boxed{l^* > l^{**} > l_d}$

①
→ l^* → <u>activation scale</u> – i.e. polymer stretched

$$\frac{v(l)}{l} = \frac{\epsilon^{1/3}}{l^{2/3}} \sim 1/\tau_Z \qquad \text{size } (l^* > l_d)$$

$$\therefore \quad \boxed{l^* = \left(\epsilon \tau_Z^3\right)^{1/2}} \sim N^{2.7} \epsilon^{1/2}$$

$$v^* = \left(\epsilon \tau_Z\right)^{1/2} \sim N^{.9} \epsilon^{1/2}$$

and obviously need $\ell^* > \ell_d$ so

$Re(\ell^*) > 1$

$$\Rightarrow \boxed{\epsilon \tilde{\tau}_Z^2/\nu > 1}$$

$\Rightarrow N^{3.6}\epsilon$ → favors big polymers!
(degradation?!).

Point: $\ell < \ell^*$ $\quad \frac{v(\ell)}{\ell} > \frac{1}{\tau_Z}$

[redacted]

⇒ coil/polymer 'frozen into' flow

⇒ coil follow deformations of local volume element

∴ defines:

$$\ell^* > \ell > \ell^{**}$$

→ "Passive range" (?!)

Hypothesize:

$$\lambda(r) \simeq (\ell^*/\ell)^n$$

→ unknown exponent

↓ polymer elongation on scale ℓ (in passive range)

$n = 1, 2$

↓ based on extensional flow

$n \sim 1$

This brings us to:

→ ℓ^{**} → elastic scale.

Flow stretches polymers, so:

$$F \simeq k_b T \lambda^{5/2}$$

→ deviates from harmonic due shape, repulsion changes (i.e. why not 2)

↳ elongation factor

F ↓ elastic free energy → single polymer coil

or Free energy / volume

$$\boxed{F_{el} = \frac{c}{N} k_b T \lambda^{5/2} = \boxed{G} \lambda^{5/2}}$$

G ↓ elastic modulus $\sim c$, linearly!

so $f_1 \simeq \frac{kT}{R_G} \lambda^{3/2}$ → restoring force on 1 spring

and $\tau \simeq \frac{c}{N} f_1 \lambda R_G \to F_{el}$

τ ↓ stress due c/N springs / vol.

Begs the question → what happens at energy balance?

i.e. $\boxed{\ell^{**}}$ is where:

$$\boxed{\mathcal{G}\left[\underline{X}(\ell^{**})\right]^{5/2} = \rho\, V(\ell^{**})^2}$$

⇒ scale of energy equipartition)

Usual K41-ology ⇒

$$\frac{\ell^{**}}{\ell^{*}} = \left[\frac{\mathcal{G}}{\rho V(\ell^*)^2}\right]^{r} \equiv X^{r}$$

$$\underbrace{X \equiv \frac{\mathcal{G}}{\rho V(\ell^*)^2} \sim \underline{\underline{C}}}\; N^{-2.8} \, \epsilon^{-1}$$

raising c raises $\ell^* \leftrightarrow \ell^{**}$ interval

⇓

X (and thus $\mathcal{G}$) is natural measure of concentration effects.

$$r = \left(\frac{5n}{2} + 2/3\right)^{-1} \qquad n = 1, 2$$

$$r \sim 1/3,\ 1/6$$

⇒ Point: Higher concentration favors favors k_i, E_s energy equipartition (i.e. departure from 'passivity') at larger ℓ.

of course, need (for "elastization"):

$$l^{**} > l_d.$$

Now, $\dfrac{l^*}{l_d} \simeq (Re^*)^{3/4}$

Need, $\boxed{X > (Re^*)^{-3/4\nu}}$

⇒ imposes a critical c_p for onset of drag reduction phenomena!

⇒ $\boxed{C_{crit} \sim N^{2.8-2.7/\nu}\, \epsilon^{1-3/4\nu}}$

$1/\nu \rightarrow 3 < \nu < 6$

C_{crit} increases sharply with $\underline{\underline{N}}$!

[redacted]

[redacted] So, now have:

$$l_0 > l_* > l_{**} > l_d$$

$\downarrow$ K41 range

$\downarrow$ passive range

$\downarrow$ elastic range.

What happens in elastic range?
what is fate of the energy?

P.G. deG:

"At the scales $\ell = \ell^{**}$, the liquid should behave like a strongly distorted rubber, carrying elastic waves (longitudinal and transverse) with comparable kinetic and elastic wave energies."

"On the whole, it is tempting to assume that the formation of new eddies is strongly restricted for $\ell < \ell^{**}$. ... this would then lead to a <u>truncation</u> in the cascade at $\ell = \ell^{**}$."

N.B. Recalling $\frac{u_*^3}{x} = \epsilon$

in pipe, can map $\ell \leftrightarrow x$, etc.

But: why truncation ?!?

⇒ Better (?!) → Conversion to elastic wave cascade

→ akin MHD turbulence!

To pg. 33 — Alfven wave.

→ and where is ℓ^{**}?
K41 modified by elasticity

Key: Cascade can persist, in wave channel, with drag reduction via stress cancellation.

Re: Models

(cf. Ogilvie & Proctor, Hinch notes)

Maxwell:
$$(\text{stress}) + \tau \frac{d(\text{stress})}{dt} = \underset{(\text{visc})}{\eta} \frac{d}{dt}(\text{strain})$$

$\tau / T \ll 1$, $D \ll 1$
$$\Rightarrow \text{stress} \approx \eta \frac{d}{dt}(\text{strain})$$
$$\sigma \approx -\eta \nabla V \quad (\text{fluid})$$

$\tau / T > 1$, $D > 1$
$$\Rightarrow (\text{stress}) \approx \underbrace{\frac{\eta}{\tau}}_{E} (\text{strain}) \quad (\text{elastic solid})$$

$\Rightarrow$ ~~[illegible]~~ Viscoelastic Fluid:

$$\boxed{\underline{\underline{T}} + \tau\left[\frac{\partial \underline{\underline{T}}}{\partial t} + \underline{v} \cdot \nabla \underline{\underline{T}} - (\nabla \underline{v})^T \cdot \underline{\underline{T}} - \underline{\underline{T}} \cdot \nabla \underline{v}\right] = \eta_p\left[(\nabla \underline{v}) + (\nabla \underline{v})^T\right]}$$

– Oldroyd B model (Dumbell).

Now MHD:

$$\frac{\partial \underline{B}}{\partial t} + \underline{v}\cdot\underline{\nabla}\,\underline{B} = \underline{B}\cdot\underline{\nabla}\,\underline{v} + \eta\,\nabla^2\underline{B}$$

Now;

$$\underline{\underline{T}}_{poly} = \underline{\underline{T}} + \frac{\mu_p}{\tau}\underline{\underline{I}}$$

$$\frac{\partial \underline{\underline{T}}_p}{\partial t} + \underline{u}\cdot\underline{\nabla}\,\underline{\underline{T}}_p - \underline{\underline{\nabla u}}\cdot\underline{\underline{T}}_p - (\underline{\underline{\nabla u}})^T\cdot\underline{\underline{T}}_p$$

$$= \frac{1}{\tau}\left(\underline{\underline{T}}_p - \frac{\mu_p}{\tau}\underline{\underline{I}}\right)$$

and $T_M = \frac{\underline{B}\,\underline{B}}{4\pi}$ (by construction)

magnetic

$$\partial_t \underline{\underline{T}}_M + \underline{v}\cdot\underline{\nabla}\,\underline{\underline{T}}_M - (\underline{\nabla u})^T\cdot\underline{\underline{T}}_M - \underline{\underline{T}}_M\cdot\underline{\nabla u}$$

$$= \eta\left[\underline{B}\,\nabla^2\underline{B} + (\nabla^2\underline{B})\,\underline{B}\right]$$

and clear $\lim\limits_{\tau\to\infty}(\text{Oldroyd-B}) = \lim\limits_{R_m\to\infty} \text{MHD}$