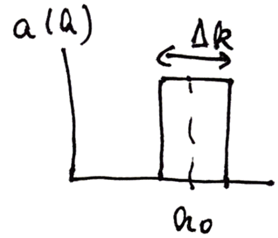
Physics 2D

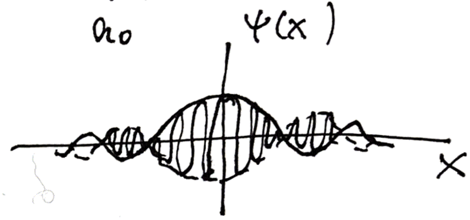
Quiz 4 Solutions

Fall 2020

Problem 1



$$\psi(x) = \int a(k) e^{ikx}\, dk$$



$a(k) = A$ for $k_0 - \Delta k < k < k_0 + \Delta k$

$k_0 = 0.1\ \text{Å}^{-1}$, $\Delta k = 0.02\ \text{Å}^{-1}$

Uncertainty relation for wavepackets:

$$\Delta x \Delta k \sim 1 \Rightarrow \Delta x = \frac{1}{\Delta k} = \frac{1}{0.02\ \text{Å}^{-1}}$$

$$\Rightarrow \boxed{\Delta x \sim 50\ \text{Å}} \quad (a)$$

(b) $p = \hbar k$, $\Delta p = \hbar \Delta k = \dfrac{\hbar c\, \Delta k}{c} = 1973\,\text{eV}\,\text{Å} \times 0.02\,\text{Å}^{-1}$

$$\Rightarrow \boxed{\Delta p \cong 39\,\frac{\text{eV}}{c}}$$

(c) The speed of the electron, assuming it's non-relativistic (check that later) is

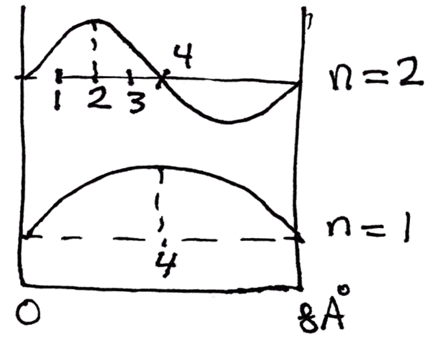
$$v = \frac{p}{m_e},\quad p = \hbar k_0 \Rightarrow v = \frac{\hbar k_0}{m_e} \Rightarrow \frac{v}{c} = \frac{\hbar c k_0}{m_e c^2}$$

$$\Rightarrow \frac{v}{c} = \frac{1973\,\text{eV}\,\text{Å} \cdot 0.1\,\text{Å}^{-1}}{511{,}000\ \text{eV}} = 3.9 \times 10^{-4} \Rightarrow \text{it is non-relativistic}$$

$$\Rightarrow v = 3.9 \times 10^{-4} c = 116\ \text{km/s}$$

So after 1s, the position of the electron is $\boxed{x \sim 116\ \text{km}}$ (initially the electron is at $x \sim 0$)

Problem 2



Clearly, for n = 2 the electron is equally likely to be at x = 1 and at x = 3.

Its energy is

$$E_2 = \frac{\hbar^2 \pi^2}{2mL^2} \cdot 2^2 = \frac{3.81\,eV\,\mathring{A}^2\,\pi^2}{8^2\,\mathring{A}^2} \cdot 2^2 = 2.35\,eV$$

(a) $\boxed{E = 2.35\,eV,\ n = 2}$

(b) The wavefunction is $\psi_2(x) = \sqrt{\frac{2}{L}}\,\sin\left(\frac{2\pi x}{L}\right)$

$$\frac{P(2A)}{P(1A)} = \frac{\psi_2(2A)^2}{\psi_2(1A)^2} = \frac{\sin^2\frac{\pi}{2}}{\sin^2\frac{\pi}{4}} = \boxed{2 \Rightarrow \text{twice as likely}}\ (b)$$

(c)

$$P(0.96 < x < 1.04) \simeq |\psi(x=1)|^2 \times (1.04 - 0.96) =$$

$$= |\psi(x=1)|^2 \times 0.08 = \frac{2}{8} \cdot \sin^2\frac{\pi}{4} \times 0.08 = \frac{1}{50} \times \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{100}$$

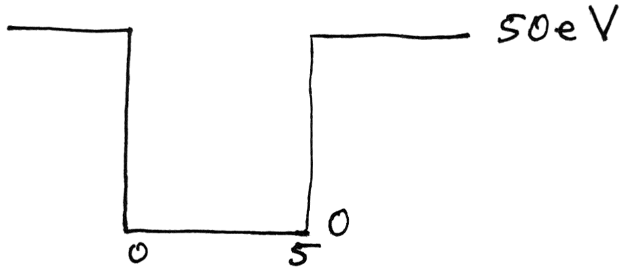
$$\Rightarrow \boxed{P(0.96 < x < 1.04) = 1\,\% = 0.01}$$

The classical value would be a uniform distribution

$$P_{cl} = \Delta x \cdot \frac{1}{L} = \frac{0.08}{8} = 1\,\%$$

$\Rightarrow$ $\boxed{\text{it is the same}}$

Problem 3



$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$\Rightarrow E_n = 1.504\,\text{eV} \cdot n^2$

For $n=5$, $E_5 = 37.6$, for $n=6$, $E_6 = 54.15$

So for sure there are at least $\boxed{\text{5 states}}$, and probably $\boxed{\text{6 states}}$

because the energy of the electron in the finite well is lower than in the ∞ well, and the energy has to be $< 50\,\text{eV}$.

(b) The wavefunction for $x > 5\,\text{Å}$ is

$\psi(x) \sim e^{-\alpha x}$, with $\alpha = \sqrt{\frac{2m}{\hbar^2}(U_0 - E_1)}$

for the lowest state. Assuming $E_1 = \frac{\hbar^2\pi^2}{2mL^2} 1^2 = 1.504\,\text{eV}$

$\Rightarrow \alpha = \sqrt{\frac{50 - 1.504}{3.81}}\ \text{Å}^{-1} \Rightarrow \alpha = 3.57\ \text{Å}^{-1}$

$\psi(x) \sim e^{-\alpha x} = e^{-x/\delta}$, electron penetrates a distance

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{3.57\ \text{Å}^{-1}} = 0.28\ \text{Å}}\ \text{(b)}$$

(c) So the effective length of the well is

$L_{eff} \cong L + 2\delta = 5.56\ \text{Å}$, so the energy

is $E_1' = \frac{\hbar^2\pi^2}{2m(L+2\delta)^2} = 1.22\,\text{eV}$

so energy is lower by $\boxed{E_1 - E_1' \cong 0.29\,\text{eV}}$