Physics 4E   Quiz 1 Solutions   April 13, 2020

Problem 1

$\lambda_1 = 3000\,Å$, $\lambda_2 = 2000\,Å$, ϕ = work function

$\frac{hc}{\lambda_1} - \phi = eV_0$ $\Rightarrow$ $eV_0 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$ $\Rightarrow$

$\frac{hc}{\lambda_2} - \phi = 2eV_0$ $\quad$ $eV_0 = \frac{\cancel{hc}\,12{,}400}{2000}\,eV - \frac{12{,}400}{3000}\,eV$

$\Rightarrow eV_0 = 2.067\,eV \Rightarrow \boxed{V_0 = 2.067\,\text{Volts}}$ (b)

$\phi = \frac{hc}{\lambda_1} - eV_0 = \frac{12{,}400}{3000}\,eV - 2.067\,eV \Rightarrow$

$\boxed{\phi = 2.066\,eV}$ (c)

(more precisely, $\phi = eV_0$ because $\lambda_2 = \frac{2}{3}\lambda_1$)

Maximum wavelength λ_m:

$\frac{hc}{\lambda_m} = \phi \Rightarrow \lambda_m = \frac{hc}{\phi} = \frac{12{,}400}{2.066}\,Å$

$\Rightarrow \boxed{\lambda_m = 6002\,Å}$ (a) (more precisely, $\lambda_m = 2\lambda_1 = 6000\,Å$)

Problem 2

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta) \; ; \quad \lambda_c = \frac{h}{m_e c} = 0.0243\,\text{Å}$$

$\lambda' = 0.5\,\text{Å}, \quad \theta = 45° \Rightarrow$

$$\lambda = \lambda' - \lambda_c (1 - \cos\theta) = 0.5\,\text{Å} - \lambda_c \left(1 - \frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow \boxed{\lambda = 0.493\,\text{Å}} \text{ (a)}$$

(b) K_e = kinetic energy of electron

$$K_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 12{,}400\,\text{eV} \left(\frac{1}{0.493} - \frac{1}{0.5} \right)$$

$$\Rightarrow \boxed{K_e = 352\,\text{eV}} \text{ (b)}$$

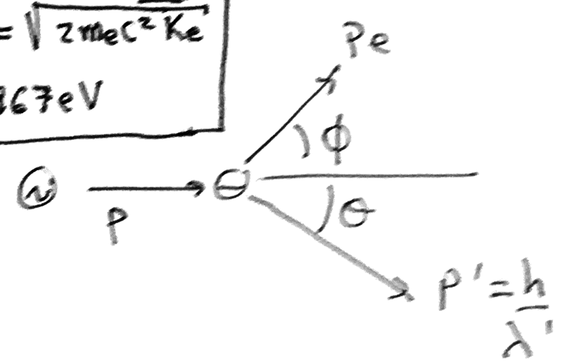
(c) $K_e = \sqrt{p_e^2 c^2 + m_e^2 c^4} - m_e c^2 \Rightarrow p_e^2 c^2 + m_e^2 c^4 = (K_e + m_e c^2)^2$

$$\Rightarrow p_e c = \sqrt{(K_e + m_e c^2)^2 - (m_e c^2)^2} = 18{,}970\,\text{eV}$$

$$\boxed{p_e c = 18{,}970\,\text{eV}} \text{ (c)}$$

or:
$K_e = \frac{p_e^2}{2m_e} \Rightarrow p_e c = \sqrt{2 m_e c^2 K_e}$
yields $p_e c = 18{,}967\,\text{eV}$

(d) Momentum conservation in y direction:



$$p_e \sin\phi = p' \sin\theta \Rightarrow$$

$$\Rightarrow \sin\phi = \frac{p'}{p_e} \sin\theta = \frac{h}{\lambda' p_e} \sin\theta = \frac{hc}{\lambda' p_e c} \sin\theta$$

$$\Rightarrow \sin\phi = \frac{12{,}400}{0.5 \times 18{,}970} \sin\theta = 1.31 \sin\theta = 0.924$$

$$\Rightarrow \boxed{\phi = 67.5°} \text{ (d)}$$

Problem 3

(a) temperature at surface is given by Wien's law

$$T = \frac{2.898 \times 10^{-3}\, m \cdot K}{30{,}000 \times 10^{-10}\, m} = \boxed{966K} \text{ (a)}$$

Temperature at center plays no role, is unknown

(b) $A = 4\pi R^2$ is area of sphere

Power emitted: $P = A\sigma T^4 \Rightarrow A = P/\sigma T^4 \Rightarrow$

$$A = \frac{10^5\, W m^2 K^4}{5.67 \times 10^{-8}\, W \times 966^4 K^4} = \boxed{2.025 m^2} \Rightarrow \boxed{R = 40cm} \text{ (b)}$$

(c) Relation between power emitted per unit area and energy density:

$$J = u \frac{c}{4} \quad \text{, with } c = \text{speed of light}$$

$$u = \frac{8\pi}{\lambda^4} \bar{E} \quad , \quad \bar{E} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$$

since $\frac{hc}{\lambda k T} = \frac{12{,}400\, eVA}{10^{10} A \cdot \frac{1}{11{,}600} \frac{eV}{K} \cdot 966 K} \approx 10^{-5} \ll 1$, classical regime

or, we know since $\lambda = 10^{10} A \gg \lambda_m = 30{,}000 A$, $\Rightarrow$ classical regime

$\Rightarrow \boxed{\bar{E} \approx k_B T}$

Power emitted with λ in interval $\Delta\lambda$: $\Delta\lambda = 1.05m - 0.95m = 0.1m$

$$P = \frac{c}{4} \cdot u \cdot \Delta\lambda \cdot A \Rightarrow P = \frac{3\times 10^8}{4} \frac{m}{s} \times \frac{8\pi}{\lambda^4} \cdot k_B T \cdot \Delta\lambda \cdot A =$$

$$= \frac{3 \times 10^8}{4} \frac{\cancel{m}}{s} \times \frac{8\pi}{\cancel{(1m)^4}} \times \frac{1}{11{,}600} \frac{eV}{\cancel{K}} \times 966 \cancel{K} \times 0.1 \cancel{m} \times 2.025 \cancel{m^2} \Rightarrow$$

$$\Rightarrow P = \frac{3 \times 10^8 \times 8\pi \times 966 \times 2.025}{4 \times 11{,}600} \frac{eV}{s} \times 1.6 \times 10^{-19} \frac{J}{eV} \times 0.1 = \boxed{5.1 \times 10^{-12} W}$$

(c)