3-8 Using $E = hf$ with $h = 4.136 \times 10^{-15}$ eV gives

(a) for $f = 5 \times 10^{14}$ Hz, $E = 2.07$ eV

(b) for $f = 10$ GHz, $E = 4.14 \times 10^{-5}$ eV

(c) for $f = 30$ MHz, $E = 1.24 \times 10^{-7}$ eV

3-9 Use $E = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{E}$ (where $hc = 1\,240$ eV nm) to find

(a) $\lambda = 600$ nm

(b) $\lambda = 0.03$ m

(c) $\lambda = 10$ m

3-10 The energy per photon, $E = hf$ and the total energy E transmitted in a time t is Pt where power $P = 100$ kW. Since $E = nhf$ where n is the total number of photons transmitted in the time t, and $f = 94$ MHz, there results $nhf = (100\text{ kW})t = (10^5\text{ W})t$, or

$$\frac{n}{t} = \frac{10^5\text{ W}}{hf} = \frac{10^5\text{ J/s}}{6.63 \times 10^{-34}\text{ J s}}(94 \times 10^6\text{ s}^{-1}) = 1.60 \times 10^{30}\text{ photons/s}.$$

3-12 As in Problems 3-9 and 3-10,

$$\frac{n}{t} = \frac{P}{hf} = \frac{P\lambda}{hc} = (10\text{ W})\frac{589 \times 10^{-9}\text{ m}}{1.99 \times 10^{-25}\text{ J m}} = 3.0 \times 10^{19}\text{ photons/s}.$$

3-13 $K = hf - \phi = \frac{hc}{\lambda - \phi}$

$$\phi = \frac{hc}{\lambda - K} = \frac{1\,240\text{ eV nm}}{250\text{ nm}} - 2.92\text{ eV} = 2.04\text{ eV}$$

3-14 (a) $K = hf - \phi = \frac{hc}{\lambda - \phi} = \frac{1\,240\text{ eV nm}}{350\text{ nm}} - 2.24\text{ eV} = 1.30\text{ eV}$

(b) At $\lambda = \lambda_c$, $K = 0$ and $\lambda = \frac{hc}{\phi} = \frac{1\,240\text{ eV nm}}{2.24\text{ eV}} = 554\text{ nm}$

3-15 (a) At the cut-off wavelength, $K = 0$ so $\frac{hc}{\lambda} - \phi = 0$, or

$\lambda_{\text{cut-off}} = \frac{hc}{\phi} = \frac{1\,240\text{ eV nm}}{4.2\text{ eV}} = 300\text{ nm}$. The threshold frequency, f_0 is given by

$$f_0 = \frac{c}{\lambda_{\text{cut-off}}} = \frac{3.0\times 10^8 \text{ m/s}}{3.0\times 10^2 \times 10^{-9} \text{ m}} = 1.0\times 10^{15} \text{ Hz}.$$

(b) $eV_s = K = hf - \phi = \frac{hc}{\lambda} - \phi$

$$V_s = \frac{hc}{\lambda e} - \frac{\phi}{e} = \frac{1\,240 \text{ eV nm}}{200 \text{ nm e}} = -4.2 \text{ eV/e} = 2.0 \text{ V}$$

3-18 (a) $K_{max} = eV_s = s(0.45 \text{ V}) = 0.45 \text{ eV}$

(b) $\phi = \frac{hc}{\lambda - K} = \frac{1\,240 \text{ eV nm}}{500 \text{ nm}} - 0.45 \text{ eV} = 2.03 \text{ eV}$

(c) $\lambda_c = \frac{hc}{\phi} = \frac{1\,240 \text{ eV nm}}{2.03 \text{ eV}} = 612 \text{ nm}$

3-20 $K_{max} = hf - \phi = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - K_{max}$;

First Source: $\phi = \frac{hc}{\lambda} - 1.00 \text{ eV}$.

Second Source: $\phi = \frac{hc}{\frac{\lambda}{2}} - 4.00 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV}$.

As the work function is the same for both sources (a property of the metal), $\frac{hc}{\lambda} - 100 \text{ eV} = \frac{2hc}{\lambda} - 4.00 \text{ eV} \Rightarrow \frac{hc}{\lambda} = 3.00 \text{ eV}$ and $\phi = \frac{hc}{\lambda} - 1.00 \text{ eV} = 3.00 \text{ eV} - 1.00 \text{ eV} = 2.00 \text{ eV}$.

3-25 $E = 300 \text{ keV}$, $\theta = 30°$

(a) $\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta) = (0.002\,43 \text{ nm})[1 - \cos(30°)] = 3.25\times 10^{-13} \text{ m}$

$= 3.25\times 10^{-4} \text{ nm}$

(b) $E = \frac{hc}{\lambda_0} \Rightarrow \lambda_0 = \frac{hc}{E_0} = \frac{(4.14\times 10^{-15} \text{ eVs})(3\times 10^8 \text{ m/s})}{300\times 10^3 \text{ eV}} = 4.14\times 10^{-12} \text{ m}$; thus,

$\lambda' = \lambda_0 + \Delta\lambda = 4.14\times 10^{-12} \text{ m} + 0.325\times 10^{-12} \text{ m} = 4.465\times 10^{-12} \text{ m}$, and

$E' = \frac{hc}{\lambda'} \Rightarrow E' = \frac{(4.14\times 10^{-15} \text{ eV s})(3\times 10^8 \text{ m/s})}{4.465\times 10^{-12} \text{ m}} = 2.78\times 10^5 \text{ eV}$.

(c) $\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$, (conservation of energy)

$$K_e = hc\left(\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right) = \frac{(4.14\times 10^{-15} \text{ eV s})(3\times 10^8 \text{ m/s})}{\frac{1}{4.14\times 10^{-12}} - \frac{1}{4.465\times 10^{-12}}} = 22 \text{ keV}$$

3-28 (a) From conservation of energy we have $E_0 = E' + K_e = 120 \text{ keV} + 40 \text{ keV} = 160 \text{ keV}$.

The photon energy can be written as $E_0 = \frac{hc}{\lambda_0}$. This gives

$$\lambda_0 = \frac{hc}{E_0} = \frac{1\,240 \text{ nm eV}}{160 \times 10^3 \text{ eV}} = 7.75 \times 10^{-3} \text{ nm} = 0.007\,75 \text{ nm}.$$

(b) Using the Compton scattering relation $\lambda' - \lambda_0 = \lambda_c(1 - \cos\theta)$ where $\lambda_c = \frac{h}{m_e c} = 0.002\,43 \text{ nm}$ and $\lambda' = \frac{hc}{E'} = \frac{1\,240 \text{ nm eV}}{120 \times 10^3 \text{ eV}} = 10.3 \times 10^3 \text{ nm} = 0.010\,3 \text{ nm}$. Solving the Compton equation for $\cos\theta$, we find

$$-\lambda_c \cos\theta = \lambda' - \lambda_0 - \lambda_c$$

$$\cos\theta = 1 - \frac{\lambda' - \lambda_0}{\lambda_c} = 1 - \frac{0.010\,3 \text{ nm} - 0.007\,5 \text{ nm}}{0.002\,43 \text{ nm}} = 1 - 1.049 = -0.049$$

The principle angle is $87.2°$ or $\theta = 92.8°$.

(c) Using the conservation of momentum Equations 3.30 and 3.31 one can solve for the recoil angle of the electron.

$$p = p' \cos\theta + p_e \cos\phi$$

$p_e \sin\phi = p' \sin\theta$; dividing these equations one can solve for the recoil angle of the electron

$$\tan\phi = \frac{p' \sin\theta}{p - p' \cos\theta} = \left(\frac{h}{\lambda'}\right) \frac{\sin\theta}{\frac{h}{\lambda_0} - \frac{h}{\lambda' \cos\theta}} = \left(\frac{hc}{\lambda'}\right) \frac{\sin\theta}{\frac{hc}{\lambda_0} - \frac{hc}{\lambda' \cos\theta}}$$

$$= \frac{120 \text{ keV}(0.998\,8)}{160 \text{ keV} - 120 \text{ keV}(-0.049)} = 0.723\,2$$

and $\phi = 35.9°$.

3-30 Maximum energy transfer occurs when the scattering angle is 180 degrees. Assuming the electron is initially at rest, conservation of momentum gives

$$hf + hf' = p_e c = \sqrt{\left(m_e c^2 + K\right)^2 - m^2 c^4} = \sqrt{(511 + 50)^2} = 178 \text{ keV}$$

while conservation of energy gives $hf - hf' = K = 30 \text{ keV}$. Solving the two equations gives $E = hf = 104 \text{ keV}$ and $hf = 74 \text{ keV}$. (The wavelength of the incoming photon is $\lambda = \frac{hc}{E} = 0.012\,0 \text{ nm}$.

3-31 (a) $E' = \frac{hc}{\lambda'}$, $\lambda' = \lambda_0 + \Delta\lambda$

$$\lambda_0 = \frac{hc}{E_0} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{0.1\ \text{MeV}} = 1.243\times10^{-11}\ \text{m}$$

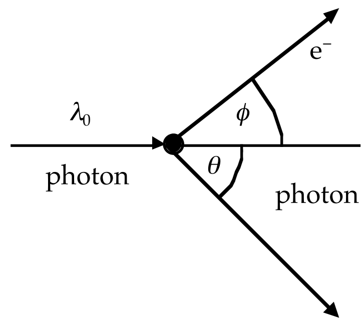
$$\Delta\lambda = \left(\frac{h}{m_e c}\right)(1-\cos\theta) = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(1-\cos 60°\right)}{\left(9.11\times10^{-34}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)} = 1.215\times10^{-12}\ \text{m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 1.364\times10^{-11}\ \text{m}$$

$$E' = \frac{hc}{\lambda'} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3\times10^{8}\ \text{m/s}\right)}{1.364\times10^{-11}\ \text{m}} = 9.11\times10^{4}\ \text{eV}$$

(b) $$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + K_e$$

$$K_e = 0.1\ \text{MeV} - 91.1\ \text{keV} = 8.90\ \text{keV}$$



(c) Conservation of momentum along x: $\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'}\right)\cos\theta + \gamma m_e v\cos\phi$. Conservation of momentum along y: $\left(\frac{h}{\lambda'}\right)\sin\theta = \gamma m_e v\sin\phi$. So that

$$\frac{\gamma m_e v\sin\phi}{\gamma m_e v\cos\phi} = \left(\frac{h}{\lambda'}\right)\sin\theta\left[\left(\frac{h}{\lambda_0}\right) - \left(\frac{h}{\lambda'}\right)\cos\theta\right]$$

$$\tan\phi = \frac{\lambda_0\sin\theta}{\left(\lambda' - \lambda_0\right)\cos\theta}$$

Here, $\theta = 60°$, $\lambda_0 = 1.243\times10^{-11}\ \text{m}$, and $\lambda' = 1.364\times10^{-11}\ \text{m}$. Consequently,

$$\tan\phi = \frac{\left(1.24\times10^{-11}\ \text{m}\right)\left(\sin 60°\right)}{\left(1.36 - 1.24\cos 60°\right)\times10^{-11}\ \text{m}} = 1.451$$

$$\phi = 55.4°$$

3-33 Substituting equations 3-33 and 3-34 of the text, $E_e = h\left(f_0 - f'\right) + m_e c^2$ and

$$p_e^2c^2 = h^2\left(f'^2 + f_0^2\right) - 2h^2 f' f_0\cos\theta$$

into the relativistic energy expression $E_e^2 = p_e^2c^2 + \left(m_e c^2\right)^2$ yields

$$h^2\left(f'^2 + f_0^2 - 2f_0 f'\right) + m_e^2 c^4 + 2h(f_0 - f')m_e c^2 = h^2\left(f'^2 + f_0^2\right) - 2h^2 f_0 f' \cos\theta^2 + \left(m_e c^2\right)^2.$$

Canceling and combining there results

$$\left(f'^2 + f_0^2 - 2f_0 f'\right) + \frac{2m_e c^2(f_0 - f')}{h} = f'^2 + f_0^2 - 2f_0 f' \cos\theta$$

which reduces to $\frac{m_e c^2(f_0 - f')}{h} = f_0 f'(1 - \cos\theta)$. Using $\lambda f = c$ one obtains $\lambda' - \lambda_0 = \frac{h(1 - \cos\theta)}{m_e c}$, which is the Compton scattering or Compton shift relation.

3-43 (a) A 4000 Å wavelength photon is backscattered, $\theta = \pi$ by an electron. The energy transferred to the electron is determined by using the Compton scattering formula $\lambda' - \lambda_0 = \left(\frac{hc}{E_e}\right)(1 - \cos\theta)$ where we take $E_e = m_e c^2$ for the rest energy of the electron

and so $E_e \approx 0.511\ \text{MeV}$. Upon substitution, one obtains

$$\Delta\lambda = 2(0.002\ 43\ \text{nm}) = 0.004\ 86\ \text{nm}.$$

The energy of a photon is related to its wavelength by the relation $E = \frac{hc}{\lambda}$, so the change in energy associated with a corresponding change in wavelength is given by $\Delta E = -\left(\frac{hc}{\lambda^2}\right)\Delta\lambda$. Upon making substitutions one obtains the magnitude $\Delta E = 6.037\ 9 \times 10^{-24}\ \text{J}$ and using the conversion factor 1 Joule of energy is equivalent to $1.602 \times 10^{-19}\ \text{eV}$. The result is $\Delta E = 3.77 \times 10^{-5}\ \text{eV}$.

(b) This may be compared to the energy that would be acquired by an electron in the photoelectric effect process. Here again the energy of a photon of wavelength λ is given by $E = \frac{hc}{\lambda}$. With $\lambda = 400\ \text{nm}$, one obtains

$$E = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.0 \times 10^8\ \text{m/s})}{400 \times 10^{-9}\ \text{m}} = 4.97 \times 10^{-19}\ \text{J}$$

and upon converting to electron volts, $E = 3.10\ \text{eV}$. $\frac{\Delta E}{E_{\text{photon}}} \approx 10^{-5}$. The maximum energy transfer is about five orders of magnitude smaller than the energy necessary for the photoelectric effect.

(c) Could "a violet photon" eject an electron from a metal by Compton scattering? The answer is no, because the maximum energy transfer occurring at $\theta = \pi$ is not sufficient.

3-44 Each emitted electron requires an energy

$$hf = \frac{1}{2}mv^2 + \phi = \left(\frac{9.11\times10^{-31}\ \text{kg}}{2}\right)\left(4.2\times10^{5}\ \text{m/s}\right)^2 + \left(3.44\ \text{eV}\right)\left(1.6\times10^{-19}\ \text{J/eV}\right)$$

$\Delta E = 6.3\times10^{-19}\ \text{J}$ per emitted electron.

Therefore, with an incident intensity of $\frac{0.055\ \text{J/m}^2}{\text{s}} = \frac{5.5\times10^{-6}\ \text{J/cm}^2}{\text{s}}$, the number of electrons emitted per cm^2 per second is

$$\text{electron flux} = \frac{\frac{5.5\times10^{-8}\ \text{J/cm}^2}{\text{s}}}{6.3\times10^{-19}\ \text{J/emitted electron}} = \frac{8.73\times10^{12}}{\text{cm}^2/\text{s}}.$$