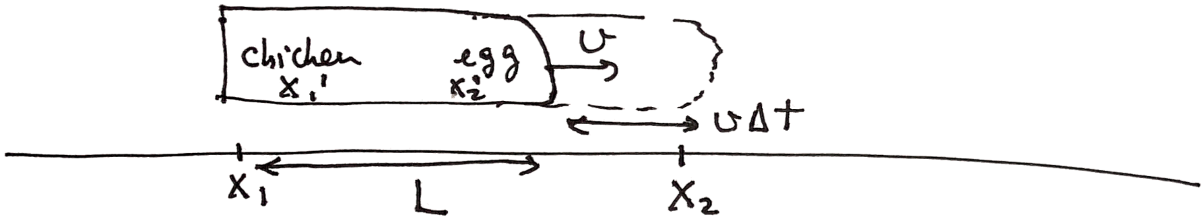
Physics 2D — Quiz 1 Solutions — Fall 2020



Events in S': (x_1', t_1') ; (x_2', t_2') ; simultaneous $\Rightarrow t_1' = t_2'$

Events in S : (x_1, t_1) ; (x_2, t_2) ; $\Delta t = t_2 - t_1 = 1\,\mu s$

L is length of ship seen from ground $= 600\,m$

In time $t_2 - t_1$, ship travels distance $d = v\cdot(t_2 - t_1) = v\Delta t$

$$\Rightarrow \boxed{x_2 = x_1 + L + d = x_1 + L + v\Delta t}$$

Lorentz:

$$\left.\begin{aligned} t_1' &= \gamma\left(t_1 - \frac{v}{c^2}x_1\right) \\ t_2' &= \gamma\left(t_2 - \frac{v}{c^2}x_2\right)\end{aligned}\right\} t_1' = t_2' \Rightarrow$$

$$t_2 - \frac{v}{c^2}x_2 = t_1 - \frac{v}{c^2}x_1 \Rightarrow t_2 - t_1 = \Delta t = \frac{v}{c^2}(x_2 - x_1)$$

$$\Rightarrow \Delta t = \frac{v}{c^2}(L + v\Delta t) = \frac{v}{c}\cdot\frac{L}{c} + \left(\frac{v}{c}\right)^2 \Delta t \Rightarrow$$

$$\Rightarrow \Delta t\left(\frac{v}{c}\right)^2 + \frac{L}{c}\left(\frac{v}{c}\right) - \Delta t = 0.$$ Solve quadratic eq. for v/c

$$\frac{v}{c} = -\frac{L}{2c\Delta t} + \sqrt{\left(\frac{L}{2c\Delta t}\right)^2 + 1} = -1 + \sqrt{2} \Rightarrow$$

$$\Rightarrow \boxed{\frac{v}{c} = 0.414} \text{ (a)}$$

$$\frac{L}{2c\Delta t} = \frac{600\,m}{2\times 3\times 10^8 \frac{m}{s} \times 10^{-6}\,s} = 1$$

(b) Length as seen on spaceship is proper length L_p:

$$L_p = \gamma L \; . \; \gamma = \frac{1}{\sqrt{1-0.414^2}} = 1.099 \; , \; L = 600\,m \Rightarrow$$

$$\Rightarrow \boxed{L_p = 659.2\,m}$$

(c) First find new $L = L_p/\gamma$ with the new $\gamma = \frac{1}{\sqrt{1-0.8^2}} = \frac{5}{3}$

$$\Rightarrow L = 395.5\,m$$

Then, from eqs. above:

$$t_1' - t_2' = \gamma\left(t_1 - t_2 + \frac{v}{c^2}(x_2 - x_1)\right) =$$

$$= \gamma\left(t_1 - t_2 + \frac{v}{c^2}(L + v\Delta t)\right) =$$

$$= \frac{5}{3}\left(-10^{-6}s + \frac{0.8}{3\times10^8}(395.5 + 0.8\times3\times10^8\times10^{-6})s\right) =$$

$$= \frac{5}{3}\left(-10^{-6}s + 10^{-6}s \cdot \frac{0.8}{3}\left(\frac{395.5+240}{100}\right)\right) = +1.1585\times10^{-6}$$

$\Rightarrow$ $\boxed{\text{in spaceship, chicken (back) happens } 1.158\,\mu s \text{ after egg (front)}}$

Alternative (easier) solution: use reverse Lorentz transformation:

$$t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right) \Rightarrow t_1 - t_2 = \gamma(t_1' - t_2') + \frac{\gamma v}{c^2}(x_1' - x_2')$$

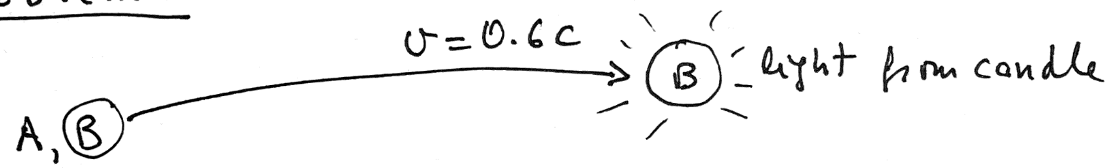
$$t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right)$$

$$\Rightarrow t_1' - t_2' = \frac{1}{\gamma}(t_1 - t_2) + \frac{v}{c^2}(x_2' - x_1') \Rightarrow$$

$$t_1' - t_2' = \frac{3}{5}(-10^{-6}s) + \frac{0.8\times659.2}{300}\times10^{-6}s = 1.158\times10^{-6}s$$

$\Rightarrow$ $\boxed{\text{in spaceship, back event happens } 1.158\,\mu s \text{ after front event}}$

Problem 2



(a) B lits candle after 1 year in her frame ⇒ 1 year is proper time

$\Delta t_p = 1$ year, $\Delta t = \gamma \Delta t_p$. $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = 1.25$

⇒ $\Delta t = 1.25$ years ⇒ $\boxed{\text{A is 21.25 years old when B lits candle}}$ (a)

(b) The distance that B traveled, in A's reference frame, is

$$d = v\Delta t = 0.6c \cdot \Delta t$$

Light emitted from B's candle travels back to A in a time

$$\Delta t_2 = \frac{d}{c} = 0.6\,\Delta t = 0.75 \text{ years}.$$

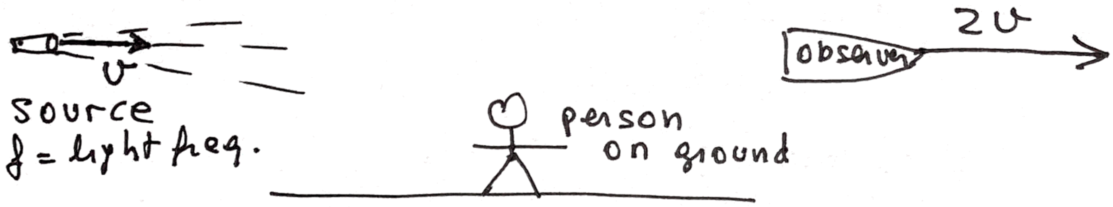
Therefore, light reaches A at time $\Delta t + \Delta t_2 = 2$ years after B departed

⇒ $\boxed{\text{A is exactly 22 years old when light from B reaches A}}$ (b)

(c) The situation is completely symmetric (ignoring acceleration)

⇒ $\boxed{\text{B is exactly 22 years old when light from A reaches B}}$ (c)

Problem 3



(a) Find relative speed.

Put frame S' on source, find speed of observer in frame S'

$v =$ speed of frame S', $u_x = 2v$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{2v - v}{1 - \frac{2v^2}{c^2}} = \boxed{\frac{v}{1 - \frac{2v^2}{c^2}} = u'_x} \quad \boxed{\text{(a) } u'_x = 0.2857c}$$

(b) Use Doppler formula with speed $v = -u'_x$:

$$f' = f\sqrt{\frac{1 - u'_x/c}{1 + u'_x/c}} \qquad \frac{1 - \frac{u'_x}{c}}{1 + \frac{u'_x}{c}} = \frac{1 - \frac{2v^2}{c^2} - \frac{v}{c}}{1 - \frac{2v^2}{c^2} + \frac{v}{c}} \Rightarrow$$

$$\Rightarrow f' = f\sqrt{\frac{1 - \frac{2v^2}{c^2} - \frac{v}{c}}{1 - \frac{2v^2}{c^2} + \frac{v}{c}}} = f\sqrt{\frac{1 - 2\times 0.25^2 - 0.25}{1 - 2\times 0.25^2 + 0.25}} = 0.745f$$

$$\Rightarrow \boxed{f' = 0.745f} \quad \text{(b) Or, simply } \boxed{f' = f\sqrt{\frac{1 - 0.2857}{1 + 0.2857}} = 0.745f}$$

(c) $$f_g = f\sqrt{\frac{1 + v/c}{1 - v/c}} = f\sqrt{\frac{1.25}{0.75}} = 1.29f$$ since source is <u>approaching</u>

then $$f'' = f_g\sqrt{\frac{1 - 2v/c}{1 + 2v/c}} = f_g\sqrt{\frac{1 - 0.5}{1 + 0.5}} = 1.29 \times 0.577f = 0.745f$$

$$\boxed{f'' = 0.745f} \text{ (c)}$$

result has to be same as in (b) since person on ground emits same frequency as it receives from source by assumption.