Justify all your answers to all 3 problems. Write clearly.

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p / \gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$ $\gamma = 1/\sqrt{1 - v^2/c^2}$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c} \;/\; \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m\vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Atomic mass unit: $1\, u = 931.5\, MeV/c^2$; electron volt: $1\text{eV} = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda)\times \overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4}\times\dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\; eV\,A$; $k_B = (1/11{,}600)eV/K$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function;

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$; $\dfrac{h}{m_e c} = 0.0243A$ $ke^2 = 14.4eVA$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n(\theta) = C\dfrac{Z^2}{K_\alpha^2}\dfrac{1}{\sin^4(\theta/2)}$; $b = \dfrac{kq_\alpha Q}{2K_\alpha}\cot(\theta/2)$

Hydrogen: $\dfrac{1}{\lambda_{mn}} = R(\dfrac{1}{m^2} - \dfrac{1}{n^2})$; $R = \dfrac{1}{911.8A}$; $\hbar c = 1973\text{eVA}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -E_0\dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \dfrac{m_e v^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_e ke^2} = 0.529A$; $L = m_e vr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j \cos(k_j x - \omega_j t)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg : $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Probability: $P(x)dx = |\Psi(x)|^2 \, dx$; $P(a \le x \le b) = \int_a^b dx P(x)$ $\hbar c = 1973 \; eVA$

Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar \frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x) e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation : $-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx \; \psi^* \psi = 1$

∞ square well : $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2 \hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Expectation value of$[Q]$: $< Q > = \int \psi^*(x)[Q]\psi(x) \, dx$; Momentum operator : $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$