PHYSICS 4E | FINAL EXAM | SPRING QUARTER 2020
PROF. HIRSCH | 8 problems | JUNE 9

<u>Open book</u>. <u>Show all steps in your calculations.</u> <u>Justify all answers.</u> Write clearly.

$hc = 12,400eVA,\ \ \hbar c = 1973eVA\ ,\ m_e c^2 = 511,000eV\ ,\ k_B = 1/11,600eV/K$

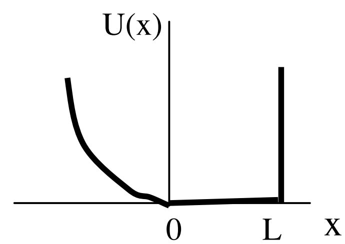
$ke^2 = 14.4eVA\ ;\ \ 1A = 10^{-10}m\ ;\ \ c = 3\cdot 10^8 m/s\ ;\ \ \hbar^2/m_e = 7.62eVA^2$

$m_p/m_e = 1836\ \ ;\ \ \mu_B = 5.79\times 10^{-5}eV/T\ \ ;\ \ \int_0^\infty dx x^n e^{-\lambda x} = n!/\lambda^{n+1}$

Problem 1 (10 pts)
An electron is in a Bohr orbit of a hydrogen-like ion that has radius 0.529A. The kinetic energy of this electron is 340eV.
(a) What is its angular momentum, in units of $\hbar$, according to the Bohr model?
(b) What is the wavelength of the photon emitted when this electron makes a transition to the lowest energy state of this ion? (in A)
(c) What is the de Broglie wavelength of this electron, in A?

Problem 2 (10 pts + 3pts extra credit)
An electron is in the ground state of a finite one-dimensional square well of depth U=8eV that extends from x=-a to x=+a. Its energy is 7eV (measured from the bottom of the well)
(a) Find the value of a, in A.
(b) What would be the ground state energy of the electron in an infinite well of this width? Explain qualitatively why it is very different from that in the case considered here.
(c) How much more likely is it to find this electron at x=0 than at x=a?
(d) How much more likely is it to find this electron at x=0 than at x=2a?

Problem 3 (10 pts +4pts extra credit)
An electron is in a stationary state of the potential shown in the figure, given by:



$U(x) = \frac{1}{2} m\omega^2 x^2 \qquad x<0$

$U(x)=0 \qquad 0<x<L$

$U(x) = \infty \qquad x>L$

m is the electron mass, and $\hbar\omega = 3eV$. Its wavefunction in the region x<0 is

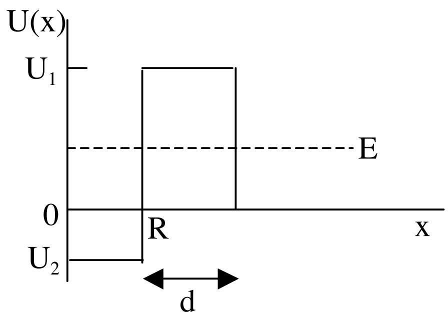
$\psi(x) = Axe^{-(\lambda/2)x^2}$ where A and λ are constants.

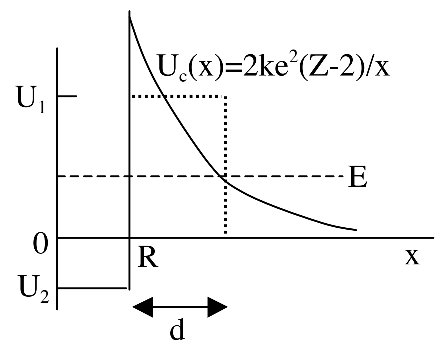
(a) What is the energy of this electron, in eV, and the value of λ in A^{-2}?
(b) What is the smallest value that L can have, in A? Justify your answer.
(c) For the L found in (b), make a qualitative plot of the wavefunction of the electron in the entire region shown in the figure given above.
(d) For the L found in (b), how much more likely is it to find the electron in the region x<0 than in the region x>0? Justify your answer.
<u>Hint:</u> use continuity of the wavefunction and its derivative at x=0.

<u>Hint 2:</u> $\int_0^\infty dx x^2 e^{-\lambda x^2} = \frac{1}{4}\sqrt{\frac{\pi}{\lambda^3}}$

Problem 4 (10 pts)
There are 21 electrons in a three-dimensional cubic box of side length L=6A. Electrons have spin 1/2 and obey the Pauli principle, for this problem we assume that they don't interact with each other and that there is no spin-orbit coupling. Assume the system is in the ground state.
(a) What is the longest wavelength photon that can be absorbed by this system?
(b) At what temperature will the electron that is in the highest energy state in the ground state be equally likely to be in the next highest energy state?
(c) If you use the Fermi-Dirac distribution to describe this system, what is the value of the Fermi energy of this system, in eV?

Problem 5 (10 pts)





Consider the simplified model for α decay shown in the figure on the left. The α particle moves in a one-dimensional potential well of length R and has energy E. Its potential energy is $U(x)=U_2$ for $0<x<R$, $U(x)=U_1$ for $R<x<R+d$, $U(x)=0$ for $x>R+d$. Initially, it is inside the nucleus, i.e. in the region $0<x<R$.
A more realistic model for α decay is given in the figure on the right, where the barrier is given by the Coulomb potential $U_c(x)$. The distance d is the distance that the particle has to tunnel through to escape.

Assume $R=3\text{x}10^{-5}$A and U_1=15MeV. Use that $m_\alpha=7295m_e$
Assume that $U_2=0$.
(a) Calculate E in MeV. You may assume for this purpose that U_1=oo for simplicity, i.e. an infinite square well. Assume the α particle is in the ground state.
(b) Obtain d from the diagram on the right, in A. Note that R+d is the distance of closest approach for the reverse process, a particle incident from the right with kinetic energy E on a nucleus of charge (Z-2)e. Assume Z=48.
(c) With this d, calculate the probability for the α particle to tunnel out of this nucleus in one attempt assuming the simplified potential of the left figure and using the simplified expression for tunneling probability discussed in class (Eq. 7.10 in textbook).
(d) Calculate the attempt frequency (in s^{-1}) assuming that the particle travels a distance 2R between attempts, with speed determined by its kinetic energy.
(e) Estimate the lifetime of this nucleus as the inverse of the tunneling probability per unit time. Give your answer in seconds.

Problem 6 (10 pts)
Consider problem 5 again, assume now U_2=-2MeV, repeat all the above steps and give answers for each point (a) through (e) for this case. For (e), give your answer in years.

Problem 7 (10 pts)
Consider a hydrogen atom in a magnetic field B=15T pointing in the z direction. Assume the electron is in a stationary state with principal quantum number n=3. Take into account spin but ignore spin orbit coupling.
(a) How many different stationary states can this electron be in? List all the states by giving their quantum numbers.
(b) What is the energy difference between the state of lowest and highest energy of the list given in (a)? Give their quantum numbers and the energy difference in eV.
(c) Find an expression for the uncertainty in the x-component of the angular momentum, ΔL_x. Give the quantum nubers of the state(s) with highest and lowest values of ΔL_x (excluding 0) among the states listed in (a), and give the value of ΔL_x for those states, as a number multiplying $\hbar$.
Hint: note that $\Delta L_x = \Delta L_y$.

Problem 8 (10 pts)
Consider a system of N_A one-dimensional harmonic oscillators, where $N_A = 6.02 \times 10^{23}$, Avogadro's number. All oscillators have the same frequency ω. At temperature T=12K, this system has average energy $U = 1.0001 U_0$, where U_0 is the energy of the system at T=0.
(a) Find the ground state energy per oscillator, give your answer in eV.
(b) Find the Einstein temperature of this system, T_E, in K ($k_B T_E = \hbar\omega$).
(c) Estimate the heat capacity of this system at 4000K, in J/K. Justify your answer.
(d) Find the ratio of the heat capacity of this system at 40K to its heat capacity at 4000K.