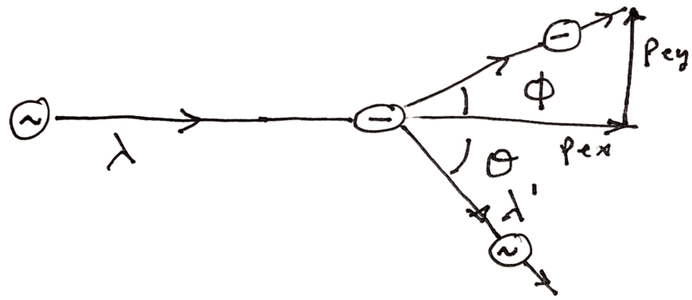
Physics 2D   Quiz 3 Solutions   Fall 2020



$\lambda = 0.5\,Å$, $\lambda' = 0.51215\,Å$; $\lambda' - \lambda = \lambda_c(1-\cos\theta)$; $\lambda_c = 0.0243\,Å$

$$\Rightarrow 1-\cos\theta = \frac{\lambda'-\lambda}{\lambda_c} = \frac{0.01215}{0.0243} = 0.5 \Rightarrow \cos\theta = 0.5 \Rightarrow$$

$\boxed{\theta = 60°}$ (a)

(b) $$K_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = 12{,}400\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \text{eV Å} \Rightarrow$$

$$\Rightarrow \boxed{K_e = 588.3 \text{ eV}}$$

(c) $$p_{e,y} = p'_y = \frac{h}{\lambda'}\sin\theta = \frac{hc}{\lambda' c}\sin\theta = \frac{12{,}400}{0.51215}\sin 60° \cdot \frac{1}{c}$$

$$\Rightarrow \boxed{p_{e,y} = 20{,}967.9 \text{ eV/c}}$$

(d) $$p_x = p'_x + p_{e,x} \Rightarrow p_{e,x} = p_x - p'_x \Rightarrow$$

$$p_{e,x} = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta = 12{,}400\left(\frac{1}{0.5} - \frac{1}{0.51215}\cos 60°\right)\frac{\text{eV}}{c}$$

$$\Rightarrow \boxed{p_{e,x} = 12{,}694.2 \text{ eV/c}}$$

(e) $$\tan\phi = \frac{p_{e,y}}{p_{e,x}} = \frac{20{,}967.9}{12{,}694.2} = 1.652$$

$$\Rightarrow \boxed{\phi = 58.8°}$$

Problem 2

$$\Delta n(\theta) = \frac{C}{\sin^4\frac{\theta}{2}} \Rightarrow \Delta n(\theta_2) = \frac{\sin^4\theta_1/2}{\sin^4\theta_2/2}\,\Delta n(\theta_1)$$

$\Delta n(\theta = 120°) = 1{,}777$, hence

$$\Delta n(\theta = 90°) = \frac{\sin^4 60°}{\sin^4 45°} \times 1{,}777 = 3{,}998$$

$$\Delta n(\theta = 180°) = \frac{\sin^4 60°}{\sin^4 90°} \times 1{,}777 = 1{,}000$$

$\boxed{\text{So we expect } 3{,}995\ \alpha\text{-ppm at } 90°,\ 1{,}000\ \alpha\text{-ppm at } 180°}$

(b) Rutherford formula predicts 1,000, if only 800 are seen it means the α-particle are penetrating the nucleus of radius R.

The distance of closest approach, d_{min}, assuming the α-particle stays outside the nucleus, is given by: $K_\alpha = \frac{2ke^2Z}{d_{min}} \Rightarrow d_{min} = \frac{2ke^2Z}{K_\alpha}$

$Z = 30$, $K_\alpha = 10\ MeV \Rightarrow d_{min} = \frac{2 \times 14.4 \times 30}{10 \times 10^6}\ Å = 8.64 \times 10^{-5}\ Å$

So we can conclude that $\boxed{R > 8.64 \times 10^{-5}\ Å}$ (b)

(c) To find R, we would use α-particles of lower kinetic energy and find the largest value of K_α for which the Rutherford formula holds for $\theta = 180°$, i.e. for which 1,000 α-particles are seen at 180° for every 3,998 scattered at 90° or 1,777 at 120°. Calling that value $\bar{K}_\alpha$ ($\bar{K}_\alpha < 10\ MeV$), we would estimate the radius of the nucleus to be $\boxed{R \approx \frac{2ke^2Z}{\bar{K}_\alpha}}$ (c)

Problem 3

We are given the radius of the orbit, $r = 1.058\,\mathring{A}$.

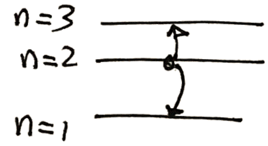
For atomic number Z, the n-th orbit has radius

$$r_n = \frac{a_0}{Z} n^2, \quad a_0 = 0.529\,\mathring{A} \Rightarrow \frac{Z}{n^2} = \frac{a_0}{r_n} = \frac{0.529}{1.058} = 0.5$$

So $\frac{Z}{n^2} = \frac{1}{2}$. Z has to be integer, so $n=1$ is not possible.

$Z = \frac{n^2}{2}$, smallest possible is $n = 2$, $\boxed{Z = 2}$. (a)

next smallest possible is $n = 4$, $\boxed{Z = 8}$



(b) largest wavelength photon it can absorb is $n=2$ to $n=3$ transition:

$$\frac{hc}{\lambda} = E_0 Z^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = E_0 Z^2 \cdot \frac{5}{36}$$

$$\Rightarrow \lambda = \frac{hc}{E_0 Z^2} \cdot \frac{36}{5} = \frac{12{,}400}{13.6 \times 4} \times \frac{36}{5}\,\mathring{A} \Rightarrow \boxed{\lambda_{abs} = 1641.2\,\mathring{A}}$$

largest wavelength (and only) photon it can emit is $n = 2$ to $n = 1$ transition. $\frac{1}{1^2} - \frac{1}{2^2} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow$

$$\lambda = \frac{hc}{E_0 Z^2} \frac{4}{3} = 303.9\,\mathring{A} \qquad \boxed{\lambda_{emit} = 303.9\,\mathring{A}}$$

(c) Use angular momentum quantization, $L = n\hbar$

$$L = m_e v r = p \cdot r \Rightarrow L_n = n\hbar = P_n \cdot r_n \Rightarrow P_n = \frac{n\hbar}{r_n}$$

here $n = 2$, $r_n = 1.058\,\mathring{A} \Rightarrow P_n = \frac{2\hbar}{1.058\,\mathring{A}}$

$$\Rightarrow P_n c = \frac{2\hbar c}{1.058\,\mathring{A}} = \frac{2 \times 1973\,eV\mathring{A}}{1.058\,\mathring{A}} \Rightarrow \boxed{P_n = 3{,}729.7\ eV/c}$$