PHYSICS 2D QUIZ 5 FALL QUARTER 2020
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Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c}\ /\ \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\ MeV/c^2$ Proton: $m_p = 938.26\ MeV/c^2$ Neutron: $m_n = 939.55\ MeV/c^2$

Atomic mass unit: $1\ u = 931.5\ MeV/c^2$; electron volt: $1\text{eV} = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_B T)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda)\times \overline{E}(\lambda, T) = \dfrac{8\pi}{\lambda^4}\times \dfrac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV\,A$; $k_B = (1/11{,}600) eV/K$

Photoelectric effect: $eV_s = K_{\max} = hf - \phi$, $\phi \equiv$ work function;

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243A$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n = C\dfrac{Z^2}{K_\alpha^2}\dfrac{1}{\sin^4(\phi/2)}$ $ke^2 = 14.4\ eV\,A$

Hydrogen spectrum: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = 1.097\times 10^7\ m^{-1} = \dfrac{1}{911.3A}$; $\hbar c = 1973\ eV\ A$

Bohr atom: $E_n = -\dfrac{ke^2 Z}{2r_n} = -E_0\dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k \Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Probability: $P(x)dx = |\Psi(x)|^2\, dx$; $P(a \le x \le b) = \int_a^b dx P(x)$ $\hbar c = 1973\ eVA$

Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\Psi}{\partial x^2} + \mathrm{U(x)}\Psi(\mathrm{x,t}) = \mathrm{i}\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\mathrm{x,t}) = \psi(\mathrm{x})e^{-\mathrm{i}\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2} + \mathrm{U(x)}\psi(\mathrm{x}) = \mathrm{E}\psi(\mathrm{x})$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Harmonic oscillator: $\Psi_n(x) = H_n(x)e^{-\frac{m\omega}{2\hbar}x^2}$; $E_n = (n+\frac{1}{2})\hbar\omega$; $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2 A^2$

Expectation value of$[Q]$: $<Q> = \int \psi^*(x)[Q]\psi(x)\ dx$; Momentum operator: $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$

Eigenvalues and eigenfunctions: $[Q]\ \Psi = q\ \Psi$ (q is a constant) ; uncertainty: $\Delta Q = \sqrt{<Q^2> - <Q>^2}$

Step potential: reflection coef : $R = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$, $T = 1 - R$; $k = \sqrt{\frac{2m}{\hbar^2}(E - U)}$

Tunneling: $\psi(x) \sim e^{-\alpha x}$; $T = e^{-2\alpha\Delta x}$; $T = e^{-2\int_{x_1}^{x_2}\alpha(x)dx}$; $\alpha(x) = \sqrt{\frac{2m[U(x) - E]}{\hbar^2}}$

Schrodinger equation in 3D: $-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi + \mathrm{U}(\vec{\mathrm{r}})\Psi(\vec{\mathrm{r}},\mathrm{t}) = \mathrm{i}\hbar\frac{\partial\Psi}{\partial t}$; $\Psi(\vec{\mathrm{r}},\mathrm{t}) = \psi(\vec{\mathrm{r}})e^{-\mathrm{i}\frac{E}{\hbar}t}$

3D square well: $\Psi(\mathrm{x,y,z}) = \Psi_1(x)\Psi_2(y)\Psi_3(z)$; $\mathrm{E} = \frac{\pi^2\hbar^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2})$

Spherically symmetric potential: $\Psi_{n,\ell,m_\ell}(r,\theta,\phi) = R_{n\ell}(r)Y_\ell^{m_\ell}(\theta,\phi)$; $Y_\ell^{m_\ell}(\theta,\phi) = P_\ell^{m_\ell}(\theta)e^{im_\ell\phi}$

Angular momentum: $\vec{\mathrm{L}} = \vec{r}\times\vec{p}$; $[L_z] = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$; $[L^2]Y_\ell^{m_\ell} = \ell(\ell+1)\hbar^2 Y_\ell^{m_\ell}$; $[\mathrm{L_z}]Y_\ell^{m_\ell} = m_\ell\hbar Y_\ell^{m_\ell}$

Radial probability density: $P(r) = r^2|R_{n\ell}(r)|^2$; Energy: $\mathrm{E_n} = -\frac{ke^2}{2a_0}\frac{Z^2}{n^2}$

Ground state of hydrogen-like ions: $\Psi_{1,0,0} = \frac{1}{\pi^{1/2}}(\frac{Z}{a_0})^{3/2}e^{-Zr/a_0}$; $\int_0^\infty dr\ \mathrm{r}^n e^{-\lambda r} = \frac{n!}{\lambda^{n+1}}$

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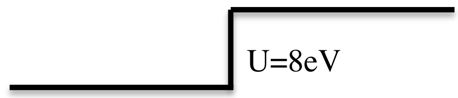
Problem 1 (10 points)



Electrons coming from the left, all with the same kinetic energy, are incident on the step potential shown in the figure, of height 8eV. 1/4 of the incident electrons are reflected, 3/4 are transmitted.

(a) What is the kinetic energy of the reflected electrons, in eV? What is the kinetic energy of the transmitted electrons, in eV?

(b) How much slower or faster do the transmitted electrons move compared to the reflected electrons? Give the ratio of their speeds.

(c) Assume now instead that electrons are incident from right to left, so they go from higher potential to lower potential. If again 1/4 of the incident electrons are reflected and 3/4 are transmitted, what is the kinetic energy of the reflected and transmitted electrons in eV? Justify your answers.

Problem 2 (10 points)

Consider an electron in a three-dimensional box of side lengths $L_1=L_2=L$, $L_3=2L$. The energy of the lowest state is 2.25 eV.

(a) Find the quantum numbers and degeneracy of the four lowest energy levels in this box.

(b) Give the energy of these energy levels, in eV (the lowest one is 2.25 eV, give the other three).

(c) Assume now L_3 is reduced to a value smaller than 2L, L_1 and L_2 stay the same (=L). For which value of L_3 larger than L will one of the four lowest energy levels be triply degenerate? Give L_3 in terms of L.

Problem 3 (10 points)

An electron in a hydrogen-like ion has wavefunction

$$\psi(r,\theta,\phi) = Cre^{-r/a_0}\cos\theta$$

where C is a constant and a_0 is the Bohr radius.

(a) Give the values of the quantum numbers n, ℓ and m_ℓ and of the ionic charge Z. Justify each of your answers.

(b) Find the uncertainties in (i) L_z and in (ii) L_x, the z and x-components of the angular momentum, denoted by ΔL_z and ΔL_x, as numbers multiplying $\hbar$. Justify your answers.

(c) Calculate the average value of r for the electron in this state. Give your answer in terms of a_0.