3-2 Assume that your skin can be considered a blackbody. One can then use Wien's displacement law, $\lambda_{\max} T = 0.289\,8 \times 10^{-2}\ \text{m} \cdot \text{K}$ with $T = 35\ {}^{0}\text{C} = 308\ \text{K}$ to find

$$\lambda_{\max} = \frac{0.289\,8 \times 10^{-2}\ \text{m} \cdot \text{K}}{308\ \text{K}} = 9.41 \times 10^{-6}\ \text{m} = 9\,410\ \text{nm} .$$

3-4 (a) From Stefan's law, one has $\frac{P}{A} = \sigma T^{4}$. Therefore,

$$\frac{P}{A} = \left(5.7 \times 10^{-8}\ \text{W}/\text{m}^{2} K^{4}\right)(3000\ \text{K})^{4} = 4.62 \times 10^{6}\ \text{W}/\text{m}^{2} .$$

(b) $$A = \frac{P}{4.62 \times 10^{6}\ \text{W}/\text{m}^{2}} = \frac{75\ \text{W}}{4.62 \times 10^{6}\ \text{W}/\text{m}^{2}} = 16.2\ \text{mm}^{2} .$$

3-5 (a) Planck's radiation energy density law as a function of wavelength and temperature is given by $u(\lambda, T) = \frac{8\pi hc}{\lambda^{5}\left(e^{hc/\lambda_B T} - 1\right)}$. Using $\frac{\partial u}{\partial \lambda} = 0$ and setting $x = \frac{hc}{\lambda_{\max} k_B T}$, yields an extremum in $u(\lambda, T)$ with respect to λ. The result is

$$0 = -5 + \left(\frac{hc}{\lambda_{\max} k_B T}\right)\left(e^{hc/\lambda_{\max} k_B T}\right)\left(e^{hc/\lambda_{\max} k_B T} - 1\right)^{-1} \text{ or } x = 5\left(1 - e^{-x}\right).$$

(b) Solving for x by successive approximations, gives $x \cong 4.965$ or

$$\lambda_{\max} T = \left(\frac{hc}{k_B}\right)(4.965) = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K} .$$

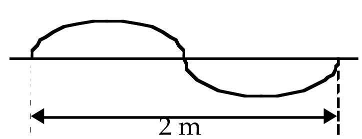
3-7 (a) In general, $L = \frac{n\lambda}{2}$ where $n = 1, 2, 3, \ldots$ defines a mode or standing wave pattern with a given wavelength. As we wish to find the number of possible values of n between 2.0 and 2.1 cm, we use $n = \frac{2L}{\lambda}$

$$n(2.0\ \text{cm}) = (2)\frac{200}{2.0} = 200$$

$$n(2.1\ \text{cm}) = (2)\frac{200}{2.1} = 190$$

$$|\Delta n| = 10$$

As n changes by one for each allowed standing wave, there are 10 standing waves of different wavelength between 2.0 and 2.1 cm.



(b) The number of modes per unit wavelength per unit length is

$$\frac{\Delta n}{L \Delta \lambda} = \frac{10}{0.1}(200) = 0.5\ \text{cm}^{-2} .$$

(c) For short wavelengths n is almost a continuous function of λ. Thus one may use calculus to approximate $\frac{\Delta n}{L\Delta \gamma} = \left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right)$. As $n = \frac{2L}{\lambda}$, $\left|\frac{dn}{d\lambda}\right| = \frac{2L}{\lambda^2}$ and $\left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right) = \frac{2}{\lambda^2}$. This gives approximately the same result as that found in (b): $\left(\frac{1}{L}\right)\left(\frac{dn}{d\lambda}\right) = \frac{2}{\lambda^2} = \frac{2}{(2.0\ \text{cm})^2} = 0.5\ \text{cm}^{-2}$.

(d) For short wavelengths n is almost a continuous function of λ, $n = \frac{2L}{\lambda}$ is a discrete function.