

Justify all your answers to all 3 problems. Write clearly.

Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma \Delta t_p$; $L = L_p / \gamma$; $c = 3 \times 10^8 \text{ m/s}$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \rightarrow -v$

Velocity transformation: $u_x' = \frac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \rightarrow -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$ $\gamma = 1/\sqrt{1 - v^2/c^2}$

Relativistic Doppler shift: $f_{obs} = f_{source} \sqrt{1 + v/c} / \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m \vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2 c^2 + m^2 c^4}$

Electron: $m_e = 0.511 \text{ MeV}/c^2$ Proton: $m_p = 938.26 \text{ MeV}/c^2$ Neutron: $m_n = 939.55 \text{ MeV}/c^2$

Atomic mass unit: $1 u = 931.5 \text{ MeV}/c^2$; electron volt: $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67 \times 10^{-8} \text{ W}/\text{m}^2 \text{K}^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda, T) d\lambda$; Wien's law: $\lambda_m T = \frac{hc}{4.96 k_B}$

Boltzmann distribution: $P(E) = C e^{-E/(k_B T)}$

Planck's law: $u_\lambda(\lambda, T) = N_\lambda(\lambda) \times \bar{E}(\lambda, T) = \frac{8\pi}{\lambda^4} \times \frac{hc/\lambda}{e^{hc/\lambda k_B T} - 1}$; $N(f) = \frac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12,400 \text{ eV \AA}$; $k_B = (1/11,600) \text{ eV}/\text{K}$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, ϕ = work function;]

Compton scattering: $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta)$; $\frac{h}{m_e c} = 0.0243 \text{ \AA}$ $ke^2 = 14.4 \text{ eV \AA}$

Coulomb force: $F = \frac{kq_1 q_2}{r^2}$; Coulomb energy: $U = \frac{kq_1 q_2}{r}$; Coulomb potential: $V = \frac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Rutherford scattering: $\Delta n(\theta) = C \frac{Z^2}{K_\alpha^2} \frac{1}{\sin^4(\theta/2)}$; $b = \frac{kq_\alpha Q}{2K_\alpha} \cot(\theta/2)$

Hydrogen: $\frac{1}{\lambda_{mn}} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$; $R = \frac{1}{911.8 \text{ \AA}}$; $hc = 1973 \text{ eV \AA}$

Bohr atom: $E_n = -\frac{ke^2 Z}{2r_n} = -E_0 \frac{Z^2}{n^2}$; $E_0 = \frac{ke^2}{2a_0} = \frac{m_e (ke^2)}{2\hbar^2} = 13.6 \text{ eV}$; $K = \frac{m_e v^2}{2}$; $U = -\frac{ke^2 Z}{r}$

$hf = E_i - E_f$; $r_n = r_0 n^2$; $r_0 = \frac{a_0}{Z}$; $a_0 = \frac{\hbar^2}{m_e ke^2} = 0.529 \text{ \AA}$; $L = m_e v r = n\hbar$ angular momentum

Problem 1 (10 points)

In a Compton scattering experiment, the incident photon has wavelength 0.5\AA and the scattered photon has wavelength 0.51215\AA .

- Find the angle at which the photon was scattered relative to the direction of incidence, in degrees.
- Find the kinetic energy of the scattered electron, in eV.
- Find the component of the electron momentum perpendicular to the direction of incidence, in eV/c .
- Find the component of the electron momentum parallel to the direction of incidence, in eV/c .
- Find the angle at which the electron was scattered relative to the direction of incidence, in degrees. Your answer should be accurate to at least one decimal point.

Problem 2 (10 points)

In a Rutherford scattering experiment with α particles of kinetic energy 10MeV incident on a foil of zinc ($Z=30$), $1,777$ α particles per minute are detected at angle 120° .

- How many α particles per minute do you expect will be detected at angles (i) 90° and (ii) 180° ?
- Assume 800 α particles per minute are detected at angle 180° . What can you conclude about the radius of this nucleus?
- Assume you have at your disposal α particles of kinetic energy lower than 10MeV but not higher. Explain how you could get more information about the radius of this nucleus beyond what you learned from (b).

Problem 3 (10 points)

In a hydrogen-like ion, the electron is in a stationary state where the radius of the orbit is 1.058\AA .

- What is the smallest possible value for Z (atomic number) for this ion? What is the next to smallest possible value for Z ?
Assuming the smallest Z found in (a), answer (b) and (c):
- (i) What is the largest possible wavelength photon that this ion can absorb? (ii) What is the largest possible wavelength photon that this ion can emit? Give the answers in \AA .
- What is the magnitude of the linear momentum of the electron in this orbit, p ? Give your answer in eV/c (equivalently, give the value of pc in eV).